

Discriminatory Trade Promotions in Consumer Search Markets*

Maarten Janssen[†] and Edona Reshidi[‡]

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Abstract

This paper shows that in consumer search markets discriminatory trade promotions create more profits for manufacturers than uniform pricing. The mechanism relies on consumers having heterogeneous search cost and applies even if they have identical demand. By giving some, but not all, retailers a trade promotion, manufacturers create more competition between retailers and boost demand. Relative to uniform pricing retailers who receive the trade promotion sell to a disproportionately larger share of low search cost consumers who are more price-sensitive making these retailers compete stronger. Retailers that do not receive the trade promotion lower their margins as well as they serve a smaller customer base and are keen to prevent more consumers from leaving.

Keywords: Trade Promotions; Consumer Search; Vertical Relations.

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[†]Department of Economics, University of Vienna and CEPR; maarten.janssen@univie.ac.at.

[‡]Bank of Canada, Banking and Payments Department; ereshidi@bank-banque-canada.ca.

1 Introduction

In industries that have relatively stable cost and demand patterns large manufacturers typically sign detailed long-term contracts with their retailers. These contracts usually specify that the retailers will regularly get a temporary reduction on the wholesale price (e.g., four times per year you get a 25% discount to induce consumers to try our product). Such price discounts from manufacturers to retailers are known as trade promotions.¹ The total spending on such price discounts has increased drastically over the past decades. In the grocery industry, for instance, the costs for trade promotions have risen from \$8 billion in 1980 to around \$75 billion in 1998 (Merli [1999]). In 2015, Nielsen estimated that every year \$500 billion is spent on trade promotions globally, with manufacturers in the fast-moving consumer goods sector spending around 20% of their revenue on these promotions.²

Trade promotions are widespread from consumer packaged goods markets to durable goods industries. In the automobile industry, for instance, trade promotions by manufacturers often surpass \$1,000 per automobile sold (Bruce et al. [2005]). Many of these markets are characterized by significant informational frictions on the side of consumers. For example, Moraga-González et al. [2021] provide empirical evidence that search frictions are significant in the Dutch automobile industry, Dubois and Perrone [2018] show that consumers face high search costs in supermarket purchases, while Pires [2016] finds significant search costs in storable goods markets.³ The literature on trade promotions, however, has largely concentrated on markets where consumers are fully informed about retail prices. In this paper we take into account these frictions, and analyse manufacturer's incentives to offer trade promotions and their impact on consumer welfare.

We argue that in consumer search markets an important distinction needs to be made between discriminatory and uniform trade promotions. We will say that trade promotions are *discriminatory* if at a particular point in time a manufacturer gives a trade promotion to some, but not to all, of her retailers, while she gives a *uniform* trade promotion if all retailers receive it. One way to think of how discriminatory trade promotions are used in stable long-term contracts is that from an *ex ante* perspective the manufacturer treats retailers equally, but simply offers them trade promotions at different points in time, where the contract specifies how often a retailer gets a sales price and how high the discount is. In this way, (many) consumers are likely not to know who the cheapest retailer is at any particular point in time, although they may well realize that there are regular prices and sales prices and that the identity of the cheapest retailer fluctuates.

¹Trade promotions should be distinguished from other types of sales promotions, such as consumer or retail promotions, where promotions are offered directly to consumers by the manufacturer or a retailer (see, Blattberg and Neslin [1990]).

²<https://www.nielsen.com/uk/en/press-releases/2015/most-grocery-trade-promotions-lose-money-for-suppliers/>

³Examples of other markets where search costs are significant are online book markets (De Los Santos et al. [2012]), car insurance markets (Honka [2014]), markets for MP3 players (Santos et al. [2017]) etc.

