Intermediated Signaling and Delegated Selling*

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Abstract

Asymmetric information about product quality can create incentives for a privately informed manufacturer to sell to uninformed consumers through a retailer and to maintain secrecy of upstream pricing. Delegating retail price setting to an intermediary generates pooling equilibria that avoid signaling distortions associated with direct selling. We define a class of intermediated signaling games for which we develop a notion of equilibrium refinement that is motivated by considerations similar to the Intuitive Criterion for standard signaling games. Whereas pooling equilibria do not satisfy the Intuitive Criterion under direct selling, pooling outcomes are consistent with the new refinement under delegated selling. Expected profit, consumer surplus and social welfare can all be higher with delegated selling. However, if secrecy of upstream pricing cannot be maintained, selling through a retailer can only lower the expected profit of the manufacturer.

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1 Introduction

Manufacturers often sell their products through retailers instead of selling directly to consumers; they delegate to retailers the task of setting prices faced by consumers. Consumers rarely observe the upstream pricing scheme used by manufacturers (when selling to retailers). In this paper we argue that this kind of delegation may be an optimal response to asymmetric information about product quality, a pervasive feature in many markets. Bagwell and Riordan (1991) have shown that when a manufacturer sells directly to consumers, the price charged can signal private information about product quality. However, signaling may introduce significant distortions and the resulting equilibrium may actually be quite inefficient as in order to deter imitation by a low quality type, the high quality price should be sufficiently distorted upward. In a setting similar to that in Bagwell and Riordan (1991), we show that selling through a retailer while maintaining secrecy of the vertical pricing scheme can help avoid the signaling distortions that arise when the manufacturer sells directly. This may not only increase the expected profit of the manufacturer but can also lead to a more efficient market outcome.

The key idea behind our main result is simple. When a manufacturer sells through a retailer and the price charged by the manufacturer to the retailer is not observed by consumers, the manufacturer can *hide* information about quality by selling to the retailer at a price that is independent of quality and as a result, the retail price does not convey any information about quality to the buyers. We show that the resulting pooling equilibrium not only eliminates the signaling distortion that arises when the manufacturer directly sells to buyers, but may also avoid the well-known double marginalization problem associated with linear wholesale pricing. Note that delegation of selling to a retailer involves long-run commitment on the part of the manufacturer and for this reason we focus on the manufacturer's ex ante incentives prior to the actual realization of quality. The delegation outcome may yield higher ex ante expected industry profit as well as higher consumer welfare relative to direct selling; when consumers are in the dark about product quality, they may be better off *not being able to infer* quality from prices.

The strategic interaction in Bagwell and Riordan (1991) falls within the class of standard signaling games where a Sender with private information sends a message that is observed by a Receiver who then chooses an action. When the privately informed manufacturer delegates selling to a retailer and consumers, who are the receivers, do not observe the vertical contract or the price at which the manufacturer sells to the retailer, the game is no longer a standard signaling game. We define a class of *intermediated signaling games* where a Sender with private information chooses an action that is only observed by an uninformed Intermediary, while the Receiver (who is also uninformed) only observes the action chosen by the Intermediary; as in standard signaling games, the private information of the Sender is pay-off relevant for both the Sender and the Receiver, but not for the Intermediary. Intermediated signaling games are not only useful for understanding vertical delegation in product markets, which is the prime application we focus on in this paper, but may also be relevant in other settings including labour markets where intermediaries play a role.

Equilibrium refinements play an important role in the signaling literature, and a significant element in the analysis of Bagwell and Riordan (1991) is to show that pooling equilibria do not satisfy the Intuitive Criterion (Cho and Kreps, 1987). In intermediated signaling games, the Sender's action is unobserved by the Receiver and therefore, the Intuitive Criterion cannot be applied as the Receiver does not observe deviations by the privately informed Sender: he only observes deviations from the equilibrium action of the Intermediary and has to consider whether the Sender or the Intermediary has unilaterally deviated from equilibrium play.[1] We develop a new equilibrium refinement for intermediated signaling games that is based on considerations similar to the Intuitive Criterion for standard signaling games. Like the Intuitive Criterion, this new refinement is based on the notion of certain actions of the Sender or the Intermediary being equilibrium dominated given the strategy of the other player. We use this notion to impose constraints on what the Receiver may reasonably believe after observing a deviation. Importantly, in the game where the manufacturer delegates the retail price setting decision to a retailer, pooling equilibria may survive the new refinement. A key part in the argument is that even though buyers may rule out that certain high retail prices stem from a unilateral deviation of the low quality manufacturer, they will not be able to rule out that the retailer unilaterally deviates to these prices allowing them to have the same beliefs about quality as the retailer.

Secrecy of the manufacturer's pricing plays an important role in generating this kind of attractive pooling outcome under delegation to a retailer. If the price set by the manufacturer is publicly observable, final buyers may infer product quality directly from the wholesale price and this leads to a signaling outcome that is qualitatively similar to that in the vertically integrated industry except that in addition to the signaling distortion in

[1] See Ekmekci and Kos (2021) for some other issues with equilibrium refinement in a signaling game when the sender decides on unobserved acquisition of private information prior to signaling.

wholesale price there is an added distortion due to double marginalization by the retailer. We show that even if the manufacturer uses a two-part tariff pricing scheme and extracts all the surplus earned by the retailer, his expected profit with observable vertical contracts can never exceed that under direct selling. Thus, we provide a new economic explanation for secrecy of vertical contracts.[2]

If the price set by the manufacturer is publicly observable, consumers can update their beliefs about quality based on the manufacturer's price in a way that is similar to the case where the manufacturer sells directly. In thinking about the manufacturer's gain from any deviation for certain beliefs of the consumers, one needs to think about the retailer's reaction to such a deviation which will depend on the latter's "second-order" belief about the beliefs of the consumers. We outline modified restrictions on these "second-order" beliefs for our specific game to show that the high quality manufacturer has an incentive to deviate from a pooling outcome in case its actions are publicly observable.

Our paper contributes to the literature on the role of intermediaries in markets with asymmetric information about quality that has largely focused on information or certification intermediaries that use their own information, skill or reputation to provide information to buyers (Biglaiser 1993, Lizzeri 1999, Albano and Lizzeri 2001 and Glode and Opp 2016). In our framework, the intermediary retailer has no skill or market reputation and in fact, may have no more information about product quality than the uninformed consumer. In contrast to this literature, our key result is based on the beneficiary role of using a retailer to *hide* information from final consumers.

A number of papers have analyzed the role of leasing of new durable goods in reducing the extent of the lemons problem in the used goods markets. The leasing firm's opportunity cost of selling the used good (at the end of the lease) is determined prior to the realization of actual quality or performance of the used good and therefore independent of it; see, among others, Lizzeri and Hendel (2002) and Johnson and Waldman (2003). One may view leasing as delegation of reselling of the used good to the leasing firm. Further, the timing of actions rules out the possibility of signaling. Unlike this literature, our paper focuses on private information about producer's quality. The manufacturer is informed about quality before he sets the terms under which the retailer acquires the good and

[2]As in our settings two-part tariffs do not increase firms' profits, our paper may also be interpreted as providing an explanation for why the lump-sum component in actual wholesale contracts is small relative to the overall payment between firms (see, e.g., Blair and Lafontaine 2015 and Kaufmann and Lafontaine 1994).

he chooses whether or not the retailer's cost of acquiring the good varies with quality. Signaling by the manufacturer is potentially possible, but the manufacturer abstains from doing so. Further, in this setting observability of the terms of the vertical contract by final consumers affects the market outcome significantly, whereas this does not play a role in the leasing literature.

Our paper also contributes to a large literature on informational factors behind vertical integration and separation. In particular, beginning with Arrow (1975), a significant body of theoretical literature has argued that information frictions create private and social incentives for vertical integration by facilitating exchange or monitoring of information between the integrating firms (see, among others, Crocker 1983 and Riordan and Sappington 1987) or by concealing information from rival firms (see, Choi 1998). In contrast, our paper provides an argument why information frictions can create incentives for vertical separation. We focus on the information revealed or concealed *to consumers* rather than on discovery or revelation of information among firms.

Finally, we contribute to the literature initiated by Bonanno and Vickers (1988), Katz (1989) and Hart and Tirole (1990) on the strategic use of vertical contracts. That literature showed, among other things, that observable contracts with downstream firms create a strategic advantage in the presence of market competition. In contrast, we highlight the strategic advantage of keeping vertical contracts secret within a supply chain irrespective of the interaction with competitors.[3]

The rest of the paper is organized as follows. Section 2 outlines a class of intermediated signaling games and develops and motivates our proposed refinement for this class of games. Section 3 introduces delegated selling through a retailer in a model that is otherwise very similar to that in Bagwell and Riordan (1991) and shows that an equilibrium satisfying our new refinement always exists. Section 4 contains our main result that when the manufacturer sells through a retailer with secret vertical pricing, pooling equilibria exist that avoid signaling distortions and yield more profit and social surplus than direct selling. Section 5 analyzes the outcome when vertical pricing is observable. Section 6 concludes. Proofs are contained in the Appendix.

[3] Fershtman and Kalai (1997) and Ok and Kockesen (2004), among others, study the effect of strategic delegation with unobservable contracts in games of perfect information.

2 Intermediated Signaling

An *intermediated signaling game* is a dynamic game of incomplete information with three players: a Sender (S), an Intermediary (I) and a Receiver (R), and the following extensive form:

1. Nature draws a type t for the sender from a finite set of types T according to a probability distribution $\beta(t), t \in T$, where $\beta(t) > 0, \sum_{t \in T} \beta(t) = 1$.

2. The Sender observes t and then chooses a message m from a set M of messages.

3. The Intermediary observes m (but not t) and then chooses an action a from a set of feasible actions A.

4. The Receiver observes a (but not m or t) and then chooses a response r from a set of feasible responses ρ.

5. Payoffs for the Sender, the Intermediary and the Receiver are given by $U_S(t, m, r), U_I(m, a, r)$ and $U_R(t, a, r)$ respectively.

The standard signaling game analyzed widely in the existing literature has two players - a Sender and a Receiver. The Sender has private information about its type and chooses a message which is observed directly by the Receiver who then chooses a response. The payoffs of both players may depend on the type of the Sender, the message and the response of the Receiver.

