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Dynamic Insurance Contracts and Adverse Selection

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Abstract

We take a dynamic perspective on insurance markets under adverse selection and study a dynamic version of the Rothschild and Stiglitz model. We investigate the nature of dynamic insurance contracts by considering both conditional and unconditional dynamic contracts. An unconditional dynamic contract has insurance companies offering contracts where the terms of the contract depend on time, but not on the occurrence of past accidents. Conditional dynamic contracts make the actual contract also depend on individual past performance (such as in car insurances). We show that dynamic insurance contracts yield a welfare improvement only if they are conditional on past performance. With conditional contracts, the first-best can be approximated if the contract lasts long. Moreover, this is true for any fraction of low-risk agents in the population.

Introduction

Adverse selection is potentially a serious problem in any type of insurance market (see, e.g., the seminal papers by Akerlof (1970) and Rothschild and Stiglitz (1976)).[1] Even though the probability of an accident is a recurrent one in most markets (with life insurance as an exception), the typical model considers a static environment where agents incur a loss only once. This modeling assumption may be justified if we want to explain the behavior of insurance companies as quite a few insurance contracts are essentially static (with car insurance as a notable exception): the terms of the insurance contract are independent of the time period and past history. In this article, we ask a normative question, namely whether Pareto-improvements can be achieved if some kind of dynamic insurance would be provided.

We consider two types of dynamic insurance contracts. The first type, which we call *conditional* dynamic contracts, allows insurance conditions in future periods to depend

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[1] A recent empirical confirmation can be found in Oosterbeek, Godfried, and van Tulder (2001).

on an agent's accidental history. In such contracts, agents that from an *ex ante* point of view take identical contracts may view different insurance terms in later periods when their accidental history differs. The second type of dynamic contract is *unconditional*, as an insurer is not able or not allowed to use an agent's past accidental history. Unconditional contract can still have a dynamic nature as the terms of the contract may depend on the time period.

We consider these two types of dynamic contracts for the following reasons. Conditional dynamic contracts are observed in the car insurance market with the infamous bonus/malus rules. It is important to understand the welfare implications of such contracts. We do not know of markets where unconditional dynamic contracts are offered, but they may be considered in markets where conditional contracts are politically not viable, such as in some health insurance markets. In such markets it may be considered unfair if someone has to pay a high premium because she simply had bad luck and got many health accidents in a row.

The model we consider is a dynamic version of the well-known Rothschild and Stiglitz (1976) world, where agents and insurance companies are committed to contracts that last for a finite number of periods. Agents discount future utility and profit levels at a given discount rate. There are two types of risk-averse agents: low-risk and high-risk. The probability that an accident happens to an individual is constant and the same in every time period. This means that we abstract from moral hazard issues. Low-risk agents have lower accidental probability than high-risk agents. We consider competitive Nash equilibria in which insurance companies offer a set of dynamic contracts such that each type of agent chooses an optimal contract from this set and no insurance company can unilaterally benefit by adding contracts to this set. We analyze the properties of these equilibria in two different dynamic settings.

We have several results. First, in both settings, competitive Nash equilibria only exist for a relatively small fraction of low-risk agents in the population, though the existence conditions cannot be easily compared. Second, high-risk agents get full insurance, in any separating equilibria. Third, when they do exist, equilibria under conditional dynamic contracts yield a Pareto-improvement over static equilibrium contracts and the optimal contract charges lower premiums to agents with better accidental histories. The main reason is that the probability of having a better accidental history is larger for low-risk agents than for high-risk agents allowing insurance companies to screen the two types of agents more easily. When utility functions are unbounded below and the number of periods gets large, the welfare achieved through conditional dynamic contracts approaches first-best welfare levels even if agents discount the future. Moreover, this is true for any fraction of low-risk agents in the population. Fourth, unconditional dynamic contracts do not provide any welfare improvement over static contracts.

It is important to emphasize the importance of commitment on the part of the parties involved. If they had not been committed to the contract, a company could have offered better insurance conditions after some periods to those low-risk agents that have unluckily been prone to get many accidents, e.g., they could have offered to start almost the same contract from the beginning (as if there had been no bad accidental history) and made a profit. However, if high-risk agents anticipate this behavior on the part of the insurance companies, they would not opt for the full-insurance contract designed for them.

The article is related to many different branches of literature, not all of which can be discussed at great length.[2,3] The literature that is closest to this article are papers by Dionne and Lasserre (1985) and Cooper and Hayes (1987) on the use of experience ratings in multiperiod self-selection models.[4] This is the setting we study when considering *conditional* dynamic contracts. Dionne and Lasserre (1985) study infinite horizon contracts where agents maximize *average* per period utility. They show that in such a world, insurance companies can screen agents in such a way that the first-best outcome is achieved. Cooper and Hayes (1987) study a similar problem in a two-period model. Their main focus is on the differences in equilibrium outcomes under monopoly and perfect competition. The part of our article dealing with conditional dynamic contracts analyzes a similar situation as modeled by these two studies, but considers the intermediate case of finite horizon contracts where, in addition, agents discount future utility. Compared to Cooper and Hayes (1987), we derive a new results, namely that in order to get close to the first-best, agents may discount future payoffs. In addition, we study unconditional dynamic contracts, in order to better understand the reasons why welfare improvements are possible under dynamic contracts.

Above we have emphasized the importance of commitment. There is also a literature (see, e.g., Laffont and Tirole (1990), Dionne and Doherty (1994), and Nilssen (2000)) dealing with dynamic adverse selection problems where parties are not committed to fulfill their contract terms.

The rest of the article is organized as follows. The section "Preliminaries" discusses definitions and notations that we will use in the rest of the article. The section "Analysis" analyzes the dynamic world of conditional and unconditional contracts. The section "Conclusion" concludes with a discussion of the results. Proofs are contained in the appendix.