Our paper shows that manufacturer brand managers can increase the profitability of trade promotions by making them discriminatory as this increases within-brand competition between retailers selling the good of the same manufacturer. Discriminatory trade promotions squeeze retail margins and give the manufacturer incentives to set higher wholesale prices. The mechanism is based on three features. First, at a particular point in time manufacturers may choose to discriminate between different retailers by giving only some of them a trade promotion. Our focus will be on the static aspects of discriminatory trade promotions. Second, retailers that receive a trade promotion will react by passing on part of it to consumers in the form of lower retail prices. Third, in the presence of discriminatory trade promotions consumers are aware that some retailers are likely to have lower prices than others and this will affect their search behaviour. By creating asymmetries between retailers and uncertainty for consumers regarding which retailer received a price promotion at a particular point in time, manufacturers induce consumers to actively search, reducing the market power of their retailers and increasing their profits as a consequence.

To gain insight into the mechanism, suppose that a manufacturer gives a trade promotion on its wholesale price to some retailers, but not to others, resulting in low and high retail prices in the downstream market. We compare this to the alternative scenario where a manufacturer does not discriminate between retailers and sets uniform wholesale prices, resulting in uniform retail prices. Expecting some price dispersion under trade promotions, consumers with different search costs will follow different search paths. This will affect retailers' demand as follows. High cost retailers that did not receive a trade promotion serve fewer consumers as consumers with lower search cost who happen to come to their store find it optimal to continue searching for lower prices. Thus, retailers that received a trade promotion serve relatively more consumers and, importantly, they get a relatively larger share of low search cost consumers who are more price sensitive. Competition between these low cost retailers for these more price sensitive consumers will induce them to lower their margins. But high cost retailers will also lower their margins. It is true that they will sell to relatively many high search cost consumers, but the important point is that they have a smaller customer base so that further raising their price will lead to a proportionally larger share of consumers leaving the firm to search for lower prices: from the consumers that search them first, given their higher wholesale price they do not have an incentive to prevent the consumers with the lowest search cost continuing to search, but given that these consumers leave, they have an increased incentive not to have also the consumers with moderate search costs leaving. Thus, both types of retailers, those that do and those that do not receive trade promotions, have lower margins. As lower retail margins *ceteris paribus* increase manufacturer profit, the manufacturer is better off by creating discriminatory trade promotions, while the market power of retailers is reduced.

For our mechanism to work it is important that (*i*) manufacturers are committed

to their wholesale prices through long-term contracts⁴ with their retailers so that they cannot quickly and opportunistically adjust their wholesale contracts without consumers and retailers noticing it, and that (*ii*) consumers with low search cost visit low cost retailers disproportionately more often. If the manufacturer was not committed to her wholesale prices through long-term contracts, she would have an incentive to (secretly) charge all retailers the same low wholesale price as she makes more profit per consumer over low cost retailers than over high cost retailers.

Given the existence of long-term contracts, a next issue is what consumers and retailers believe these contracts to be. In this paper we follow the standard approach and assume that both consumers and retailers have correct beliefs. Together with the manufacturer being committed to long-term contracts, correct beliefs imply that in the formal model we assume that retailers and consumers observe the structure of the wholesale arrangements the manufacturer offers. It is clear that certainly as far as consumers are concerned, this assumption will not literally hold in practice, but we see it as a reasonable approximation. In practice, consumers may learn through experience that there are discriminatory trade promotions and may well believe that the manufacturer engages in trade promotions guaranteeing that some retailers have lower prices than the regular price that is offered in the market. This belief is enough for low search cost consumers to continue to search when visiting a retailer with a regular price, while high search cost consumers will not. This creates the mechanism we are after where both low and high cost retailers compete more strongly and the manufacturer makes more profit than under uniform wholesale pricing. To formally determine the optimal wholesale arrangement, we use the assumption that consumers observe or are able to correctly learn the wholesale price arrangements between the manufacturer and retailers as there is no obvious way how to model incorrect beliefs and we do not want to create the impression that our mechanism depends on retailers and consumers having wrong beliefs.⁵

That relatively more consumers with low search cost visit low cost retailers is natural in many settings. The way we generate this in our model is that given consumer beliefs, the manufacturer is indifferent between any permutation of a given set of wholesale prices across retailers. Thus, the manufacturer may well randomize with equal probability over all permutations and randomly choose which retailer gets lower wholesale prices. Accordingly, consumers are uncertain about which retailers are low-cost and which retailers are high-cost and the consumer belief that any retailer could be a low-cost retailer with

⁴For example, Costello [2013] finds that especially in markets where relationship specific investments are important, firms engage in long-term contracts to avoid the hold-up problem.