The intermediated signaling game outlined above differs from this standard signaling game in several respects. First, there are three players including an Intermediary who moves after the Sender and before the Receiver. Like the Receiver, the Intermediary does not observe the type of the Sender. Second, the message sent by the Sender is observed only by the Intermediary, while the Receiver only observes the action chosen by the Intermediary. Third, the Receiver's payoff does not depend directly on the message sent by the Sender to the Intermediary,but only depends on the action chosen by the Intermediary. Finally, the Sender's payoff does not depend directly on the action chosen by the intermediary (though the latter may influence the Sender's payoff through the Receiver's response).

In this paper we focus on pure strategy Perfect Bayesian Equilibrium (PBE) of this game. The equilibrium strategies are as follows: (a) the Sender's equilibrium strategy is a function $m^* : T \to M$; Sender of type t chooses message $m^*(t), t \in T$, (b) the Intermediary's equilibrium strategy is a function $a^* : M \to A$; following any message m from the Sender the Intermediary chooses action $a^*(m)$ and (c) the Receiver's strategy is a function $r^*(a) : A \to R$; following any action a by the Intermediary, the receiver chooses response $r^*(a)$.

On the equilibrium path, the message sent by the Sender lies in the set M^* where

$$M^* = \{m^*(t) : t \in T\}$$

and the action chosen by the intermediary lies in the set A^*

$$A^* = \{a^*(m^*(t)) : t \in T\}.$$

Let $U_s^*(t)$ denote the equilibrium payoff of type t sender. For $m^* \in M^*$, let $U_I^*(m^*)$ denote the equilibrium payoff of the Intermediary in the continuation game after the manufacturer chooses m^*. For $a \in A/A^*$, let the probability distribution $\mu_a(t) \geq 0, t \in T$, $\sum_{t\in T} \mu_a(t) = 1$, be the out-of-equilibrium belief of the Receiver when it observes action a of the intermediary.

Consider any out-of-equilibrium action $a \in A/A^*$of the Intermediary. Let Σ_T be the set of all probability distributions on T. Further, for any $\widehat{\mu} \in \Sigma_T$ let

$$BR(\widehat{\mu}, a) = \arg\max_{r\in\rho} \sum_{t\in T} U_R(t, a, r)\widehat{\mu}(t)$$

be the set of best responses of the Receiver after observing a when it has belief $\widehat{\mu}$; let $BR(T, a)$ be the set of all such best responses for all possible beliefs of the Receiver i.e.,

$$BR(T, a) = \cup_{\widehat{\mu}\in\Sigma_T} BR(\widehat{\mu}, a).$$

In line with the Intuitive Criterion (Cho and Kreps, 1987) for the standard signaling game and in accordance with the principle of focusing on unilateral deviations when accounting for an observed out-of-equilibrium action, we introduce two definitions of equilibrium domination:

Definition 1 *Given the equilibrium strategy $m^* : T \to M$ of the Sender, an out-of-equilibrium action $a \in A/A^*$ is said to be equilibrium dominated for the Intermediary who has observed message $\widetilde{m} \in M^*$if*

$$\max_{r\in BR(T,a)} U_I(\widetilde{m}, a, r) < U_I^*(\widetilde{m}).$$

Definition 2 *Given the equilibrium strategy* $a^* : M \to A$ *of the Intermediary, for* $t \in T$*, a message* $\overline{m} \in M/M^*$ *is said to be equilibrium dominated for the Sender of type* t *if*

$$\max_{r \in BR(T, a^*(\overline{m}))} U_S(t, \overline{m}, r) < U_S^*(t).$$

The belief formation process by the Receiver after observing an out-of-equilibrium action $a \in A/A^*$ has two components. First, the Receiver assigns a probability $x_a^I \in [0, 1]$ that the observed out-of-equilibrium action a results from a unilateral deviation by the Intermediary (given the equilibrium strategy of the Sender) and a probability $x_a^S = 1 - x_a^I$ that the out-of-equilibrium action a results from a unilateral deviation by the Sender (given the equilibrium strategy of the Intermediary). Second, conditional on a unilateral deviation by the Intermediary, the Receiver assigns a probability $y_a^I(t)$ to the Sender being of type $t, y_a^I(t) \geq 0, \sum_{t \in T} y_a^I(t) = 1$, while conditional on a unilateral deviation by the Sender, the Receiver assigns a probability $y_a^S(t)$ to the Sender being of type $t, y_a^S(t) \geq 0, \sum_{t \in T} y_a^S(t) = 1$. Note that even when considering a unilateral deviation by the Intermediary, one assigns probabilities $y_a^I(t)$ to the Sender being of type t as only the Sender and not the Intermediary can be of different types. Nevertheless, as explained in more detail below, depending on the type of equilibrium (pooling, semi-separating or separating), a deviation by the Intermediary can reveal some information to the Receiver.

Next, we use the above definitions of equilibrium domination to define when an out-of-equilibrium action can be attributed to a unilateral deviation by the Intermediary and/or the Sender.

Definition 3 *An out-of-equilibrium action* $a \in A/A^*$ *of the Intermediary can be attributed to a* unilateral *deviation by the Intermediary if given the equilibrium strategy* $m^* : T \to M$ *of the Sender,* a *is* not *equilibrium dominated for the Intermediary for some equilibrium message* $\widetilde{m} \in M^*$ *from the Sender.*

Definition 4 *An out-of-equilibrium action* $a \in A/A^*$ *of the Intermediary can be attributed to a* unilateral *deviation by the Sender if* $a = a^*(\overline{m})$ *for some* $\overline{m} \in M$ *and further, given the equilibrium strategy of the Intermediary, there exists* $t \in T$ *such that* $\overline{m}$ *is* not *equilibrium dominated for the Sender of type* t*.*

We now outline a set of restrictions on the out-of-equilibrium belief $\mu_a(t), t \in T$, that adapt the Intuitive Criterion for standard signaling games to our game of intermediated signaling.[4]

Condition 1 ***Adapted Intuitive Criterion (AIC)****: Consider an out-of-equilibrium action $a \in A/A^*$ of the intermediary that can be attributed to a unilateral deviation by either the Sender or the Intermediary (or both). Then, the out-of-equilibrium belief $\mu_a(t), t \in T$ satisfies the Adapted Intuitive Criteria (AIC) if the following restrictions are satisfied:*

(i)

$$\mu_a(t) = x_a^I y_a^I(t) + (1 - x_a^I) y_a^S(t), t \in T$$

and further, if action a cannot be attributed to a unilateral deviation by the Sender (Intermediary), then $x_a^I = 1$ (0).

(ii) Suppose that the action a can be attributed to a unilateral deviation by the Intermediary. Let $T_0(a) = \{t \in T : a$ is equilibrium dominated for the Intermediary when the Sender is of type t and sends equilibrium message $m^*(t)\}$. Then, $y_a^I(t) = 0$ for all $t \in T_0(a)$. Further, for any $\widehat{t} \in T/T_0(a)$, $\tau(\widehat{t}) = \{t \in T/T_0(a) : m^*(t) = m^*(\widehat{t})\}$, the following holds:

$$\frac{y_a^I(\widehat{t})}{\sum_{t \in \tau(\widehat{t})} y_a^I(t)} = \frac{\beta(\widehat{t})}{\sum_{t \in \tau(\widehat{t})} \beta(t)}.$$

(iii) Suppose that the action a can be attributed to a unilateral deviation by the Sender. If $a = a^*(\overline{m})$ and further, given the equilibrium strategy of the Intermediary, the message $\overline{m}$ is equilibrium dominated for the Sender of type t, then $y_a^S(t) = 0$.

The first part of the definition considers when to attribute an out-of-equilibrium action $a \in A/A^*$ exclusively to the retailer or the manufacturer or whether unilateral deviations by both players can account for the deviation (given the equilibrium strategy of the other); for any such attribution to a unilateral deviation by a player, the second and third requirements impose restrictions on assignment of beliefs to different types of the Sender. The third requirement simply adjusts the implications of the original Intuitive Criterion

[4] The criterion follows the principles that if observations can be accounted for by unilateral deviations, they should get priority. There are, however, out-of-equilibrium actions $a \in A/A^*$ that cannot be accounted for by unilateral deviations. In that case, any belief is permissible. Considering whether some restrictions on out-of-equilibrium beliefs are reasonable to impose by analyzing multiple deviations, one encounters the following difficulty. As the Receiver only observes action a, it has to infer which message m could have lead the Intermediary to also deviate and choose action a (which depends on the Intermediary's belief about the Receiver's response) and which type of Sender could then have had an incentive to deviate to such a message. We have abstained from considering the possible restrictions from such joint deviations, also because in the context of our delegated pricing game they do not seem to lead to any further meaningful restrictions on out-of-equilibrium beliefs (see footnote 13 for more details).

to the setting of intermediated signaling where the Receiver does not observe the Sender's message and can only infer which Sender could have deviated given the Intermediary's equilibrium strategy.

The second requirement is more involved and a few examples may clarify. If, for example, one considers a candidate pooling equilibrium and an out-of-equilibrium action $a \in A/A^*$ that is not equilibrium dominated for the retailer, then $y_a^I(\widehat{t}) = \beta(\widehat{t})$, i.e., conditional on a unilateral deviation by Intermediary, as the Receiver cannot infer any information from the incentive of the Intermediary to deviate (given the deviation is not based on any learning of the type of the Sender), the Receiver should assign the beliefs to be identical to the prior for all types. In other words, as the Intermediary does not have acquire any additional information after observing the Sender's message, to the extent that the Receiver blames out of equilibrium action on a unilateral deviation by the Intermediary it should assign the same beliefs as the Intermediary has at that stage. A similar logic applies if a subset of Sender types pool on a message and one looks at the event of unilateral deviation by the intermediary conditional on this pooled message: the relative likelihood of each type that pools should be as in the prior belief. Finally, if an out-of-equilibrium action $a \in A/A^*$ is equilibrium dominated for the Intermediary given the Sender's equilibrium strategy $m^*(t)$ for only a strict subset of Sender types, then conditional on attributing attributing a to a unilateral deviation by the Intermediary, the Receiver should assign probability zero to this subset and probability one to the complement of this subset. Overall, the second requirement is a conservative way to implement the considerations underlying the Intuitive Criterion: the Receiver attributes deviations in such a way that they are consistent with the information the Intermediary may have had when deviating.