Preliminaries

The static version of the environment studied here is taken from Rothschild and Stiglitz (1976). Individual agents come in two types, high-risk agents "H" and low-risk agents "L." Everyone is endowed with some income level in every period, which is normalized to 1, and is subjected to a potential loss of amount e, $0 < e < 1$. Each

[2] Another literature studies the way probationary periods are used to separate agents with different risk profiles (see, e.g., Eeckhoudt et al. (1988)). A probationary period is a possible outcome in our framework. The literature on probationary periods considers, however, a situation where agents incur only one loss over a certain time period where the timing of the loss may be different for different types of agents. In contrast, our model considers situations in which in any given period, agents have a certain given probability of getting an accident.

[3] There is also some similarity to Janssen and Roy (2002) and Janssen and Karamychev (2002) who show in second-hand car markets waiting time before selling can act as a screening device in dynamic competitive markets with adverse selection.

[4] There is also a paper by Henriet and Rochet (1986) who study whether an adverse selection or a moral hazard problem is at the core of experience ratings in insurance contracts.

type $i \in \{H, L\}$ is characterized by a probability of an accident q_i, $0 < q_L < q_H < 1$. The probability of an accident is private knowledge and constant through time. All agents are risk averse, they have the same state independent strictly concave and increasing utility function u and for the sake of convenience we assume that $u(1) = 0$. Let $\alpha \in (0, 1)$ denote the share of low-risk agents in the population.

On the supply side of the market, there is a number of risk-neutral insurance companies competing with each other. These companies are not able to discriminate between the different types. In what follows we will use the superscripts "S" and "D" to refer to static and dynamic variables, respectively, and we will compare the welfare implications of two types of insurance contracts: static and dynamic. A *static* insurance contract $\Theta^S = (P^S, D^S)$ consists of a constant premium P^S and a constant deductible D^S such that in case of an accident an insured individual receives $\max\{e - D^S, 0\}$ from the insurance company. By $\Theta_0^S = (0, \infty)$ we denote an artificial contract, which gives no insurance at all. The expected utility of type i under contract Θ^S is $U_i^S(\Theta^S) \equiv q_i u(1 - P^S - D^S) + (1 - q_i)\, u(1 - P^S)$.

A dynamic contract Θ^D lasts T time periods and consists of T parts, each part specifying the terms of the contract in that time period. Unlike a static contract, dynamic contracts may offer different insurance conditions for an agent in time periods $t = 2, \ldots, T$ depending on her previous accidental history h_t. Thus, a dynamic contract's term in time period t is a set of 2^{t-1} insurance policies that correspond to every $h_t \in H_t$, where H_t is a set of all possible history realizations up to period t. For example, in period 1 a dynamic contract Θ^D offers a simple static insurance policy $\Theta_1 = (P_1, D_1)$, in period 2 a (static) policy $\Theta_2^{(1)} = (P_2^{(1)}, D_2^{(1)})$ applies if there was an accident, and $\Theta_2^{(0)} = (P_2^{(0)}, D_2^{(0)})$ applies if there was no accident. Hence, $\Theta_2 = \{\Theta_2^{(0)}, \Theta_2^{(1)}\}$. We will call such a contract $\Theta^D = (\Theta_1, \Theta_2, \ldots, \Theta_T)$.

The *ex ante* expected *per period* utility of type i under a contract Θ^D is given by

$$U_i^D(\Theta^D) \equiv \frac{1 - \delta}{1 - \delta^T} \sum_{t=1}^{T} \Bigg\{ \delta^{t-1} \sum_{h_t \in H_t} \Pr_i(h_t)\big(q_i u(1 - P_t^{(h_t)} - D_t^{(h_t)}) + (1 - q_i) u(1 - P_t^{(h_t)})\big) \Bigg\},$$

where $\delta \in (0, 1)$ is the common discount factor and $\Pr_i(h_t)$ is agent i's probability to end up with a history h_t at time period t. For example, for $h_4 = (0, 1, 0)$, i.e., no accidents in time periods 1 and 3 and an accident in time period 2, $\Pr_i(h_4) = q_i(1 - q_i)^2$. One can see that for a dynamic contract with constant insurance conditions, i.e., for $\Theta_t^{(h_t)} = \Theta^*$, $U_i^D(\Theta^D) = U_i^S(\Theta^*)$. This allows us to make welfare comparisons between static and dynamic contracts.

As explained in the introduction, in certain cases an insurer is not able, or not allowed, to use the information, which is obviously available to him, about an agent's past accidents. In this case the contract terms Θ_t are no longer sets of policies but simply a sequence of static contracts $\Theta_t = (P_t, D_t)$ and the expression for the expected per period utility simplifies to

$$U_i^D(\Theta^D) = \frac{1-\delta}{1-\delta^T}\sum_{t=1}^{T}\left\{\delta^{t-1}(q_i u(1-P_t-D_t)+(1-q_i)u(1-P_t))\right\}.$$

We will call such a contract an *unconditional* dynamic contract.