⁵Another interpretation of our analysis is that it is part of a repeated game. If there is a (possibly small) probability that consumers find out whether or not the manufacturer engages in discriminatory trade promotions and would not believe the manufacturer discriminates in the future once it is discovered it has not done so in the past and if the manufacturer cares about long-term profits, then the manufacturer does not want to deviate from discriminatory trade promotions as the future loss in profits of not being able to discriminate outweighs the short-term gains of deviating.

equal probability is fulfilled. In practice, the manufacturer may achieve this by varying discounts across retailers over time. Consumers do not observe retail prices without incurring a search cost. This is a standard assumption in large parts of the consumer search literature and it is likely to be relevant in brick-and-mortar markets or in markets where price advertisement does not work well, because either consumers do not pay attention to these advertisements, or manufacturers impose minimum advertised prices (MAPs) preventing retailers to advertise low prices (see, e.g., Asker and Bar-Isaac [2020]). Thus, in our model, all consumers with a low search cost buy from low cost retailers. However, for the general mechanism to hold we only need that under discriminatory trade promotions a disproportionate share of consumers with low search cost visit low cost retailers.

In a static setting, one can interpret discriminatory trade promotions as wholesale price discrimination, which is customary in many important markets.⁶ In this way, our paper can also be seen as making a contribution to the policy debate on whether or not wholesale price discrimination should be forbidden. The European Union is strict and forbids wholesale price discrimination in most cases,⁷ while the Robinson-Patman Act, the main piece of legislation in the U.S., considers these practices to be illegal (only) if their effect “may be to substantially lessen competition”. Even though (some) retailers may complain that they are treated “unfairly”, our paper offers an explanation of why manufacturers may be shielded from being accused of anti-competitive behaviour as a numerical analysis shows that consumers, on average, may benefit from this unequal treatment. In addition, there is an additional positive welfare effect in case retailers differ in their intrinsic cost levels.

Two papers are closest in spirit to ours. Cui et al. [2008] focus on the ability of a manufacturer to exploit differences in retailers’ storage costs. By offering trade promotions to all, large retailers with lower storage costs will be able to buy more products at lower wholesale prices so that over time they will be able to impose a competitive constraint on their smaller competitors. Our explanation differs from this form of second-degree price discrimination in that we predict that the manufacturer purposefully picks out some, but not all, retailers to offer them a trade promotion and creates consumer uncertainty regarding which retailers currently received the trade promotion by choosing different retailers over time. Our explanation does not rely on retailer heterogeneity and also holds if retailers are identical. In addition, if retailers have different retail cost, we show that manufacturers may exacerbate these differences by setting a higher wholesale price to the more inefficient retailer.

⁶Empirical studies dealing with wholesale price discrimination are scarce due to the fact that the wholesale arrangement between manufacturers and retailers is typically not publicly observed. The few studies that explicitly study wholesale price discrimination include research on the coffee market in Germany (Villas-Boas [2009]) and on gasoline markets in the U.S.A. (Hastings [2009]).

⁷Article 102 (c) of the Treaty of the European Union forbids dominant firms to apply “dissimilar conditions to equivalent transactions with other trading parties, thereby placing them at a competitive disadvantage”.

Garcia and Janssen [2018] also study a setup with consumer search and retail discrimination. Our paper differs, however, in terms of the model setup, mechanism and results. Garcia and Janssen [2018] focuses on the interaction between one manufacturer and two retailers who sell to consumers that have the same positive search cost. Thus, under discriminatory trade promotions, the low cost retailer knows he is the lowest cost retailer and this gives him the power to choose for any wholesale price the retail monopoly price, while under uniform wholesale pricing the Diamond paradox arises at the retail level resulting in double marginalization in the overall market. The high cost retailer, on the other hand, gets a wholesale price that is equal to consumers' reservation price and he is fully squeezed. The competition effect of discriminatory trade promotion on low cost retailers that we focus on in this paper, is absent in Garcia and Janssen [2018]. This has important implications for the results. First, Garcia and Janssen [2018] show that it is optimal for the manufacturer to discriminate between retailers if the search cost and the fraction of shoppers is not too high. Importantly, uniform pricing is optimal if these conditions do not hold. In contrast, we show that discriminatory trade promotions are *always* optimal as long as the search cost distribution is continuous and there are more than two retailers. Second, with three retailers we show the conditions under which it is optimal to discriminate by giving one retailer a higher wholesale price than the other two.