The criterion outlined above confines attention to incentives for unilateral deviations. If an out-of-equilibrium action cannot be accounted for by unilateral deviations, then our criterion does not impose any restriction on the out of equilibrium belief.[5]

[5] In applications with more specific structure, one may think about reasonable restrictions on out-of-equilibrium beliefs based on the possibility of deviations by multiple players. We briefly discuss such restrictions in the context of our specific price signaling model and show in Section 4 (footnote 14) that they do not affect the equilibria we focus on.

3 A Simple Model of Delegated Selling

In the rest of this paper, we consider intermediated signaling in the specific framework used by Bagwell and Riordan (1991) to analyze price signaling of product quality by a monopolist. The monopolist, who we shall henceforth refer to as the manufacturer, produces a good whose quality can be either high (H) or low (L). The unit cost of production is constant and depends only on the quality of the good; in particular, high quality has a unit cost of $c > 0$ while the cost of low quality is normalized to zero. There is a unit mass of consumers. All consumers have unit demand. They have identical valuation $v_L > 0$ for low quality, while their valuation of high quality is uniformly distributed on $[v_L, 1 + v_L]$. Thus, if the consumers face a price p and assign probability μ to high quality, then the quantity demanded $d(p, \mu)$ is given by:

$$\begin{aligned} d(p,\mu) &= 0, \text{ if } p \geq \mu + v_L \\ &= 1 - \frac{p - v_L}{\mu}, \text{ if } p \in [v_L, \mu + v_L] \\ &= 1, \text{ if } p \leq v_L \end{aligned} \tag{1}$$

The prior probability that quality is high is common knowledge and denoted by $\alpha \in (0, 1)$. The realized quality of the good is observed only by the manufacturer. The manufacturer maximizes expected profit and each consumer maximizes her expected net surplus.

Our focus is on markets where in a signaling equilibrium a high quality firm has to distort its price relative to the full information outcome. This is the case if

$$v_L + c < 1. \tag{2}$$

Bagwell and Riordan (1991) fully characterize the equilibria when the manufacturer sells directly to buyers. In particular, the manufacturer sets a price p after observing the true (realized) product quality; buyers use this price to update their belief and make their purchase decision. There is a unique Perfect Bayesian Equilibrium (hereafter, PBE) outcome that can be supported by beliefs that satisfy the Intuitive Criterion. It is the least distortionary of all separating PBE outcomes.[6] Under restriction (2), the high quality man-

[6] Note that while there are pooling outcomes that can be sustained as PBE, they are eliminated once beliefs are restricted to satisfy the Intuitive Criterion. Pooling equilibria that satisfy the Intuitive Criterion can only exist if a significant proportion of buyers directly observe actual product quality. We focus on the case where all buyers are ex ante uninformed.

ufacturer charges a price $p_H^I = 1$ that exceeds his full information optimal price and earns profit equal to $\upsilon_L(1-c)$, while the low quality manufacturer charges his full information optimal price $p_L^I = \upsilon_L$ (which is also his profit) and is indifferent between charging this price and imitating the high quality price. Thus, the ex ante expected equilibrium profit of the manufacturer is given by

$$\pi^I = \upsilon_L(1 - \alpha c).$$

We shall refer to this as the *direct selling* outcome. Note that from the manufacturer's perspective, the signaling distortion is relatively large if c is small, α is large and υ_L is small. Pooling equilibria cannot be sustained with beliefs satisfying the Intuitive Criterion as after observing a deviation to a sufficiently high price p, buyers would infer that only the high quality type (with higher marginal cost) could possibly gain from this deviation and the criterion then suggests that the out-of-equilibrium belief $\mu(p)$ should equal 1 which would in turn make it gainful for the high quality type to deviate.

In subsequent sections, we analyze the consequences of the manufacturer selling exclusively through an intermediary retailer. The retailer has no specific expertise and its outside option is zero. For our analysis, it is irrelevant whether the retailer knows the quality of the good provided by the manufacturer; we make no assumption in this respect. The only cost incurred by the retailer is what he pays the manufacturer for the good; his payoff is his expected profit net of this payment. The manufacturer sets a linear wholesale price w at which it sells to the retailer. It is observed by the retailer before setting the retail price p at which it sells to consumers.

As it takes time to set up a retail distribution channel, we view the decision whether or not to delegate to a retailer as a long-term commitment; a manufacturer that delegates to a retailer no longer has a distribution network to sell directly to consumers at a later stage. The decision whether or not to delegate is evaluated by comparing the ex ante pay-offs to the manufacturer. In this section and the next, we consider the case where the wholesale price (or, the upstream contract) is secret, i.e., observed only by the retailer and not by the consumers.[7]

This delegated selling game is a special case of the general intermediated signaling game considered in the previous section, with the manufacturer in the role of Sender, the retailer in the role of Intermediary and the buyer in the role of Receiver. We apply AIC to this specific setting.

[7] In Section 5 we contrast our results with the case where the wholesale price is observed by consumers.

Our first Proposition states that a PBE satisfying AIC always exists in this delegated selling game.

Proposition 1 *In the intermediated signaling game where the manufacturer sells through a retailer with secret wholesale pricing, there exists a (separating) perfect Bayesian equilibrium where the beliefs satisfy AIC.*

The proof is by construction. Interestingly, under delegated selling, the vertically integrated outcome cannot be sustained as a PBE outcome even if there are no restrictions on beliefs.[8]

4 Hiding Information through Delegated Selling

Having defined the refinement for intermediated signaling games and the price signaling game we consider, we now characterize a class of *pooling* PBE satisfying AIC where the retailer is fully squeezed, i.e., makes zero profits. We also show that there exist other pooling PBE that do not satisfy AIC. Finally, we show that some of the *pooling* PBE that satisfy AIC yield higher expected profit to the manufacturer compared to the vertically integrated outcome.

Consider pooling equilibria where the manufacturer sets a wholesale price w^* regardless of product quality and the retailer follows up by selling at a retail price $p^* = w^*$. After observing the retail price p^*, a buyer's updated belief is identical to her prior belief, i.e., $\mu(p^*) = \alpha$, while the manufacturer and retailer sell a quantity $d(p^*, \alpha)$. We focus on outcomes where $p^* = w^* < \alpha + v_L$ so that the manufacturer sells a strictly positive quantity. Further,

$$p^* = w^* \geq \max\{v_L, c\} \tag{3}$$

as the retailer will always want to deviate if p^* were smaller than v_L,[9] while the high

[8] In such an outcome, $p_H = 1, p_L = v_L$. For the low cost retailer to not imitate the high cost retailer we need

$$\begin{aligned} v_L - w_L &\geq (p_H - w_L)(1 + v_L - p_H) \\ &= (1 - w_L)v_L \end{aligned}$$

which can only hold if $w_L = 0$. But in that case, the low quality manufacturer earns zero profit and will always want to imitate the high quality manufacturer unless $w_H = w_L = 0$ which cannot happen in a separating equilibrium.

[9] Note that (3) ensures that $p^* = w^* > c$ so that both types of the manufacturer earn strictly positive profit.

quality manufacturer would not agree to set $w^* < c$.

We will show that under some additional restrictions, this outcome can be sustained as a perfect Bayesian equilibrium satisfying AIC by focusing on the following retailer strategy:

$$\begin{aligned} p(w) &= w, \text{ if } w \geq p^* \\ &= p^*, \text{ if } p^* \geq w > \max\{p^* - \alpha, 0\} \\ &= \upsilon_L, \text{ if } 0 \leq w \leq p^* - \alpha. \end{aligned} \tag{4}$$

If these strategies are part of a PBE, then the equilibrium pay-offs of the low, respectively high, quality manufacturer are given by

$$\pi_L^* \equiv \left(1 - \frac{p^* - \upsilon_L}{\alpha}\right) p^* \text{ and } \pi_H^* \equiv \left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)(p^* - c),$$

while the retailer's equilibrium profits equals 0.

Given these equilibrium strategies we now consider the implications of AIC. First, consider $p \in (p^*, 1 + \upsilon_L)$. Given that the equilibrium pay-off of the retailer is 0, it is clear that no such p is equilibrium dominated for the retailer (given the equilibrium strategy of the manufacturer). This immediately implies that any such price can, in principle, be accounted for by some unilateral deviation by the retailer and/or the manufacturer. Thus, AIC applies. Consider then the question whether such a price is equilibrium dominated for the low quality manufacturer given the equilibrium strategy of the retailer. It is clear that this is the case if, and only if, $\pi_L^* > (1 - (w - \upsilon_L))\, w$. Define $\widetilde{p}(p^*) > p^*$ such that $\pi_L^* \equiv (1 - (\widetilde{p}(p^*) - \upsilon_L))\, \widetilde{p}(p^*)$,which yields

$$\widetilde{p}(p^*) = \frac{1 + \upsilon_L + \sqrt{(1 + \upsilon_L)^2 - 4\pi_L^*}}{2} > p^*$$

Note that $\widetilde{p}(p^*)$ is well defined.[10] Further, as $\widetilde{p}(p^*) \geq \frac{1+\upsilon_L}{2}$, $(1 - (p - \upsilon_L))\, p$ is strictly decreasing in p for all $p > \widetilde{p}(p^*)$. Given the retailer's equilibrium strategy of setting $p(w) = w$ for $w \geq p^*$, any $w > \widetilde{p}(p^*)$ is equilibrium dominated for the low quality manufacturer. Thus, if a type of manufacturer may have an incentive to unilaterally deviate and induce a retail price $p \in (\widetilde{p}(p^*), 1 + \upsilon_L)$ is is the high quality manufacturer. As the retailer also has an incentive to unilaterally deviate to such a price, AIC requires the out-of-equilibrium

[10]Note that π_L^* is maximized at $p^* = \frac{\alpha+\upsilon_L}{2}$. Thus, if $\alpha < \upsilon_L$ then $\pi_L^* \leq \upsilon_L$ and if $\alpha > \upsilon_L$ it is smaller than or equal to $(\alpha + \upsilon_L)^2 / 4\alpha$. In both cases, $4\pi_L^* \leq (1 + \upsilon_L)^2$.