Having offered a contract Θ^D, an insurance company gets a profit

$$\pi_i(\Theta^D) = \sum_{t=1}^{T}\left(\delta^{t-1}\sum_{h_t\in H_t}\Pr_i(h_t)(P_t^{h_t} - q_i(e - D_t^{h_t}))\right)$$

from an agent of type i. Average per-agent profit of contract Θ^D depends on the share of low-risk people α_{Θ^D} among those who buy Θ^D:

$$\pi(\Theta^D) = \alpha_{\Theta^D}\pi_L(\Theta^D) + (1-\alpha_{\Theta^D})\pi_H(\Theta^D).$$

Let Σ_T^D be the set of all T-period dynamic (conditional or unconditional, depending on the context) insurance contracts. The set Σ^S, which is the set of all static insurance contracts, coincides with Σ_1^D so that any static contract Θ^S can be treated as a one-period dynamic contract. We describe the welfare properties and existence conditions of a competitive Nash equilibrium over the set Σ_T^D for an arbitrary but fixed $T > 1$. The formal definition of a (competitive Nash) equilibrium is as follows:

Definition 1: *A T-period competitive Nash equilibrium is a subset of T-period insurance contracts, $\Psi_T \subset \Sigma_T^D$, present in the market satisfying the following conditions:*

(a) *Each agent chooses an insurance contract that maximizes her per period utility, i.e., every type $i \in \{H, L\}$ chooses the contract $\Theta_i \in \arg\max_{\Theta\in\Psi_T} U_i(\Theta)$.*

(b) *Any equilibrium contract is bought by at least one type, i.e., for any $\Theta' \in \Psi_T$ there exists $i \in \{H, L\}$ such that $\Theta' = \Theta_i$.*

(c) *Any equilibrium contract yields nonnegative profit to an insurer, i.e., $\pi(\Theta) \geq 0$ for any $\Theta \in \Psi_T$.*

(d) *No insurance company can benefit by unilaterally offering a different insurance contract, i.e., any insurance company offering a contract $\Theta' \in \Sigma_T^D\backslash\Psi_T$, such that $U_i(\Theta') > \max_{\Theta\in\Psi_T} U_i(\Theta)$ for some $i \in \{H, L\}$, makes strictly negative profit: $\pi(\Theta') < 0$.*

Parts (a), (c), and (d) are part of any standard definition of equilibrium. Part (b) is introduced in order to get rid of a multitude of uninteresting equilibria. Standard arguments rule out any *pooling* contract Θ_P to be a Nash equilibrium. For static contracts, the argument is given by Rothschild and Stiglitz (1976). In a dynamic world a similar argument holds true: for any (partial) pooling contract there exists a contract that differs from it in only one time period in such a way that only low-risk agents prefer the latter contract. This implies that the deviation yields strictly positive profit.

On the other hand, a *separating* Nash equilibrium (static or dynamic), which involves two contracts Θ_H and Θ_L, may not exist if there exists a *profitable* pooling contract Θ_P that gives a higher utility level to the low-risk agent than Θ_L. Hence, the existence of a separating Nash equilibrium is guaranteed if any pooling contract yielding nonnegative profit, Θ_P, gives less utility to low-risk type agents than Θ_L, i.e., $U_l(\Theta_P) \leq U_l(\Theta_L)$.

Due to competition, every contract must generate zero profit. We study two types of settings in more detail, one where the overall profit is equal to zero, and the other, where insurance companies are forced to price contract in such a way that they yield zero profit in *every* time period. We also briefly make some observations on the case where contracts have to satisfy a zero-profit condition after each history. Both settings yield qualitatively similar results although in the latter setting, we are able to establish more properties of the equilibrium contracts. Finally, agents are not allowed to transfer wealth between periods.

ANALYSIS

In this section, we first study the properties and existence conditions of competitive Nash equilibria in a setting where insurance companies offer conditional dynamic contracts. In second instance, we study unconditional contracts. With conditional contracts, insurance conditions may depend on the time period *and* on the accidental history of insured agents. We first allow insurance companies to transfer profits between different time periods as well as from one accidental history to another within one time period, i.e., competition between insurance companies results in a zero-profit condition of the form:

$$\sum_{t=1}^{T}\left(\delta^{t-1}\sum_{h_t \in H_t} \Pr_t(h_t)(P_t^{h_t} - q_L(c - D_t^{h_t}))\right) = 0. \qquad (1)$$

As we will see, insurers may want to distinguish between agents who (by pure chance) have a different accidental history so in order to better screen high- and low-risk agents.

The proposition below states the main result for conditional dynamic contracts. Whenever competitive Nash equilibria exist, they yield a Pareto-improvement over the static equilibrium contracts: high-risk agents also get full insurance in every period independent of their accidental history and low-risk agents get better insurance conditions than under a static contract. These equilibria exist whenever the fraction of low-risk agents is small enough so that no company wants to deviate by offering a pooling contract. Finally, if the utility level associated with very low income levels falls dramatically, formally when $\lim_{m \to 0} u(m) = -\infty$, and the time horizon is large enough, then it is possible to offer lower-risk agents almost full insurance even if the fraction of low-risk agents is high.

Proposition 1 (conditional contracts, overall zero-profit condition): *With zero-profit condition (1), for any T there exists an $\alpha_C^D \in (0, 1)$ such that*

(a) *For all $\alpha \in (0, \alpha_C^D)$, there exist a separating competitive Nash equilibrium. All competitive Nash equilibria are separating. Moreover, all competitive Nash equilibria involve the* *same equilibrium payoffs for type L and the same equilibrium contract* Θ_H^D *for type H, which coincides with* Θ_H^S.[5]

(b) *If* $\alpha < \min\{\alpha^S, \alpha_C^D\}$, *then* $U_L^D(\Theta_L^D) > U_L^S(\Theta_L^S)$.

(c) *If* $\lim_{m \to 0} u(m) = -\infty$, *then for any fixed* $\alpha \in (0, 1)$ $\lim_{T \to \infty} U_L^D(\Theta_L^D) = u(1 - q_L e)$ *and* $\lim_{T \to \infty} \alpha_C^D = 1$.

Part (b) of the result basically says that if we had a T-fold repetition of the static equilibrium contract for low-risk agents, then (i) there are unused gains from trade across states in a certain time period, and (ii) there are unused gains from intertemporal trade. The reason is that in the accident state of a certain period, the agent is worse off than he expects to be in future periods and in the no-accident state he is better off than he expects to be in future periods. In the absence of the per-period break-even condition on insurers, we should expect each of these intertemporal effects on its own to introduce some spreading over all periods of the impact of the accident/no accident revelation in any time period, i.e., have a lower deductible and a bonus/malus system. As $q_L < q_H$, the terms of the insurance contract after one or a few accidents gives a relatively smaller weight in the low-risk agent's overall evaluation of an insurance contract than in the evaluation of the same contract by a high-risk agent. Hence, it is possible to design an incentive compatible dynamic contract that low-risk agents prefer to the static equilibrium contract, whereas high-risk agents still prefer the full insurance contract at fair odds.