The paper is also related to several strands of literature that are not yet mentioned. First, there is a recent literature on vertically related industries with consumer search. Janssen and Shelegia [2015] show that markets can be quite inefficient if consumers search sequentially while not observing the wholesale arrangement between the manufacturer and retailers. Importantly, and in contrast to our paper, the manufacturer always sets the same wholesale price to all retailers and retailers know this. Garcia et al. [2017] show that the inefficiency of vertical markets with consumer search continues to hold if there are many manufacturers and retailers engage in sequential search among these manufacturers. Lubensky [2017] shows that a manufacturer can use RRPs to signal his production cost to searching consumers. Asker and Bar-Isaac [2020] study different potential roles of minimum advertised prices (MAPs).

Finally, while most papers in the search literature assume at most two different levels of search cost (see, e.g., Stahl [1989]), there do exist some papers that consider more general forms of heterogeneity in consumers' search costs, such as Stahl [1996], Chen and Zhang [2011] and Moraga-González et al. [2017]. In contrast to these papers, however, we focus on vertically related industry structures and this paper is the first to consider general forms of search cost heterogeneity in such settings.

The main body of the paper analyses markets with linear wholesale pricing given that two-part tariffs, despite their theoretical appeal, are not often used in actual business transactions. Blair and Lafontaine [2015] state that, even in situations when two-part tariffs are adopted, the fixed component seems to be a relatively small part of the overall payment between firms (see, also, Kaufmann and Lafontaine [1994]). Differences in de-

mand expectations, in risk attitude, the possibility of ex-post opportunism by the supplier and wealth constraints by the retailers are mentioned among reasons why two-part tariffs are not often implemented in actual transactions. In an online Appendix, we show that our analysis is robust to manufacturers setting a fixed fee extracting part, but not all, of the retail profits.

The remainder of this paper is organized as follows. The next section presents the model, while Section 3 focuses on the main effects of discriminatory trade promotions on the retail market. Section 4 presents our main results on the profitability of discriminatory trade promotions. Section 5 determines the optimal wholesale contract for the case of three retailers. Section 6 discusses the equilibrium selection rule we applied in the retail market and shows that similar results continue to hold if another equilibrium selection rule would be applied. Section 7 concludes, while the proofs of the general results are presented in the Appendix.

2 The Model

We focus on a vertically related industry with a monopolist manufacturer (she) in the upstream market supplying a homogeneous product to $N \geq 3$ retailers (he/they).⁸ The manufacturer's production costs are normalized to zero. In principle, the manufacturer can charge a different wholesale price w_i to every retailer, so that formally the manufacturer's strategy is a tuple (w_1, w_2, \dots, w_N) . For given wholesale prices, an individual retailer i sets his retail price p_i , $i = 1, \dots, N$. Retailers take their wholesale price as given and do not face other costs except for the wholesale price paid to the manufacturer for each unit they sell.

There is a unit mass of consumers, each demanding $D(p)$ units of the good if they buy at price p . We make standard assumptions on the demand function so that it is well-behaved. In particular, there exists a \bar{p} such that $D(p) = 0$ for all $p \geq \bar{p}$ and the demand function is continuously differentiable and downward sloping whenever demand is strictly positive, *i.e.*, $D'(p) < 0$ for all $0 \leq p < \bar{p}$.⁹ For every $w \geq 0$, the retail monopoly price, denoted by $p^M(w)$ is uniquely defined by $D'(p^M(w))(p^M(w) - w) + D(p^M(w)) = 0$ and $D''(p)(p-w) + 2D'(p) < 0$. Note that for $w = 0$, this condition gives that the profit function of an integrated monopolist is concave. We denote by $p^M(w^M)$ the double marginalization retail price, which arises in case there would be a monopoly at both levels of the supply chain. In numerical examples, we consider demand to be linear, $D(p) = 1 - p$.