belief to be such that $\mu(p) \geq \alpha$ and in particular, allows us to set $\mu(p) = \alpha$ for all $p \in (\widetilde{p}(p^*), 1 + \upsilon_L)$.[11] For $p^* < p \leq \widetilde{p}(p^*)$ AIC permits $\mu(p) = 0$, while out-of-equilibrium beliefs do not matter for $p \geq 1 + \upsilon_L$ as demand will always be equal to 0 at these prices.[12]

Next, we consider out-of-equilibrium beliefs and the implications of AIC at retail prices $p < p^*$. For any $p \leq \upsilon_L$ out-of-equilibrium beliefs are irrelevant as consumers anyway demand one unit. For any $p \in (\upsilon_L, p^*)$, it is clear that such a price cannot be attributed to a unilateral deviation by the retailer (as it is equilibrium dominated given the manufacturer's strategy), while given the strategy of the retailer it can also not be attributed to a unilateral deviation of the manufacturer. Thus, AIC does not impose restrictions on the set of permissible beliefs and we may impose $\mu(p) = 0$ for all $p \in (\upsilon_L, p^*)$ making the retailer's strategy indeed an optimal response to the strategy of the manufacturer.[13]

Given these beliefs, the buyers' optimal strategy (summarized by the quantity $q(p)$ demanded) is as follows

$$\begin{aligned} q(p) &= 1 \text{ if } p \leq \upsilon_L \\ &= 0 \text{ if } \upsilon_L < p < p^* \text{ or if } p^* < p < \widetilde{p}(p^*) \\ &= \max\{1 - \frac{p - \upsilon_L}{\alpha}, 0\} \text{ if } p = p^* \text{ or if } p \geq \widetilde{p}(p^*). \end{aligned} \tag{5}$$

Using this strategy (that is clearly based on the beliefs that satisfy AIC), we now argue

[11] More formally, requirement (ii) of AIC in Section 2 implies that for any $p \in (p^*, 1 + \upsilon_L)$, as the retailer has a unilateral incentive to deviate to any such p then for $a = p$, $y_a^I(H)$=α while requirement (iii) implies that for any $p > \widetilde{p}(p^*)$, $y_a^S(H) = 1$, and for $p \in (p^*, \widetilde{p}(p^*)]$, $y_a^S(H) \in [0, 1]$.For $p \in [\widetilde{p}(p^*), 1 + \upsilon_L)$, if the high quality manufacturer also has a unilateral incentive to deviate to such a p, then AIC requires that $\mu(p)$ can only be a weighted average of α and 1 i.e., $\mu(p) \geq \alpha$. Further, $\mu(p) = \alpha$ if the high quality manufacturer does not have an incentive to deviate to p (as in that case $x_a^I = 1$). The simplest way to satisfy both cases is by choosing $\mu(p) = \alpha$ for all $p > \widetilde{p}(p^*)$.

[12] Note that the restrictions implied by AIC in our game of delegated selling are similar in nature to the restrictions implied by the Intuitive Criterion for the price signaling game of Bagwell and Riordan (1991) in that AIC does not impose any restrictions if we consider deviations to prices slightly larger than p^* (up to the $\widetilde{p}$ to be defined), while there are retail prices that are so large (larger than $\widetilde{p}$) that the low quality manufacturer will not have an incentive to induce these retail prices even if consumers believe quality to be high.

[13] One may want to consider joint deviations of both the manufacturer and the retailer to investigate whether $\mu(p) = 0$ is a reasonable out-of-equilibrium belief. It is easy to check that if the high quality manufacturer gains by reducing his wholesale price from w^* to $w \leq p < p^*$ for some belief of buyers μ' (after observing retail price p), *i.e.*, if $(w - c)d(p, \mu') \geq (p^* - c)d(p^*, \alpha)$, then the low quality manufacturer must strictly gain from this deviation, *i.e.*, $wd(p, \mu') > p^*d(p^*, \alpha)$ indicating that if some type of manufacturer would want to induce the retailer to set p it is certainly the low quality manufacturer that has an incentive to do so. Thus, it seems that any restriction on beliefs based on joint deviations, should allow for buyers to hold the belief $\mu(p) = 0$ at $p \in (\upsilon_L, p^*)$.

under which conditions the retailer and the manufacturer do not have incentives to deviate from their equilibrium strategy. First, consider deviations downward. Any price $p < p^*$ can only arise if the manufacturer deviates and set a lower w. Clearly, as reducing the wholesale price to $w \in (\max\{p^* - \alpha, 0\}, p^*)$ leads to the same retail price as $w = w^*$ it is not gainful. The manufacturer may consider deviating to $0 \leq w \leq \max\{p^* - \alpha, 0\}$ resulting in a retail price of υ_L and increasing the quantity sold to 1. Obviously, the best deviation here is to set $p^* - \alpha$. If such a deviation is not profitable for the low quality manufacturer, it is certainly not profitable for the high quality manufacturer. Further, this deviation is not profitable for the low quality manufacturer if, and only if, $\pi_L^* \geq p^* - \alpha$, or

$$p^* \leq \frac{\upsilon_L + \sqrt{(\upsilon_L)^2 + 4\alpha^2}}{2} \equiv \overline{p}. \tag{6}$$

On the other hand, the retailer strategy specified in (4) is optimal for any $w < w^*$ as the profit of sticking to the equilibrium price is equal to $(p^* - w)\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)$ and this is larger than or equal to the profit of setting υ_L if, and only if, $w \geq p^* - \alpha$.

Next, consider upward deviations. It is clear that for the high quality manufacturer not to have an incentive to deviate we need at least that $p^* \geq c$. In addition, if there exists a $p > p^*$ such that $q(p) > 0$, then the retailer has an incentive to deviate. Thus, equilibrium requires that $q(p) \leq 0$ for all $p > p^*$ and if this is the case, then it is clear that neither type of the manufacturer has an incentive to deviate. From (5) it is clear that $q(p) \leq 0$ for all $p > p^*$ if, and only if, $\widetilde{p}(p^*) \geq \upsilon_L + \alpha$. The next Lemma states that this condition does not impose further restrictions on the equilibrium prices that can be sustained if $\alpha + \upsilon_L \leq 1$, while if $\alpha + \upsilon_L > 1$ it does.

Lemma 2 *If $\alpha + \upsilon_L \leq 1$, then $\widetilde{p}(p^*) \geq \upsilon_L + \alpha$ for all $p^* > \upsilon_L$, while if $\upsilon_L + \alpha > 1$ then $\widetilde{p}(p^*) \geq \upsilon_L + \alpha$ for all $p > \underline{p}$, with*

$$\underline{p} = \frac{1}{2}\left(\alpha + \upsilon_L + \sqrt{(\alpha - \upsilon_L)^2 - 4\alpha^2(1 - \alpha - \upsilon_L)}\right) > \upsilon_L. \tag{7}$$

The next proposition summarizes the above discussion.

Proposition 3 *Suppose the manufacturer sells through a retailer with secret wholesale pricing and that $\overline{p} > c$. If (i) $\alpha + \upsilon_L \leq 1$, then there exists a continuum of pooling PBE with $w^* = p^* \in [\max\{\upsilon_L, c\}, \overline{p}]$ satisfying AIC, while if (ii) $\upsilon_L + \alpha > 1$ only $w^* = p^* \in$*

$\left[\max\{\underline{p}, c\}, \overline{p}\right]$ *can be supported in PBE that satisfy AIC,where* $\underline{p}$ *is defined in (7).*[14]

Thus, given certain conditions there exists a continuum of pooling equilibria that satisfy AIC where the retailer is fully squeezed. As the manufacturer is already able to fully extract the retailer's rent, it is easy to show that these pooling outcomes can also be sustained if the manufacturer uses a nonlinear pricing scheme such as two-part tariffs.

Next, we show that there exist pooling PBE outcomes where the retailer is fully squeezed that cannot be supported by beliefs satisfying AIC. To do so we first consider which requirements in the above discussion have to be satisfied by all PBE and which ones stem from AIC. It is clear that (3) and (6) have to be satisfied by all PBE. If (3) is violated either the retailer or the high quality manufacturer have an incentive to deviate, while if (6) is violated, then the low quality manufacturer will deviate downwards. The requirement that $\widetilde{p}(p^*) \geq \upsilon_L + \alpha$ and the conditions this imposes on p^* as specified in the Lemma above are, however, imposed by AIC. Without AIC, one could simply stipulate $\mu(p) = 0$ for all $p > p^*$ and this would immediately imply that neither the manufacturer nor the retailer would have an incentive to deviate upwards.

The above discussion is, however, cast in terms of specific strategies for the manufacturer and, especially, the retailer. The next Proposition shows that even if one considers other possible equilibrium strategies, an outcome where $w^* = p^*$ and $\widetilde{p}(p^*) < \alpha + \upsilon_L$ can be sustained as a PBE, but not as a PBE satisfying AIC. Lrom Lemma 2 it then follows that this applies to any $\upsilon_L < p^* < \underline{p}$.

Proposition 4 *For any* $p^* \in [\max\{\upsilon_L, c\}, \overline{p}]$, *with* $\overline{p}$ *being defined in (6), such that* $\widetilde{p}(p^*) < \alpha + \upsilon_L$, *there is a PBE with a pooling outcome where the manufacturer of both types set wholesale price* $w^* = p^*$. *However, there is no PBE satisfying AIC that can generate such an outcome.*

Finally, we state the welfare implications of PBE satisfying AIC. The proposition provides necessary and sufficient conditions under which PBE satisfying AIC yield more profit and higher consumer surplus than under direct selling.

Proposition 5 *If PBE satisfying AIC as described in Proposition 3 exist, then they generate higher ex ante expected profit for the manufacturer and higher expected consumer*

[14]Note that $\underline{p} > \overline{p}$ if, and only if $\alpha + \sqrt{(\alpha - \upsilon_L)^2 - 4\alpha^2(1 - \alpha - \upsilon_L)} > \sqrt{{\upsilon_L}^2 + 4\alpha^2}$ and in that case the interval is empty.

surplus (and therefore, higher social surplus) than in the direct selling outcome if, and only if,

$$(\alpha(1+c) - \upsilon_L)^2 > 4\alpha^2 c(1-\upsilon_L), \tag{8}$$

and

$$\alpha(1+c) + \upsilon_L + \sqrt{(\alpha(1+c) - \upsilon_L)^2 - 4\alpha^2 c(1-\upsilon_L)} \geq 2\max\{\underline{p}, \upsilon_L, c\}. \tag{9}$$

Observe that (8) holds if υ_L or c is small, or if α is large, while (9) always holds if α is large and $\underline{p} < 1$ or c is small and $\max\{\underline{p}, \upsilon_L\} < \alpha$.