The most interesting result (c), the approximation of the first-best outcome, is based on the following considerations. When T gets large, the probability of an event history with many accidents converges to zero for both types of agents. However, the convergence is exponentially faster for low-risk agents than for high-risk agents. When the utility function is unbounded below, insurance companies can exploit this fact by providing extremely unfavorable insurance conditions in case of an event with many accidents in such a way that, whereas satisfying the zero-profit and incentive compatibility conditions, these unfavorable terms is of vanishing significance for the overall evaluation of the insurance contract only for the low-risk agents. The reason that $\lim_{T \to \infty} \alpha_C^D = 1$ lies in the fact that the separating contract approaches the first-best outcome, whereas for a given $\delta < 1$ the best pooling deviation will never be close to the first-best outcome.

Now, we consider the case where insurance companies are forced to price contracts in such a way that they yield zero profit in *every* time period.[6] Then, the zero-profit condition takes the following form:

$$\sum_{h_t \in H_t} \Pr_t(h_t)\left(P_t^{h_t} - q_L\left(e - D_t^{h_t}\right)\right) = 0, \quad t = 1, \ldots, T. \tag{2}$$

[5] In what we believe to be nongeneric cases, different equilibria may involve different contracts for type L.

[6] Thus, the zero-profit condition also has to hold for possible deviations from the equilibrium contracts.

In this environment, Proposition 1 holds true, in addition more specific equilibrium properties can be derived. Proposition 2 states the result.

Proposition 2 (conditional contracts, per-period zero-profit condition): *With zero-profit condition* (2), *Proposition* 1 *holds true. In addition:*

(*a*) *For a time period t, the optimal premium and deductible given a history of k accidents,* $P^{k}_{L,t}$ *and* $D^{k}_{L,t}$, *satisfy the relation* $P^{k}_{L,t} = P^{k-1}_{L,t} + D^{k-1}_{L,t}$, *if low-risk agents get positive insurance.*

(*b*) *The profits an insurer gets at time t from low-risk agents with a history of k accidents satisfies the relation* $\pi^{t-1}_{L,t} > \cdots > \pi^{k}_{L,t} > \pi^{k-1}_{L,t} > \cdots > \pi^{0}_{L,t}$, *with* $\pi^{t-1}_{L,t} > 0 > \pi^{0}_{L,t}$.

(*c*) *For all* $\alpha \in (\alpha^{D}_{C}, 1)$, *a separating competitive Nash equilibrium* Ψ_T *does not exist.*

Proposition 2 establishes some equilibrium properties when a per-period zero-profit condition is introduced. A first result says that the price at time period t increases with the number of accidents happened in such a way that the total payment in period t of an agent having k accidents by that time is independent on whether he had all k accidents before, i.e., k accidents by time $t-1$, and no accident now, i.e., at time t, or he had $k-1$ accidents at time $t-1$ and an accident in period t. The second result basically says the profits an insurer gets at time t from a low-risk agent increases in the number of registered accidents. This is due to the fact that the most efficient way to discriminate between the two types of agents is by imposing worse insurance conditions in case of an accident. As the number of past accidents does not affect the accidental probability, this implies that the insurer makes more profits in case of histories with many accidents. Finally, $\alpha^{D}_{C} \in (0, 1)$ becomes a unique cut-off point such that the separating competitive Nash equilibrium exists for all $\alpha \leq \alpha^{D}_{C}$ and does not exist for all $\alpha > \alpha^{D}_{C}$.

The argument used bears some similarity to the argument employed by Mirrlees (1974) who, in moral hazard settings, uses penalties for very bad outcome which are much more unlikely under good than under bad behavior to approximate first-best outcomes.[7] The critique of Holmstrom and Milgrom (1987) that this technique can only be applied in case the principal has very precise knowledge about the agents including the full realized history of accidental events, is not relevant in our case. One of the main differences between our result and the one by Mirrlees (1974) is that our result holds true even if the principal can only observe time aggregates of outcomes. To see this, note that the probability that agent of type i has k accidents in T periods is given by $C^{k}_{n} q_i^{k} (1 - q_i)^{T-k}$, where C^{k}_{n} is the binominal coefficient. This implies that the relative likelihood of having k accidents in T periods of a low-risk agent compared to a high-risk agent can be written as $((\frac{q_L}{q_H})^{\frac{k}{T}} (\frac{1-q_L}{1-q_H})^{1-\frac{k}{T}})^{T}$. It is easy to see that for $k = T$, or (for somewhat larger T) for any k provided the ratio $\frac{k}{T}$ is close enough to 1, this expression converges to zero when $T \to \infty$. This indicates that even if the insurer can only observe time aggregates of outcomes and, moreover, can only do so in the very last period and even observes the number of accidents somewhat imprecisely, he is still able to offer a contract for the low-risk type that approaches the first best full-information outcome.

[7] We thank one of the referees for bringing this literature to our attention.