In order to observe prices consumers have to engage in costly sequential search with perfect recall. Consumers differ in their search cost s . Search costs are distributed accord-

⁸The effect of discriminatory trade promotions we focus on does not work with only two retailers. The reason is that if one retailer gets a lower wholesale price than the other, he has market power up to the retail price of the competitor. We will provide more details in Sections 3 and 5.

⁹Downward sloping demand is important for our analysis as under unit demand, the manufacturer always has an incentive to increase the wholesale price until it equals the consumers' willingness to pay.

ing to the distribution function $G(s)$, with $G(0) = 0$. We denote by $g(s)$ the density of the search cost distribution, with $g(s) > 0$ and consider that the search cost distribution has an increasing hazard rate, i.e., $g(s)/(1 - G(s))$ is non-decreasing in s , and that $g'(s)$ is bounded, i.e., there exists a finite M such that $-M \leq g'(s) \leq M$. In numerical examples, we take $G(s)$ to be uniformly distributed and in an online Appendix we show that our main qualitative results continue to hold for an exponential search cost distribution and for the Kumaraswamy distribution. As consumers are not informed about retail prices before they search, an equal share of consumers visits each retailer at the first search.¹⁰

A market is fully described by the number of retailers N , the demand function $D(p)$ and the search cost distribution $G(s)$. We compare uniform pricing to wholesale price discrimination in markets where the manufacturer is able to commit to wholesale prices through long-term contracts with her retailers.¹¹ This implies that under uniform pricing, the manufacturer chooses $w_i = w$ both on and off the equilibrium path. Under wholesale price discrimination the manufacturer may choose different prices to different retailers and we write \mathbf{w} for the vector of wholesale prices she chooses. As explained in the Introduction, we interpret our set-up as one where retailers and consumers *eventually* correctly anticipate the contractual arrangements set by the manufacturer. This allows us to analyse the retail market as a subgame¹² and write $p_i^*(w_i, \mathbf{w}_{-i})$ for the equilibrium retail price reaction of retailer i who has received the wholesale price w_i .

Definition 1 *An equilibrium with wholesale price discrimination is defined in two parts.¹³ First, for every \mathbf{w} we define a symmetric retail equilibrium as retail pricing strategies $p_i^*(w_i, \mathbf{w}_{-i})$ and an optimal sequential search strategy for all consumers such that (i) retailers maximize their retail profits given consumers' optimal search strategy and choose symmetric strategies in the sense that all retailers receiving the same wholesale price set the same retail price and (ii) consumers' sequential search strategy is optimal given their beliefs.¹⁴ Consumer beliefs are updated using Bayes' rule whenever possible. Second, given*

¹⁰For most part of the analysis, it does not matter whether or not the first search is costly. We proceed assuming the first search is for free and do not consider the participation constraint of consumers.

¹¹In a study of first-mover advantage, Bagwell [1995] has shown that a player's ability to commit is equivalent to the observability of his actions. In our world, with a manufacturer, multiple retailers and many consumers, the issue of commitment is more subtle as the manufacturer may commit to an individual retailer, or to retailers in general, without committing to consumers.

¹²The analysis in terms of subgames allows us to focus on the impact of discriminatory trade promotions on the retail market without having to consider the different beliefs retailers and consumers may have about wholesale contracts. The only belief that is relevant in this context is the belief of consumers about retail prices.

¹³As the equilibrium definition of the benchmark with uniform pricing is a special case, we skip that formal definition. Moreover, the definition only says that the search rule must be optimal without characterizing the rule. For the main results of the paper we only need to characterize the optimal search rule for the cases under consideration.