As indicated in the previous section, low values of υ_L and c imply that the signaling distortion is high when the manufacturer sells directly; further, when α is large, the ex ante surplus puts a higher weight on the high quality state which is where the price is distorted under direct selling. The pooling outcome generated when the manufacturer sells through a retailer with secret upstream pricing can avoid much of this signaling distortion in the high quality state. As a result, even though the high quality manufacturer faces lower demand when pooling, it can be more profitable than selling directly provided the distortion due to double marginalization can be kept to a minimal level. Keeping product quality hidden for consumers by preventing the retailer from signaling quality by setting a wholesale price that is independent of quality helps prevent double marginalization. Surprisingly, the pooling equilibria that generate higher ex ante expected profit for the manufacturer also benefit consumers through lower prices on average, though they remain uninformed about product quality before purchase.

5 Selling through a Retailer: Observable Wholesale Pricing

A key feature underlying the analysis of the previous sections is that the manufacturer's wholesale pricing (or the message of the Sender in de intermediated signaling model of Section 2) is not observed by the buyer (Receiver). In this section, we show how the results of the previous section are affected if consumers are able to observe the wholesale price in addition to the retail price. We show that with observable wholesale prices, any pure strategy PBE that satisfies reasonable restrictions on consumers' out-of-equilibrium beliefs yields less profit for the manufacturer than direct selling. Further, this holds even if the manufacturer can use two-part tariffs to extract rent from the retailer.

In particular, we consider markets where the manufacturer sells through a retailer by setting a two-part tariff that is directly observed by both the retailer and the consumers.

Let w_τ be the unit wholesale price and F_τ the fixed fee charged by manufacturer of type $\tau, \tau \in \{H, L\}$. Observe that as the manufacturer's actions are observed by the buyer, this is not a game of intermediated signaling as defined in Section 2. The considerations outlined in that section do not apply here as the Receiver can directly observe whether the Sender has deviated and thus does not need to see whether an out-of-equilibrium action by the Intermediary should be attributed to a unilateral deviation by the Sender or the Intermediary. Thus, this game is much closer to a standard signaling game where criteria like the Intuitive Criterion may apply. To apply such a criterion, the main question now is how to define the notion of equilibrium domination in view of the fact that whether a message is equilibrium dominated for the Sender may now depends on the action taken by the Intermediary and the effect this has on the Receiver's response.

In case of our price signaling game, to see which type of manufacturer may have an incentive to deviate to some out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ what matters is consumer demand $d(p, \mu(\widehat{w}, \widehat{F}))$, which depends on consumer beliefs and on the price set by the retailer, which in turn depends on $(\widehat{w}, \widehat{F})$ and on the *second-order belief* of the retailer about what consumers would believe about product quality. Note that while in any PBE, (both on and off-the-equilibrium path) the second-order beliefs of the retailer must necessarily coincide with consumers' first-order beliefs as specified in the equilibrium, the difficulty arises when we want to determine the reasonableness of *out-of-equilibrium* beliefs by looking at the relative incentives of different types of the manufacturer to choose an out-of-equilibrium wholesale price.

Whether or not a deviation is profitable depends on the relation between consumer beliefs and the retailer's *second-order belief* about consumer beliefs. Obviously, the more optimistic consumers are about product quality and the less optimistic the retailer believes the consumer is, the more incentives the manufacturer has to deviate. To give the Intuitive Criterion some bite, it is natural to impose that first- and second-order beliefs are coordinated, i.e., that the retailer holds correct beliefs about the beliefs of consumers: if after observing a deviation by the manufacturer, consumers believe with probability μ that the manufacturer sells high quality, then the retailer also believes that consumers have belief μ. Coordinated beliefs are implied by, but weaker than, the retailer and the consumer having a "common prior" about the quality the manufacturer sells in the continuation game following the manufacturer's action w. Coordinated beliefs seem natural as the manufacturer cannot control these beliefs and there does not seem to be any reason why the manufacturer should entertain the possibility that consumers' beliefs about quality should be different from the retailer's second-order beliefs of consumers' beliefs. Note that the retailer's own belief about quality does not play any role in determining his response to a deviation.

To define the Intuitive Criterion while requiring that first- and second-order beliefs are coordinated, suppose the manufacturer deviates from the equilibrium contract and chooses some out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ and that the retailer sets his retail price p assuming that demand is $d(p, \mu)$ at any retail price p, where for notational simplicity we suppress that μ may depend on $(\widehat{w}, \widehat{F})$. The optimal response of the retailer, denoted by $p(\widehat{w}, \mu)$, is then given by:

$$p((\widehat{w}, \widehat{F}), \mu) = \arg\max_{p \geq \widehat{w}} \left[(p - \widehat{w}) d(p, \mu)\right].$$

Using this price reaction of the retailer to an out-of-equilibrium contract $(\widehat{w}, \widehat{F})$, and defining the equilibrium pay-off for type τ manufacturer as $\pi^*_\tau = (w^*_\tau - c_\tau) d(p^*(w^*_\tau)) + F^*, \tau = L, H$, we can directly apply the logic of the Intuitive Criterion and require that if for some $\tau \in \{H, L\}$ the out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ is equilibrium dominated, *i.e.*,

$$\pi^*_\tau \geq (\widehat{w} - c_\tau) d(p(\widehat{w}, \mu), \mu) + \widehat{F} \text{ for all } \mu \in [0, 1],$$

while for $\tau' \in \{H, L\}, \tau' \neq \tau$ the out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ is not equilibrium dominated, *i.e.*,

$$\pi^*_{\tau'} < (\widehat{w} - c_{\tau'}) d(p(\widehat{w}, \mu), \mu) + \widehat{F} \text{ for some } \mu \in [0, 1],$$

then the out-of-equilibrium belief $\mu(\widehat{w}, \widehat{F})$ should be such that consumers believe that the manufacturer of type τ' has deviated with probability one.

We will now consider equilibria that satisfy the Intuitive Criterion with coordinated beliefs as defined above, and argue that pooling equilibria where the manufacturer sets a per unit wholesale price $w^* > v_L - \alpha$ and a fixed fee F^* regardless of his product quality, and the retailer's equilibrium strategy is $p^*_R(w)$ where $p^* = p^*_R(w^*)$ do not satisfy the Intuitive Criterion for coordinated beliefs. The proof of the next Proposition argues that for any pooling equilibrium where the pooling two-part tariff (w^*, F^*) is such that $w^* > v_L$ one can find deviations $(\widehat{w}, \widehat{F})$ such that, using Intuitive Criterion with coordinated beliefs, consumers have to believe that they come from a high quality manufacturer, making these deviations profitable. This part of the argument is similar to the argument used by Bagwell and Riordan (1991) to eliminate pooling equilibria under direct selling, but in our setting the manufacturer also has to take double marginalization by the retailer into account. If

$\widehat{w}$ is sufficiently large, low quality manufacturers would never have an incentive to deviate, while due to higher production cost one can still find wholesale prices in this range (and appropriately chosen fixed fees) that may be profitable for the high quality manufacturer (if beliefs of consumers and the retailer are coordinated). For $\upsilon_L - \alpha < w^* < \upsilon_L$, there are deviations to $\widehat{w} < w^*$ (and appropriately chosen levels of fixed fees) that have to be attributed to low quality manufacturers, again making these deviations profitable for the low type. Thus, a pooling equilibrium satisfying the Intuitive Criterion with coordinated beliefs with $w^* > \upsilon_L - \alpha$ does not exist. If pooling equilibria exist for $w^* \leq \upsilon_L - \alpha$, the manufacturer's profit cannot be larger than under direct selling.

Proposition 6 *When the manufacturer sells through a retailer with observable two-part tariff upstream pricing, any pooling equilibrium* (w^*, F^*) *satisfying the Intuitive Criterion with coordinated beliefs, satisfies the following properties:*

(i) $w^* \leq \upsilon_L - \alpha$,

(ii) the ex ante expected profit of the manufacturer is lower than the expected profit under direct selling (i.e., the vertically integrated outcome).

The next proposition focuses on separating equilibria. As, in a separating equilibrium, consumers can infer quality from wholesale prices, the retailer can mark-up the retail price without affecting consumers' beliefs about quality. This leads to a distortion due to double marginalization and an excessively high retail price for the high quality good. This argument is independent of out-of-equilibrium beliefs.

Proposition 7 *When the manufacturer sells through a retailer with observable two-part tariff upstream pricing, its ex ante expected profit in any separating perfect Bayesian equilibrium is lower than in the direct selling outcome.*

Taken together, the two Propositions in this section imply that in any pure strategy PBE satisfying the Intuitive Criterion with coordinated beliefs, the manufacturer ex ante expected profit when he sells through a retailer with observable two-part tariff upstream prices is smaller than in the direct selling outcome.

6 Discussion and Conclusion

In this paper we have argued that when consumers are uninformed about product quality, a manufacturer with private information can increase his expected profit and at the same time, increase consumer and social welfare, by delegating the task of setting the price faced by consumers to an intermediary retailer. By delegating and not imposing vertical control, while withholding information about the wholesale pricing contract between manufacturer and retailer, the manufacturer can prevent signaling distortions. We have also shown that the argument extends if a fraction of consumers is informed about product quality and that in that case, pooling outcomes that generate higher expected profit than direct selling are associated with strictly positive retail margin and retailer's profit. Interestingly, by increasing the retail margin, an increase in the fraction of informed consumers may leave all consumers worse off.

Our results have been derived for a specific demand structure that was used by Bagwell and Riordan (1991). It should be clear, however, that our results do not depend on it. In fact, delegation is more likely to be gainful if the demand for high quality is more elastic. For example, we can easily extend the demand structure in the model to one where valuation of the high quality product is distributed over $[\upsilon_L + x, \upsilon_L + 1]$ with $x \in [0, 1]$; higher x would then imply that the demand for high quality is more elastic. As the private and social cost associated with the signaling distortion is higher when demand is more elastic, larger values of x are a more fertile ground for delegation to improve profits and market efficiency through pooling. One can see this most starkly in the special case where all buyers are homogenous (for instance, if $x = 1$); delegation can then lead to a pooling outcome that is both socially efficient and generates the full information level of (ex ante) profit for the manufacturer, while direct selling remains highly costly due to signaling distortion.[15]

Our analysis points to a class of intermediated signaling games that is of clear economic interest but has not been studied extensively, namely games where the sender chooses an action that is not directly pay-off relevant to the final receiver but that potentially influences the behavior of the intermediate receiver. Our analysis indicates the different implications that arise depending on whether or not the sender's action is observed by the final receiver. Future research directed to understanding the general nature of such three-player interactions and their economic implications will be useful.