Propositions 1 and 2 have investigated the impact of an *overall* and a *per-period* zero-profit constraint on equilibrium contracts. Now we will briefly discuss what happens if a *per-history* zero-profit condition is imposed. One may easily repeat the derivations of the proof of Proposition 1 for this case and see that parts (a) and (b) of Proposition 1 continue to be true. This implies that it is *not* the intra- and intertemporal trade *per se* that makes conditional dynamic contracts welfare-superior relative to static contracts. Rather, it is the ability to discriminate between histories that have different likelihood ratios for high- and low-risk agents that is important. In its present form where it is formulated for all parameter values, part (c) of Proposition 1 remains valid only if $\delta = 1$, i.e., if there is no discounting.[8] This is because an insurer is not able to punish agents that have many accidents more than the "no insurance contract" does. This in turn implies that with $\delta < 1$, a contract offering low-risk agents (almost) full insurance during the initial time periods cannot be provided as such a contract is also preferred by high-risk agents. Finally, a per-history zero-profit constraint simplifies parts (a) and (b) of Proposition 2. First, all the profits are zero, i.e., $\pi^k_{L,t} = 0$ for all $k = 0, \ldots, t-1$, and, second, the zero-profit constraint implies that $P^k_{L,t} = q_L\,(e - D^k_{L,t})$ with $D^k_{L,t}$ being a strictly increasing function of k.

In this model, we have implicitly assumed that insurance companies *and* agents are committed to the contracts they have signed and that if an agent switches to another insurance company she will not start the contract from the beginning. In some markets, like the one for car insurance, commitment on the part of insurance companies is achieved as insurance companies share information about accidental history of their clients. It is more difficult, however, to ensure that agents are committed to the contract. If this form of commitment is not achievable, additional constraints have to be satisfied such as that after every history contract terms should be such that it is not possible to design a profitable contract that agents prefer to continuing the existing contract. In a T-period world, it is difficult to satisfy all those constraints.[9] This is another reason to consider unconditional contracts next.

It is clear from the outset that wherever both exist, unconditional contracts yield lower welfare than conditional contracts as the latter include the former. What is less clear, however, is whether unconditional contracts yield a welfare improvement over static contracts. It turns out that the best unconditional contract is just the repeated static contract. In other words, welfare gains are not possible using unconditional dynamic contracts.

Proposition 3 (Unconditional Contracts): *For all $\alpha \in (0, \alpha^S)$, there exists a unique separating competitive Nash equilibrium that has the static insurance policies Θ^S_H and Θ^S_L in any time period. For all $\alpha \in (\alpha^S, 1)$ a separating competitive Nash equilibrium does not exist.*

It is interesting to better understand the reason for this result. With unconditional contracts, an insurer can offer different insurance conditions in different time periods, but cannot condition on the number of accidents. Time-dependent contracts allow the insurance company to better screen the types. On the other hand, this screening also makes low-risk type agents worse off, making such a contract less interesting than Θ^S_L

[8] Ideas of the proof by Dionne and Lasserre (1985) apply in case $\delta = 1$.
[9] See Cooper and Hayes (1987) for some of the relevant considerations when $T = 2$.

for low-risk type agents. This result holds independently of whether the zero-profit condition takes form (1) or (2).

Proposition 3 also sheds another light on the positive result obtained for conditional dynamic contracts. There are two important differences between the two settings. First, with conditional contracts, insurance companies are able to shift profits between different accidental histories for every given time period. Second, even if expected profits are zero after every history, insurance companies may still give agents with better histories contracts with better insurance. Both screening strategies are not possible with unconditional contracts.

CONCLUSION

In this article, we studied a dynamic version of the Rothschild and Stiglitz (1976) insurance model. We showed that if in the multiperiod dynamic model a competitive Nash equilibrium exists, it is Pareto-superior to the static equilibrium if, and only if, *conditional* contracts are allowed. This has an important policy conclusion: when it is considered unfair to discriminate between people with different accidental histories, dynamic contracts cannot overcome the adverse selection problem in insurance markets and improve welfare.

Of course, this conclusion is valid only within the limits of the model we have analyzed. We have not allowed agents to transfer wealth between different periods and we have considered situations where consumers only differ in their accidental probability. Future work may investigate how to relax these conditions.

APPENDIX

Proof of Proposition 1: We prove all the statements of the proposition sequentially.

(a) We begin by deriving a set of competitive contracts $\{\Theta_H^D, \Theta_L^D\}$ satisfying the incentives compatibility constraints and maximizing $U_L^D(\Theta_L^D)$. Then, we derive a competitive pooling contract $\hat{\Theta}_{P,L}$ maximizing low-risk utility. Finally, we show that there exist an $\alpha_C^D \in (0, 1)$ such that for all $\alpha < \alpha_C^D$ ($\alpha > \alpha_C^D$) $\hat{\Theta}_{P,L}$ gives a lower (higher) utility for the low-risk type than Θ_L^D.

Contract $\Theta_H^D = (\Theta_{H,1}, \ldots, \Theta_{H,T})$ maximizes $U_H^D(\Theta^D)$ subject to the zero-profit constraint. Solving yields a unique solution $D_{H,t}^{h_t} = 0$ and $P_{H,t}^{h_t} = q_H e$, hence, $\Theta_H^D = (\Theta_H^S, \ldots, \Theta_H^S)$. Contract $\Theta_L^D = (\Theta_{L,1}, \ldots, \Theta_{L,T})$ maximizes $U_L^D(\Theta^D)$ subject to the zero-profit constraint and incentives compatibility constraint $U_H^D(\Theta^D) \leq U_H^D(\Theta_H^D) = u(1 - q_H e_H)$. Analyzing the first-order conditions

$$\begin{cases} \lambda = \left(1 - \mu \dfrac{(1 - q_H)\Pr_H(h_t)}{(1 - q_L)\Pr_L(h_t)}\right) u'\left(1 - P_{L,t}^{(h_t)}\right) \\ \lambda = \left(1 - \mu \dfrac{\Pr_H(h_t) q_H}{\Pr_L(h_t) q_L}\right) u'\left(1 - P_{L,t}^{(h_t)} - D_{L,t}^{(h_t)}\right) \end{cases} \tag{A1}$$

yields that both constraints are binding and, therefore, $D_{L,t}^{h_t} > 0$ for all t and h_t. Eliminating λ leads to

$$
\left((1 - q_H)q_L + (q_H - q_L)\left(1 - \mu \frac{q_H}{q_L}\left(\frac{\Pr_H(h_t)}{\Pr_L(h_t)}\right)\right)\right)^{-1}
$$

$$
= (1 - q_L)q_H \frac{u'(1 - P_{L,t}^{(h_{t-1})} - D_{L,t}^{(h_{t-1})})}{u'(1 - P_{L,t}^{(h_{t-1})})}.
$$

The last expression implies that the deductible $D_{L,t}^{(h_{t-1})}$ is larger in those histories that have larger relative likelihood $\frac{\Pr_H(h_t)}{\Pr_L(h_t)}$ of meeting high-risk agents rather than low-risk agents.