¹⁴It is difficult to characterize the optimal search rule for arbitrary retail prices and N retailers. For the equilibrium definition, we only need to be able to say that the rule has to be optimal without being able to characterize.

a symmetric retail equilibrium, the manufacturer chooses \mathbf{w} to maximize her profits.

This equilibrium definition does not specify consumers' out-of-equilibrium beliefs. The most natural assumption regarding beliefs in our context, and the one that has been followed in most of the consumer search literature, is that consumers have passive beliefs: after observing an out-of-equilibrium retail price consumers believe that the retailers that they have not yet visited charge their equilibrium prices. In the next section we will explain that in case of discriminatory trade promotions passive beliefs do not provide enough precision to determine consumers' optimal search behaviour as consumers also have to form expectations about the cost of the retailer that has deviated. In an online Appendix we analyse the consequences of alternatives to the assumption made in the next section that consumers believe that high cost retailers are to be blamed for deviations from equilibrium play.

3 The Retail Market

As explained in the Introduction, a manufacturer has an incentive to price discriminate between *ex ante identical* retailers as doing so creates a more competitive retail market. In this Section, we explain in detail the mechanism by characterizing the behaviour of consumers and retailers. We will do so for a specific case where the manufacturer chooses to set a low wholesale price w_L to $N - 1$ retailers and another higher wholesale price w_H to 1 retailer.¹⁵ We do this for two reasons. First, in the next section, we use this set-up to show that in general the manufacturer increases her profits by choosing discriminatory trade promotions. Second, in Section 5 we show that for $N = 3$, this is actually the optimal way to discriminate.

As a benchmark, consider first the case of uniform pricing where all retailers have the same wholesale price w^* . Let $p^*(w)$ denote the equilibrium price charged by all retailers (which is the retail price consumers expect). To determine, for a given w , the equilibrium retail price, we need to investigate how a retailer's demand depends on his own price, which in turn depends on how consumers' search behaviour reacts to a price deviation. If consumers buy at a deviation price $\tilde{p} > p^*(w)$, they get a surplus of $\int_{\tilde{p}}^{\bar{p}} D(p)dp$. Under passive beliefs, a consumer with search cost s continues to search for the equilibrium price $p^*(w)$, if the cost of doing so is smaller than the expected benefit, i.e., if $s < \int_{p^*(w)}^{\bar{p}} D(p)dp - \int_{\tilde{p}}^{\bar{p}} D(p)dp = \int_{p^*(w)}^{\tilde{p}} D(p)dp$.

¹⁵Focusing in this section only on two different wholesale prices enables us to have a tractable analysis of vertical markets. In many real-world examples the distribution of wholesale prices has been found to be bimodal with a high (regular) price and a lower (sale) price. This pattern is documented and is a salient feature of the well-known Dominick's database. For a specific example of a beer product we refer the reader to Figure 1 in Garcia et al. [2017]. The next Section argues that using this bi-modal structure, a manufacturer always does better than under uniform pricing, while Section 5 shows conditions when this structure is the best possible structure.

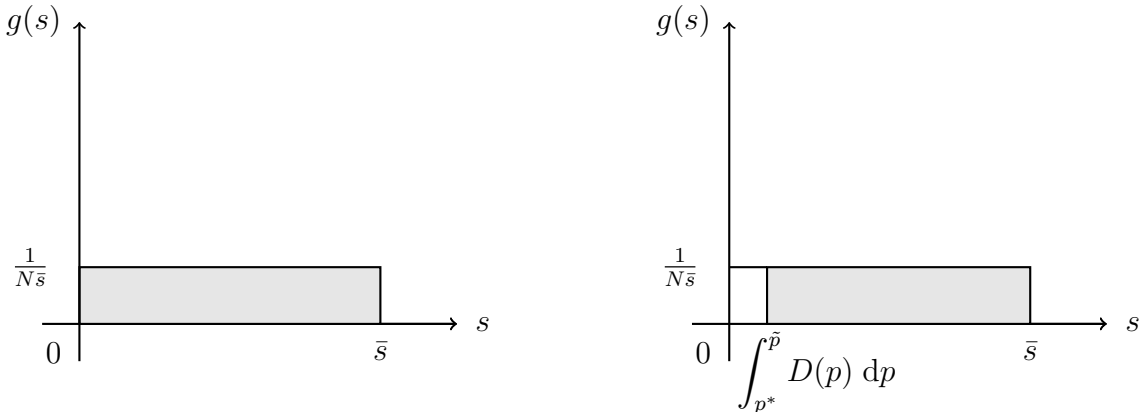


Figure 1: **Left:** Search cost composition of demand for a retailer under uniform pricing **Right:** Share of consumers that buy at the deviating retailer; where $s \sim U[0, \bar{s}]$.