[15] In this special homogenous buyers case, delegation can lead to a D1 pooling outcome where the manufacturer and the retailer charge the ex ante expected valuation and all buyers buy. In contrast, under direct selliing, there is significant quantity distortion in the high quality state.

Appendix

Proof of Proposition 1. We construct a separating equilibrium where

$$w_L^* = \frac{\upsilon_L}{2}, w_H^* = 1, p_H^* = 1 + \frac{\upsilon_L}{2}, p_L^* = \upsilon_L \tag{10}$$

and the retailer's equilibrium strategy is given by:

$$\begin{aligned} p(w) &= \frac{1+\upsilon_L+w}{2}, \text{ for all } w \in [w_H^*, 1+\upsilon_L] \\ &= p_H^* = \frac{1+\upsilon_L+w_H}{2}, \text{ for } w \in (w_L^*, w_H^*) \\ &= p_L^* = \upsilon_L, \text{ for } w \leq w_L^* \end{aligned}$$

The out of equilibrium beliefs are as follows:

$$\begin{aligned} \mu(p) &= 1 \text{ for all } p \in (p_H^*, 1+\upsilon_L] \\ &= 0 \text{ for all } p \in (p_L^*, p_H^*) \end{aligned}$$

For $p < p_L^*$, we allow any $\mu(p) \in [0,1]$. The buyers' equilibrium strategy can be summarized by the total quantity $q(p)$ bought at any retail price p :

$$\begin{aligned} q(p) &= 1+\upsilon_L - p, \text{ if } p \geq p_H^* \\ &= 0, \text{ if } p \in (p_L^*, p_H^*) \\ &= 1, \text{ if } p \leq p_L^* \end{aligned}$$

The equilibrium profits of the manufacturer of types H and L are given by $\pi_H^{M*} = (1-c)(\upsilon_L/2)$ and $\pi_L^{M*} = \frac{\upsilon_L}{2}$. The equilibrium profits of the retailer when the manufacturer is of type H and L are given by $\pi_H^{R*} = (\upsilon_L/2)^2$ and $\pi_L^{R*} = \frac{\upsilon_L}{2}$.

Using (10), it is easy to check that:

$$\pi_L^{R*} = \frac{\upsilon_L}{2} \gtreqless (p - w_L^*)(1+\upsilon_L - p) \text{ for } p \lesseqgtr p_H^* = 1 + \frac{\upsilon_L}{2} \tag{11}$$

Also, easy to verify that $q(p)$ is optimal for buyers given their belief. We now argue that given $q(p)$, the retailer's strategy $p(w)$ is optimal. To see this note that as $q(p) = 1$ for $p \leq \upsilon_L$ it is never optimal for the retailer to set $p < \upsilon_L$. Note that $w_H^* = 1 \in (p_L^*, p_H^*)$.

For $w \le w_H^*$, $(p-w)q(p) = (p-w)(1+v_L-p)]$ is strictly decreasing in p for $p > p_H^* = 1 + \frac{v_L}{2}$. Thus, $p(w) \le p_H^*$ for $w \le w_H^*$. In particular, for $w \in (w_L^*, w_H^*]$, using (11) we have $(v_L - w) < (v_L - w_L^*) = (p_H^* - w)(1+v_L-p_H^*)$, we must have $p(w) = p_H^*$. For $w \le w_L^*$, as $q(p) = 0$ for $p \in (p_L^*, p_H^*)$ and using (11), $(v_L - w) \ge (v_L - w_L^*) = (p_H^* - w_L^*)(1+v_L-p_H^*)$, $p(w) = p_L^* = v_L$ is optimal for retailer. For $w > w_H^*$,as $(1+v_L+w)/2$ maximizes $(p-w)q(p) = (p-w)(1+v_L-p)$ with respect to p for $p \ge p_H^*$ and further, $w_H^* = 1 \in (p_L^*, p_H^*)$ and $q(p) = 0$ for $p \in (p_L^*, p_H^*)$, it is optimal to set $p(w) = (1+v_L+w)/2$. Finally, one can check that $w_L^* = \frac{v_L}{2}, w_H^* = 1$ are optimal for L and H type manufacturer given $p(w)$ and $q(p)$.In particular, $\pi_L^{M*} = v_L/2 = w_H^*(1+v_L-p_H^*)$ so that the L type manufacturer is indifferent between his equilibrium action and imitating the H type's action, and further, $\pi_H^{M*} = (1-c)(v_L/2) > (w_L^* - c) = (v_L - c)$ so that the H type manufacturer strictly prefers to not imitate L type. Manufacturer of either type has no incentive to set $w < w_L^*$ (as it leads to the same retail price as charging $w = w_L^*$); further, for any $w \in (w_L^*, w_H^*]$, the retailer sets price p_H^* and the manufacturer (of either type) is better off setting w_H^* than any $w \in (w_L^*, w_H^*]$. Manufacturer of L type has no incentive to set $w > w_H^*$ if the H type manufacturer has no incentive to do so and the latter is ensured if

$$(w-c)(1+v_L-p(w)) = (w-c)(1+v_L-\frac{1+v_L+w}{2})$$

is decreasing in w for $w \ge w_H^*$ which holds if $w_H^* \ge \frac{1+v_L+c}{2}$ and this holds as $w_H^* = 1$ and $v_L + c < 1$. We have now establishes that the outlined strategies and beliefs constitute a Perfect Bayesian Equilibrium.

We now argue that the out of equilibrium belief of buyers $\mu(p)$ satisfies AIC. For $p < p_L^*$, p is equilibrium dominated for the retailer regardless of whether he observes w_L^* or w_H^* and so cannot be attributed to a unilateral deviation by the retailer; $p < p_L^*$ is not in the image of the retailer's equilibrium strategy and so cannot be attributed to a unilateral deviation by the manufacturer; so, AIC imposes no restriction on $\mu(p)$.

For $p \in (p_L^*, p_H^*)$, (11) implies $\pi_L^{R*} < (p - w_L^*)(1+v_L-p)$. Thus, the retailer facing wholesale price w_L^* will always gain by unilaterally deviating to $p \in (p_L^*, p_H^*)$ when buyers believe that quality is high with probability one and so p is not equilibrium dominated for the retailer when he faces wholesale price w_L^*; the AIC therefore allows for $\mu(p) = 0$ based on attributing p to unilateral deviation by the retailer with probability one and conditional on this, assigning probability one to L type manufacturer i.e., to the event that the retailer's deviation follows the equilibrium action w_L^* of L type manufacturer.

Finally, consider out of equilibrium retail price $p \in (p_H^*, 1+\upsilon_L]$; this set is in the image of the retailer's equilibrium strategy and every such $p = p(w)$ for $w = 2p - (1+\upsilon_L)$. Using (11), $\pi_L^{R*} = \frac{\upsilon_L}{2} > (p - w_L^*)(1+\upsilon_L - p)$ so that p is equilibrium dominated for the retailer facing wholesale price w_L^*. Thus, either (a) p is not equilibrium dominated for the retailer when he faces wholesale price w_H^* or (b) it is equilibrium dominated for the retailer facing both w_L^* and w_H^*. In case (a), AIC allows for $\mu(p) = 1$ based on attributing p to a unilateral deviation by the retailer with probability one and conditional on this, assigning probability one to H type manufacturer i.e., to the event that the retailer's deviation follows the manufacturer's action w_H^*; in case (b), either $w = 2p-(1+\upsilon_L)$ is not equilibrium dominated for the H type manufacturer so that AIC allows for $\mu(p) = 1$ (based on attributing p to a unilateral deviation by the manufacturer with probability one and conditional on this, assigning probability one to H type manufacturer), or it is equilibrium dominated for both types of the manufacturer (in addition to the retailer) so that AIC imposes no restriction on belief. This concludes the proof.[16]

Proof of Lemma 2. Given the definition of $\widetilde{p}$ it is clear that $\widetilde{p} \geq \upsilon_L + \alpha$, if and only if,

$$\frac{1+\upsilon_L+\sqrt{(1+\upsilon_L)^2 - 4\pi_L^*}}{2} \geq \upsilon_L + \alpha, \tag{12}$$

which is equivalent to

$$\begin{aligned} \sqrt{(1+\upsilon_L)^2 - 4\pi_L^*} &\geq 2(\upsilon_L+\alpha) - (1+\upsilon_L) \\ &= \upsilon_L + 2\alpha - 1 \end{aligned}$$

If $\upsilon_L + 2\alpha - 1 \leq 0$, the RHS is negative and (12) always holds. If $\upsilon_L + 2\alpha - 1 > 0$, we need

$$(1+\upsilon_L)^2 - 4\pi_L^* \geq (\upsilon_L - 1 + 2\alpha)^2.$$

This can be rewritten as

$$\pi_L^* \leq \upsilon_L - \alpha^2 + \alpha(1-\upsilon_L),$$

or

$$(p^*)^2 - (\alpha+\upsilon_L)p^* + \alpha[\upsilon_L - \alpha^2 + \alpha(1-\upsilon_L)] \geq 0.$$

[16] The separating equilibrium outlined in the proof is one where the low type manufacturer is indifferent to imitating the high type manufacturer's action and the retailer facing the low type's wholesale price is indifferent to imitating the action of the retailer facing high type's wholesale price. In fact, one can construct a continuum of separating equilibria satisfying AIC where these constraints need not bind.