It might happen that the solution Θ_L^P does not satisfy $D_{L,t}^{(h_{t-1})} \le e$. Hence, we may have to find all corner solutions imposing $\Theta_{L,t}^{h_t} = \Theta_0^S$ for some set of states of the world $H_t^0 \subset H_t$. The Lagrange function in this case differs from the former one only by the summation range $\sum_{h_t \in H_t \setminus H_t^0}$, which now takes place over the subset $h_t \in H_t \setminus H_t^0$ instead of the whole set $h_t \in H_t$. Having been calculated for all $H_t^0 \subset H_t$, a contract Θ_L^P is chosen in such a way that, first, it satisfies $D_{L,t}^{h_t} < e_t$ for all $h_t \in H_t \setminus H_t^0$, and, second, it maximizes $U_L^P(\Theta_L^P)$.

The set of competitive contracts $\{\Theta_H^P, \Theta_L^P\}$ becomes a competitive Nash equilibrium if no competitive pooling contract gives a higher utility level for the low-risk agents. We will prove that for small enough values of α this is indeed the case. The utility low-risk agents get under Θ_L^P does not depend on α whether the utility they get under a pooling contract $\Theta_p^P = (\Theta_{P,1}, \ldots, \Theta_{P,T})$ depends on it. The contract Θ_p^P maximizes $U_L^P(\Theta_p^P)$ under the average zero-profit constraint with the following first-order conditions:

$$
\begin{cases}
\lambda \dfrac{\alpha \Pr_L(h_t)(1 - q_L) + (1 - \alpha)\Pr_H(h_t)(1 - q_H)}{\Pr_L(h_t)(1 - q_L)} = u'(1 - P_{P,t}^{(h_{t-1})}) \\[2ex]
\lambda \dfrac{\alpha \Pr_L(h_t)q_L + (1 - \alpha)\Pr_H(h_t)q_H}{\Pr_L(h_t)q_L} = u'(1 - P_{P,t}^{(h_{t-1})} - D_{P,t}^{(h_{t-1})}).
\end{cases}
$$

The zero-profit constraint binds as $\lambda > 0$ and the solution to this system Θ_p^P is unique due to the global concavity of the objective function and linear constraints.

If such an interior solution has $D_{P,t}^{h_t} > e$, then we have to look at the corner solutions, where $\hat{\Theta}_{P,t}^{h_t} = \Theta_0^S$ for some set of states of the world $H_t^0 \subset H_t$. Now, the first-order conditions remain the same but now only for $h_t \in H_t \setminus H_t^0$. Solving them for $\hat{\Theta}_{P,t}$ for all $H_t^0 \subset H_t$ and taking one that maximizes $U_L^P(\Theta_p^P)$ gives the contract we are looking for.

The contract conditions, hence, $U_L^P(\hat{\Theta}_p^P)$, now become continuous (and even piecewise continuously differentiable) functions of α due to the strict concavity of the utility function. At $\alpha = 1$ we get $\lambda = u'(1 - P_{P,t}^{(h_{t-1})}) = u'(1 - P_{P,t}^{(h_{t-1})} - D_{P,t}^{(h_{t-1})})$ and, therefore, $D_{P,t}^{(h_{t-1})} = 0$, i.e., full insurance for the low-risk type. Obviously, $U_L^P(\hat{\Theta}_p^P)|_{\alpha=1} > U_L^P(\Theta_L^P)$. At $\alpha = 0$, on the other hand, we get $\frac{(1-q_L)}{(1-q_H)}u'(1 - P_{P,t}^{(h_{t-1})}) =$

$\frac{q_L}{q_H} u'(1 - P^{(h_t)}_{P,t} - D^{(h_t)}_{P,t})$ and, therefore, $P^{(h_t)}_{P,t}$ and $D^{(h_t)}_{P,t}$ do not depend on h_t. This means that the pooling contract $\hat{\Theta}_{P,t}$ at $\alpha = 0$ coincides with pooling contract in static environments and standard arguments apply to show that $U_l^D(\hat{\Theta}_P^D)|_{\alpha=0} = U_l^S(\hat{\Theta}_P^S)|_{\alpha=0} < U_l^S(\Theta_l^S) \le U_l^D(\Theta_l^D)$.

Thus, $U_l^D(\hat{\Theta}_P^D)|_{\alpha=0} < U_l^D(\Theta_l^D) < U_l^D(\hat{\Theta}_P^D)|_{\alpha=1}$ and, therefore, there exists an $\alpha_C^D \in (0,1)$ such that $U_l^D(\hat{\Theta}_P^D) < U_l^D(\Theta_l^D)$ for all $\alpha \in (0, \alpha_C^D)$ and part (a) of the proposition follows.