The roots of this inequality are given by

$$p^* = \frac{\alpha + \upsilon_L}{2}\left[1 \pm \sqrt{1 - \frac{4\alpha(1-\alpha)}{\alpha + \upsilon_L}}\right].$$

If these roots are not real numbers, then the inequality always holds and no further restrictions are in place. This is the case if $4\alpha(1-\alpha) > \alpha + \upsilon_L$ or $\upsilon_L < \alpha(3 - 4\alpha)$. In case the roots are real numbers, which arises if $\upsilon_L \geq \alpha(3-\alpha)$, then we may need to impose further restrictions. The roots are then given by

$$p^* = \frac{1}{2}\left[\alpha + \upsilon_L \pm \sqrt{(\alpha - \upsilon_L)^2 - 4\alpha^2(1 - \alpha - \upsilon_L)}\right]$$

The largest root is smaller than or equal to υ_L if, and only if $1 - \alpha - \upsilon_L \geq 0$. Note that $\alpha + \upsilon_L \leq 1$ implies that $\upsilon_L + 2\alpha - 1 \leq 0$. Thus, if $\alpha + \upsilon_L \leq 1$, then $\widetilde{p}(p^*) \geq \upsilon_L + \alpha$ for all $p^* > \upsilon_L$, while if $\upsilon_L + \alpha > 1$, $\widetilde{p}(p^*) \geq \upsilon_L + \alpha$ if, and only if, $p^* \geq \underline{p} = \frac{1}{2}\left(\alpha + \upsilon_L + \sqrt{(\alpha - \upsilon_L)^2 - 4\alpha^2(1 - \alpha - \upsilon_L)}\right)$.[17] Note that $\upsilon_L + \alpha > 1$ implies $\upsilon_L \geq \alpha(3-\alpha)$ so that the roots are indeed real numbers.

Proof of Proposition 4. The first part is easily shown by considering the retailer strategy

$$\begin{aligned} p(w) &= w, \text{ for } w \geq p^* \\ &= p^*, \text{for } p^* > w \geq \max\{\upsilon_L - \alpha, 0\} \\ &= \upsilon_L, \text{ for } w < \max\{\upsilon_L - \alpha, 0\} \end{aligned}$$

[17] Note that if $\upsilon_L + \alpha < 1$

$$p^* < \frac{1}{2}\left[\alpha + \upsilon_L - \sqrt{(\alpha - \upsilon_L)^2 - 4\alpha^2(1 - \alpha - \upsilon_L)}\right]$$

cannot be sustained as pooling prices p^* that satisfy $\widetilde{p}(p^*) \geq \upsilon_L + \alpha$ as that would require

$$\frac{1}{2}\left[\alpha + \upsilon_L - \sqrt{(\alpha - \upsilon_L)^2 - 4\alpha^2(1 - \alpha - \upsilon_L)}\right] > \upsilon_L,$$

or

$$-\sqrt{(\alpha - \upsilon_L)^2 - 4\alpha^2(1 - \alpha - \upsilon_L)} > (\upsilon_L - \alpha).$$

which can only hold if $\upsilon_L - \alpha < 0$ and in that case we would have

$$(\alpha - \upsilon_L)^2 - 4\alpha^2(1 - \alpha - \upsilon_L) < (\upsilon_L - \alpha)^2,$$

which holds only if $\upsilon_L + \alpha < 1$, a contradiction.

and out-of-equilibrium beliefs $\mu(p) = 0$ for all $p \neq p^*$.

To see the second part, consider any PBE that generates this outcome and let $\widehat{p}(w)$ be the retailer's equilibrium strategy in that equilibrium. Now, consider any out-of-equilibrium $p_0 \in (\widetilde{p}(p^*), \alpha + \upsilon_L)$. Observe that as the retailer's equilibrium profit is zero, he always has an incentive to unilaterally deviate to p_0 if buyers believe after observing this deviation that quality is high for sure. There are two possibilities: (i) there exists w_0 such that $\widehat{p}(w_0) = p_0$ and (ii) p_0 is not in the image of the retailer's equilibrium strategy $\widehat{p}(w)$.

In case (i), observe that as $w_0 \leq p_0$, the low quality manunfacturer's profit when he deviates to w_0 is at most $w_0 d(p_0, 1) \leq p_0 d(p_0, 1)$ and as $p_0 > \widetilde{p}(p^*)$, $p_0 d(p_0, 1) < p^* d(p^*, \alpha) = w^* d(p^*, \alpha)$, so that $w_0 d(p_0, 1) < w^* d(p^*, \alpha)$, his equilibrium profit. So, w_0 is equilibrium dominated for the low quality manufacturer. AIC therefore requires that the out of equilibrium belief $\mu(p_0) = \alpha$ if w_0 is also equilibrium dominated for the high quality manufacturer and otherwise, lies in the interval $[\alpha, 1]$. In case (ii), AIC requires that the deviation be attributed to the retailer with probability one so that $\mu(p_0) = \alpha$. However, these restrictions imply that the retailer's profit if he unilaterally deviates to p_0 are such that

$$(p_0 - w^*) d(p_0, \mu(p_0)) \geq (p_0 - w^*) d(p_0, \alpha) > 0,$$

which, as $p_0 < \alpha + \upsilon_L$, means that the deviation is strictly gainful for the retailer.

Proof of Proposition 5. The expected profit in a pooling equilibrium with retail price p^* exceeds that when selling directly if, and only if,

$$\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)(p^* - \alpha c) > \upsilon_L (1 - \alpha c). \tag{13}$$

There are p^* that satisfy this inequality if, and only if,

$$(\alpha(1 + c) + \upsilon_L)^2 - 4\alpha\upsilon_L(1 - \alpha c) - 4\alpha c(\alpha + \upsilon_L) > 0,$$

which can be rewritten as

$$(\alpha(1 + c) - \upsilon_L)^2 > 4\alpha^2 c(1 - \upsilon_L),$$

which is (8).

In order that some of these solutions to (13) can be supported as PBE satifying AIC we also need that the solutions to (13) satisfy $p^* \in [\max\{\underline{p}, \upsilon_L, c\}, \overline{p}]$. If there are solutions to

(13), then $p^* = \frac{\alpha(1+c)+\upsilon_L}{2}$ is always one of them. As $c < 1$, it is easy to see that $\frac{\alpha(1+c)+\upsilon_L}{2} < \overline{p}$. On the other hand, there are solutions to (13) such that $p^* \geq \max\{\underline{p}, \upsilon_L, c\}$, if and only if,

$$\alpha(1+c) + \upsilon_L + \sqrt{(\alpha(1+c) - \upsilon_L)^2 - 4\alpha^2 c(1-\upsilon_L)} > 2\max\{\underline{p}, \upsilon_L, c\},$$

which is condition (9).

When the manufacturer sells directly, the ex ante expected consumer surplus is given by $\frac{\alpha}{2}(\upsilon_L)^2$. When selling through a retailer with secret pricing, in the pooling equilibrium where $p^* = \frac{\upsilon_L+\alpha(1+c)}{2}$, the ex ante expected consumer surplus is given by

$$\frac{1}{2\alpha}(\alpha + \upsilon_L - p_S^*)^2 = \frac{1}{8\alpha}(\alpha(1-c) + \upsilon_L)^2 > \frac{\alpha}{2}(\upsilon_L)^2,$$

which is the case if $2\alpha\upsilon_L < \alpha(1-c) + \upsilon_L$ i.e., $\upsilon_L(2 - \frac{1}{\alpha}) + c < 1$ which always holds.

Proof of Proposition 6. We begin stating with some useful facts. Suppose that the (coordinated) belief is that quality is high with probability μ. Then for any unit wholesale price $w \leq \mu + \upsilon_L$ the optimal price set by the retailer (if he accepts the contract) is

$$\begin{aligned} p(w, \mu) &= \frac{\mu + \upsilon_L + w}{2}, \text{ if } w \geq \upsilon_L - \mu \\ &= \upsilon_L, \text{ if } w < \upsilon_L - \mu. \end{aligned}$$

For $w > \mu + \upsilon_L$, the retailer sells zero at any $p \geq w$ and so $p(w, \mu)$ is any price at least as large as w. The quantity sold by the retailer is then

$$\begin{aligned} d(p(w, \mu), \mu) &= \frac{\mu + \upsilon_L - w}{2\mu}, \text{if } w \in [\upsilon_L - \mu, \upsilon_L + \mu] \\ &= 1, \text{if } w \leq \upsilon_L - \mu \\ &= 0, \text{ if } w \geq \upsilon_L + \mu. \end{aligned}$$

Note that given $w \geq \upsilon_L$, $d(p(w, \mu), \mu)$ is non-decreasing in μ and for $w < \upsilon_L$, $d(p(w, \mu), \mu)$ is non-increasing in μ.

Consider a pooling equilibrium where the manufacturer sets two part tariff (w^*, F^*). Then,

$$\begin{aligned} c &\leq w^* \leq \alpha + \upsilon_L \\ F^* &\leq (p^* - w^*)d(p^*, \alpha) \end{aligned}$$

The retailer's equilibrium strategy $p_R(w, F)$ in such an outcome must be such that $p^* = p_R(w^*, F^*)$ is given by

$$\begin{aligned} p^* &= \frac{\alpha + \upsilon_L + w^*}{2}, \text{ if } w^* \in [\upsilon_L - \alpha, \upsilon_L + \alpha] \\ &= \upsilon_L, \text{ if } w^* < \upsilon_L - \alpha. \end{aligned}$$

Note that the equilibrium profits of the high and low type manufacturers would then be

$$\begin{aligned} \pi^H &= (w^* - c)d(p^*, \alpha) + F^* \\ &= \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*) + F^*, \text{ if } w^* \geq \upsilon_L - \alpha \\ &= (w^* - c) + F^*, \text{ if } w^* < \upsilon_L - \alpha, \end{aligned}$$

$$\begin{aligned} \pi^L &= w^* d(p^*, \alpha) + F^* \\ &= \frac{1}{2\alpha} w^*(\alpha + \upsilon_L - w^*) + F^*, \text{ if } w^* \geq \upsilon_L - \alpha \\ &= w^* + F^*, \text{if } w^* < \upsilon_L - \alpha. \end{aligned}$$

First, suppose that $w^* \geq \upsilon_L$. Consider any unit wholesale price $w \in (w^*, 1 + \upsilon_L)$ and an associated fixed fee $F(w)$

$$F(w) = [p(w, 1) - w]d(p(w, 1), 1).$$

The profit earned by the low type manufacturer by deviating to such $(w, F(w))$ when buyers' belief is $\mu = 1$ equals

$$\begin{aligned} g(w) &= p(w, 1)d(p(w, 1), 1) \\ &= \frac{1}{4}(1 + \upsilon_L + w)(1 + \upsilon_L - w). \end{aligned}$$