(b) As contract $\Theta^D = (\Theta_l^S, \ldots, \Theta_l^S)$ was available during the optimization procedure, it is clear that $U_l^D(\Theta_l^D) \ge U_l^S(\Theta_l^S)$. In addition, as the contract $\Theta^D = (\Theta_l^S, \ldots, \Theta_l^S)$ does not satisfies the first-order conditions (A1), we have $U_l^D(\Theta_l^D) > U_l^S(\Theta_l^S)$ unless those first-order conditions degenerate in a global corner solution $\Theta_l^D = (\Theta_0^S, \ldots, \Theta_0^S)$. This is never possible, however, as the latter contract yields even lower utility for the low-risk type than $U_l^S(\Theta_l^S)$.

(c) We will show that for any T there exists a contract $\hat{\Theta}_l^D$ satisfying both zero-profit and the incentive compatibility constraints that is such that $\lim_{T\to\infty} U_l^D(\hat{\Theta}_l^D) = u(1 - q_l e)$. As $U_l^D(\Theta_l^D) \ge U_l^D(\hat{\Theta}_l^D)$, $\lim_{T\to\infty} U_l^D(\Theta_l^D) = u(1 - q_l e)$ holds as well. Then, as $U_l^D(\Theta_P^D) < u(1 - q_l e)$ for any $\alpha < 1$, this implies that $\lim_{T\to\infty} \alpha_C^D = 1$.

Let us consider a contract $\hat{\Theta}_l^D = (\hat{\Theta}_{l,1}, \ldots, \hat{\Theta}_{l,T})$, where $\hat{\Theta}^k_{l,t} = (q_l e_l, 0)$ for all $t = 1, \ldots, T-1$, $\hat{\Theta}^k_{l,T} = (\hat{P}^k_l, \hat{D}^k_l) = (P^-, 0)$ for $k = 0, \ldots, T-2$ and $\hat{\Theta}^{T-1}_{l,T} = (P^+, 0)$ for some values of P^- and P^+. The zero-profit condition requires that $\sum_{k=0}^{T-1} \Pr_l(k)(\hat{P}^k_{l,T} - q_l e) = 0$, which makes P^- dependent on P^+:

$$P^-(P^+) = \frac{q_l e - q_l^{T-1} P^+}{1 - q_l^{T-1}}, \tag{A2}$$

hence $P^- < q_l e$ for all $P^+ > q_l e$. On the other hand, the incentives compatibility constraint requires that $U_H^D(\Theta_H^D) = U_H^D(\hat{\Theta}_l^D)$, therefore,

$$u(1 - P^-)(1 - q_H^{T-1}) + q_H^{T-1} u(1 - P^+)$$
$$= u(1 - q_l e) - (u(1 - q_l e) - u(1 - q_H e))\frac{(1 - \delta^T)}{(1 - \delta)\delta^{T-1}}.$$

Together with (A2), this equation defines unique values of $P^- < q_l e$ and $P^+ > q_l e$. To see this we substitute (A2) back into the last equation. The left-hand side, being a function of P^+ has the following properties:

$$LHS|_{P^+ = q_l e_l} = u(1 - q_l e)(1 - q_H^{T-1}) + q_H^{T-1} u(1 - q_l e) = u(1 - q_l e) > RHS.$$

If $\lim_{m \to 0} u(m) = -\infty$, then $\lim_{P^+ \to 1} LHS = (1 - q_H^{T-1}) u(\frac{1 - q_l e}{1 - q_l^{T-1}}) + q_H^{T-1} \lim_{P^+ \to 1} u(1 - P^+) = -\infty$.

$$\frac{dLHS}{dP^+} = q_H^{T-1}\left(\frac{q_l^{T-1}(1 - q_H^{T-1})}{q_H^{T-1}(1 - q_l^{T-1})} u'(1 - P^-) - u'(1 - P^+)\right) < 0.$$

Hence, $P^{\cdot}$ and P are uniquely defined. Then, $\lim_{T\to\infty} P = \lim_{T\to\infty} \times \frac{q_L e - q_L^{T-1} P^{\cdot}}{1 - q_L^{T-1}} = q_L e$, and $\lim_{T\to\infty} \delta^{T-1} q_H^{T-1} u(1 - P^{\cdot}) = -\frac{1}{(1-\delta)}(u(1 - q_L e) - u(1 - q_H e))$. Therefore,

$$\lim_{T\to\infty} U_L^P(\hat{\Theta}_L^P) = u(1 - q_L e) - \lim_{T\to\infty} \Big(u(1 - q_L e) - u(1 - q_H e)\Big) \frac{q_L^{T-1}}{q_H^{T-1}} = u(1 - q_L e). \qquad \text{Q.E.D.}$$

Proof of Proposition 2: Repeating all the derivations from the proof of Proposition 1 with the per-period zero-profit condition (2) yields that all the statements of Proposition 1 are still valid. The first-order conditions now become:

$$\begin{cases} \mu = \dfrac{\Pr_L(h_t)}{\Pr_H(h_t)} \dfrac{q_L(1 - q_L)(u'(1 - P^{h_t}_{L,t} - D^{h_t}_{L,t}) - u'(1 - P^{h_t}_{L,t}))}{q_H(1 - q_L)u'(1 - P^{h_t}_{L,t} - D^{h_t}_{L,t}) - q_L(1 - q_H)u'(1 - P^{h_t}_{L,t})} \\ \lambda_t = \dfrac{1 - \delta}{1 - \delta^T} \delta^{t-1} \dfrac{(q_H - q_L)u'(1 - P^{h_t}_{L,t} - D^{h_t}_{L,t})u'(1 - P^{h_t}_{L,t})}{q_H(1 - q_L)u'(1 - P^{h_t}_{L,t} - D^{h_t}_{L,t}) - q_L(1 - q_H)u'(1 - P^{h_t}_{L,t})}. \end{cases} \quad \text{(A3)}$$