Note that $g(w)$ is continuous (and strictly decreasing) in w on $[\upsilon_L, 1 + \upsilon_L]$. Note that as

$w \downarrow w^*$,

$$\begin{aligned} g(w) &\to p(w^*,1)d(p(w^*,1),1) = \frac{1}{4}(1+\upsilon_L+w^*)\left(1-(w^*-\upsilon_L)\right) \\ &> \frac{1}{4}(\alpha+\upsilon_L+w^*)\left(1-\frac{w^*-\upsilon_L}{\alpha}\right) \\ &= p^*d(p^*,\alpha) = w^*d(p^*,\alpha)+(p^*-w^*)d(p^*,\alpha) \\ &\geq \frac{1}{2\alpha}(w^*-c)(\alpha+\upsilon_L-w^*)+F^* = \pi^L \end{aligned}$$

On the other hand, as $w \uparrow (1+\upsilon_L)$, $g(w) \to 0$. Thus, there exists a unique $w_0 \in (w^*, 1+\upsilon_L)$ such that

$$g(w_0) = \pi^L. \tag{14}$$

We now claim that the low type manufacturer can never strictly gain by deviating to the contract $(w_0, F(w_0))$ for any belief $\mu \in [0,1]$. As noted above, $w_0 > \upsilon_L$ implies $d(p(w_0,\mu),\mu)$ is non-decreasing in μ. So, if the contract $(w_0, F(w_0))$ is feasible for belief μ (i.e., the retailer makes non-negative profit) the low type manufacturer's deviation profit:

$$\begin{aligned} & w_0 d(p(w_0,\mu),\mu) + F(w_0) \\ =\ & g(w_0) - w_0[d(p(w_0,1),1) - d(p(w_0,\mu),\mu)] \\ \leq\ & g(w_0) = \pi^L. \end{aligned}$$

If the contract $(w_0, F(w_0))$ is not feasible for belief μ, the low type manufacturer makes zero profit. Thus, regardless of the beliefs of buyers, the low type manufacturer can never gain by deviating to a contract $(w_0, F(w_0))$. Note that (14) implies

$$p(w_0,1)d(p(w_0,1),1) = w^*d(p^*,\alpha) + F^* \leq p^*d(p^*,\alpha) = \frac{\alpha+\upsilon_L+w^*}{2}d(p^*,\alpha).$$

As $\alpha < 1, w_0 > w^*$

$$p(w_0,1) = \frac{1+\upsilon_L+w_0}{2} > \frac{\alpha+\upsilon_L+w^*}{2}$$

it follows that

$$d(p(w_0,1),1) < d(p^*,\alpha). \tag{15}$$

If a high type manufacturer deviates to a contract $(w_0, F(w_0))$ and belief is $\mu = 1$ his deviation profit is:

$$\begin{aligned} & (p(w_0,1)-c)d(p(w_0,1),1) \\ = \;& g(w_0) - cd(p(w_0,1),1) = \pi^L - c\ d(p(w_0,1),1) \\ = \;& w^* d(p^*,\alpha) + F^* - c\ d(p(w_0,1),1),\ \text{using (14)} \\ = \;& (w^*-c)d(p^*,\alpha) + F^* + c[d(p^*,\alpha) - d(p(w_0,1),1)] \\ = \;& \pi^H + c[d(p^*,\alpha) - d(p(w_0,1),1)] \\ > \;& \pi^H,\ \text{using (15).} \end{aligned}$$

Intuitive Criterion with coordinated beliefs therefore requires that the out-of-equilibrium belief satisfies

$$\mu(w_0, F(w_0)) = 1,$$

which immediately implies that the high quality manufacturer has an incentive to deviate to $(w_0, F(w_0))$. Thus, there is no pooling equilibrium satisfying the Intuitive Criterion with coordinated beliefs where the marginal wholesale price $w^* \geq \upsilon_L$

Next, suppose $w^* \in (\upsilon_L - \alpha, \upsilon_L)$. Note that when the manufacturer is of low type, the total industry profit in the pooling equilibrium (w^*, F^*) is given by

$$\begin{aligned} & \frac{1}{2\alpha}(p^*-w^*)(\alpha+\upsilon_L-w^*) + \frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*) = \frac{1}{4\alpha}[(\alpha+\upsilon_L)^2 - (w^*)^2] \\ < \;& \frac{1}{4\alpha}[(\alpha+\upsilon_L)^2 - (\upsilon_L-\alpha)^2],\ \text{as } w^* > \upsilon_L - \alpha \\ = \;& \upsilon_L \end{aligned}$$

so that

$$\frac{1}{2\alpha}(p^*-w^*)(\alpha+\upsilon_L-w^*) < \upsilon_L - \frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*)$$

and as

$$F^* \leq \frac{1}{2\alpha}(p^*-w^*)(\alpha+\upsilon_L-w^*)$$

we have

$$F^* < \upsilon_L - \frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*)$$

so that there exists $h > 0$ such that

$$F^* < \upsilon_L - \left(\frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*) + \epsilon\right) \text{ for all } \epsilon \in (0,h). \tag{16}$$

Now, consider a deviation by the manufacturer to a two-part tariff $(\widehat{w}, F^*)$ where $\widehat{w} < w^*$and in particular:

$$(w^* - c)[\frac{w^* - (\upsilon_L - \alpha)}{2\alpha}] \leq w^* - \widehat{w} < w^*[\frac{w^* - (\upsilon_L - \alpha)}{2\alpha}] - \epsilon_0 \tag{17}$$

for some $\epsilon_0 \in (0, h)$. Note that as $c > 0$, the left most expression in (17) must be strictly less than the right most expression for ϵ_0 small enough.

We claim that a high type manufacturer can never gain from such a deviation regardless of belief μ. Suppose to the contrary there exists belief $\mu' \in [0, 1]$ such that the deviation is strictly gainful for the high type manufacturer. Then, for such μ', the retailer makes non-negative profit and

$$\begin{aligned} & (\widehat{w} - c)d(p(\widehat{w}, \mu'), \mu') + F^* \\ > \quad & \pi^H = \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*) + F^* \end{aligned}$$

i.e.,

$$(\widehat{w} - c)d(p(\widehat{w}, \mu'), \mu') > \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*)$$

and recalling that $\upsilon_L > w$ implies $d(p(w, \mu), \mu)$ is non-increasing in μ we have

$$(\widehat{w} - c)d(p(\widehat{w}, 0), 0) > \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*).$$

As $d(p(\widehat{w}, 0), 0) = 1$, we must have

$$\widehat{w} > \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*) + c$$

which yields,

$$w^* - \widehat{w} < (w^* - c)\frac{w^* - (\upsilon_L - \alpha)}{2\alpha}$$

This contradicts the left inequality in (17). Thus, the high type manufacturer can never gain from the deviation regardless of belief.

On the other hand, a low type manufacturer strictly gains from this deviation if belief $\mu = 0$ as the second inequality in (17) implies

$$\widehat{w} > \frac{1}{2\alpha}w^*(\alpha + \upsilon_L - w^*) \tag{18}$$

so that

$$\widehat{w} + F^* > \frac{1}{2\alpha} w^*(\alpha + v_L - w^*) + F^* = \pi^L.$$

To verify that the contract $(\widehat{w}, F^*)$ yields non-negative profit for the retailer when belief $\mu = 0$ i.e., $F^* \leq v_L - \widehat{w}$, note that (16) and $\epsilon_0 \in (0, h)$ imply

$$\begin{aligned} F^* &< v_L - \left(\frac{1}{2\alpha} w^*(\alpha + v_L - w^*) + \epsilon_0 \right) \\ &< v_L - \widehat{w}, \end{aligned}$$

using the second inequality in (17). Thus, there is no pooling equilibrium satisfying the Intuitive Criterion with coordinated beliefs where $w^* \in (v_L - \alpha, v_L)$.

This only leaves possibility of pooling equilibria where $w^* \leq v_L - \alpha$. On the equilibrium path in such an equilibrium, the retailer sets price equal to v_L (sells quantity equal to 1) and the manufacturer's ex ante expected profit is bounded above by the expected industry profit, $v_L - \alpha c$. The latter is (strictly) smaller than $v_L(1 - \alpha c)$, the ex ante expected profit of the manufacturer under direct selling. This completes the proof.

Proof of Proposition 7. Consider a separating perfect Bayesian equilibrium where the low and high type manufacturers set distinct two part tariffs (w_L, F_L) and (w_H, F_H) where $w_L \leq v_L$ and $w_H > w_L$. Given that $d(p, 0) = 1$ for all $p \leq v_L$, it is clear that the retailer's optimal strategy must be such that $p_R(w_L, F_L) = v_L$ and $p_R(w_H, F_H) = (1 + v_L + w_H)/2$. The condition that the low quality manufacturer should not have an incentive to imitate the high quality type is:

$$\frac{1}{2} w_H (1 - w_H + v_L) + F_H \leq w_L + F_L. \tag{19}$$

Suppose that the ex ante expected profit of the manufacturer in a separating equilibrium is at least as high as $\pi^I = v_L(1 - \alpha c)$. Then,

$$\begin{aligned} v_L(1 - \alpha c) &\leq (1 - \alpha)(w_L + F_L) + \alpha \left(\frac{1}{2}(w_H - c)(1 - w_H + v_L) + F_H \right) \\ &\leq w_L + F_L - \alpha c \frac{1}{2} (1 - w_H + v_L) \text{ (using (19))} \\ &= v_L(1 - \alpha c) + \alpha c v_L - \alpha c \frac{1}{2} (1 - w_H + v_L) \\ &= v_L(1 - \alpha c) + \frac{\alpha c}{2}(w_H + v_L - 1), \end{aligned}$$

which can only hold if $w_H \geq 1 - \upsilon_L$. The equilibrium retail price if the manufacturer is of high type is then $p_H = p_R^*(w_H, F_H) = (1 + \upsilon_L + w_H)/2 > 1$; as $(p - c)(1 + \upsilon_L - p)$ is strictly decreasing in p for $p \geq 1$ (by assumption (2)), the total *industry* profit in this state of the world is $(p_H - c)(1 + \upsilon_L - p_H) \leq \upsilon_L(1 - c)$. The ex ante industry profit can then not be larger than $\upsilon_L(1 - \alpha c) = \pi^I$, a contradiction.

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