Taking into account that $\Pr_t(k) = C^k_{t-1} q_L^k (1 - q_L)^{t-k-1}$, where $C^k_{t-1} = \frac{k!(t-1-k)!}{(t-1)!}$ are binomial coefficients, and getting rid of λ_t and μ leads to

$$\begin{cases} P^0_{L,t} = q_L e - \sum_{k=0}^{t-1} \Pr_L(k) \left(\sum_{i=0}^{k-1} D^i_{L,t} + q_L D^k_{L,t} \right) \\ P^k_{L,t} = P^{k-1}_{L,t} + D^{k-1}_{L,t} \end{cases}.$$

Now, part (b) can be easily proven:

$$\begin{aligned} \pi^k_{L,t} &= P^k_{L,t} - q_L(e - D^k_{L,t}) = P^{k-1}_{L,t} + D^{k-1}_{L,t} - q_L(e - D^k_{L,t}) \\ &= \pi^{k-1}_{L,t} + (1 - q_L)D^{k-1}_{L,t} + q_L D^k_{L,t} > \pi^{k-1}_{L,t}, \end{aligned}$$

which together with $\sum_{k=0}^{t-1} \pi^k_{L,t} = 0$ leads to the result.

In order to prove part (c), we need to show that $\frac{d}{d\alpha} U_L^P(\hat{\Theta}_P^P) > 0$. The first-order conditions for the best pooling contract in case of per-period zero-profit constraints

$$\begin{aligned} &\alpha \sum_{h_t \in H_t \setminus H^c} \Pr_L(h_t)(P^{h_t}_{P,t} - q_L(e - D^{h_t}_{P,t})) \\ &= -(1 - \alpha) \sum_{h_t \in H_t \setminus H^c} \Pr_H(h_t)(P^{h_t}_{P,t} - q_H(e - D^{h_t}_{P,t})), \quad t = 1, \ldots, T \end{aligned}$$

are:

$$\begin{cases} u'(1-\hat{P}^{h_t}_{p,t}) = \lambda_t\left(\alpha + (1-\alpha)\dfrac{\Pr_H(h_t)(1-q_H)}{\Pr_L(h_t)(1-q_L)}\right) \\ u'(1-\hat{P}^{h_t}_{p,t}-\hat{D}^{h_t}_{p,t}) = \lambda_t\left(\alpha + (1-\alpha)\dfrac{\Pr_H(h_t)q_H}{\Pr_L(h_t)q_L}\right). \end{cases}$$

Finally, taking the derivative $\frac{d}{d\alpha}U_L^P(\hat{\Theta}_p^P)$ and using the envelope theorem yields:

$$\frac{d}{d\alpha}U_L^P(\hat{\Theta}_p^P) = \frac{\lambda_t}{1-\alpha}\frac{1-\delta}{1-\delta^T}\sum_{t=1}^{T}\left(\delta^{t-1}\sum_{h_t\in H_H}\Pr_L(h_t)(\hat{P}^{h_t}_{p,t}-q_L(e_L-\hat{D}^{h_t}_{p,t}))\right).$$

It is easily seen that $\sum_{h_t\in H_H}\Pr_H(h_t)(\hat{P}^{h_t}_{p,t}-q_H(e-\hat{D}^{h_t}_{p,t}))<0<\sum_{h_t\in H_H}\Pr_L(h_t)\times(\hat{P}^{h_t}_{p,t}-q_L(e-\hat{D}^{h_t}_{p,t}))$, i.e., an insurer gets a positive profit from the low-risk type and a negative profit from the high-risk type. Therefore, $\frac{d}{d\alpha}U_L^P(\hat{\Theta}_p^P)>0$ as $\lambda_t>0$. Q.E.D.

Proof of Proposition 3: We first show that $\Theta_H^P\equiv(\Theta_{1,H},\ldots,\Theta_{T,H})=(\Theta_H^S,\ldots,\Theta_H^S)$. Maximizing $U_H^P(\Theta_H^P)$ with respect to all $D_{t,H}\in[0,e]$ and subject to the zero-profit condition yields $D_{t,H}=0$ and $P_{t,H}=q_He$ for all $t=1,\ldots,T$. Hence, $\Theta_{t,H}=\Theta_H^S$ and $U_H^P(\Theta_H^P)=U_H^S(\Theta_H^S)$. On the other hand, maximizing $U_L^P(\Theta_L^P)$ with respect to all $D_{t,L}\in[0,e]$ and subject to the zero-profit constraint $\sum_{t=1}^T\delta^{t-1}(P_{t,L}-q_L(e-D_{t,L}))=0$ and the incentive compatibility constraint $U_H^P(\Theta_L^P)\leq U_H^S(\Theta_H^S)$ yields that the latter binds and, in case of overall zero-profit condition (1), the first-order conditions lead to

$$\begin{cases} \lambda = \left(1-\mu\dfrac{(1-q_H)}{(1-q_L)}\right)u'(1-P_{t,L}) \\ \lambda = \left(1-\mu\dfrac{q_H}{q_L}\right)u'(1-P_t-D_{t,L}) \end{cases}$$

where λ and μ are Lagrange multipliers.

Imposing a per-period zero-profit condition, the first-order conditions can be rewritten as

$$\lambda = \frac{q_L(1-q_L)u'(1-P_{t,L}-D_{t,L})-q_L(1-q_L)u'(1-P_{t,L})}{q_H(1-q_L)u'(1-P_{t,L}-D_{t,L})-q_L(1-q_H)u'(1-P_{t,L})}\equiv\varphi(D_{t,L}),\quad t=1,\ldots,T,$$

where φ is a strictly increasing function.

Thus, all prices and deductibles, $P_{t,L}$ and $D_{t,L}$, have to be equal to each other in both settings, i.e., $P_{t,L}=P_{1,L}$ and $D_{t,L}=D_{1,L}$ for all t and, therefore, Θ_L^P is just a repetition of a static contract. But we know that the best contract for the low-risk type is Θ_L^S. Finally, Θ_L^S exists if and only if $\alpha\in(0,\alpha^S)$. Q.E.D.

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