Thus, of all consumers who visit a retailer deviating to a price $\tilde{p} > p^*(w)$, a fraction $1 - G\left(\int_{p^*(w)}^{\tilde{p}} D(p) dp\right)$ will continue buying from him. Therefore, the deviating retailer's profit in a uniform pricing equilibrium equals:

$$\pi_r(\tilde{p}, p^*) = \frac{1}{N} \left(1 - G\left(\int_{p^*(w)}^{\tilde{p}} D(p) dp\right) \right) D(\tilde{p})(\tilde{p} - w).$$

Maximizing retail profit and using the equilibrium condition $\tilde{p}(w) = p^*(w)$, yields

$$-g(0)D^2(p^*)(p^* - w) + D'(p^*)(p^* - w) + D(p^*) \leq 0. \quad (1)$$

The last two terms describe the standard monopoly condition for a retailer. The first (negative) term represents the search effect: if a retailer increases his price from the equilibrium price, some consumers, namely those with lower search cost, will walk away and buy elsewhere. Thus, for a given w the equilibrium retail price is smaller than the retail monopoly price. The impact of the search cost distribution also becomes clear: if $g(0)$ is large, monopoly considerations are relatively unimportant as the first term dominates whenever p^* is not close to w implying that the retail market is very competitive with small retail margins; if on the other hand $g(0)$ is small, the first term is relatively unimportant and retail prices are close to their monopoly levels. Note that in line with the search literature the first-order condition formally holds with an inequality only. This is due to the fact that retailers cannot attract more consumers by lowering their prices as consumers will only discover these lower prices by visiting the firm. In what follows, we focus on manufacturer-optimal equilibria where the FOC holds with equality and Section 6 discusses equilibrium selection in more detail. Finally, what is important for a later understanding of the impact of discriminating trade promotions is that the equilibrium retail price is determined by marginal considerations: if a retailer marginally increases his price, he will gain some margin, but also loses some consumers and in equilibrium these two effects offset each other.

Now we will explore the consequences of discriminatory trade promotions, where (con-

sumers expect that) one retailer gets a “regular” wholesale price w_H and the other $N - 1$ retailers buy the good from the manufacturer at a “discounted” w_L . The low and high cost retailers are expected to react to w_L and w_H by setting p_L^* and p_H^* , respectively. We first show that it will always be the case that retailers with higher wholesale prices set higher retail prices.

Lemma 2 *If retailers i and j receive wholesale prices $w_i < w_j$, then in any retail equilibrium it will be the case that $p_i^* \leq p_j^*$.*

As consumers do not know which retailer faces the higher wholesale price, they do not know which retailer has the higher retail price. The first effect of wholesale price discrimination on consumer search is that the low search cost consumers who happen to encounter the high cost retailer setting p_H^* will continue to search for lower retail prices. In particular, defining $\hat{s} = \int_{p_L^*}^{p_H^*} D(p)dp$, all consumers who happen to observe p_H^* at their first search and have a search cost $s < \hat{s}$ continue to search.

If the high-cost retailer sets the equilibrium price p_H^* his profit equals

$$\pi_r^{H*}(p_L^*, p_H^*, w_H^*) = \frac{1}{N} (1 - G(\hat{s})) D(p_H^*)(p_H^* - w_H),$$

while a low cost retailer’s profit when setting the equilibrium price is

$$\pi_r^L(p_L^*, p_H^*, w_L^*) = \frac{1}{N} \left[1 + \frac{G(\hat{s})}{(N-1)} \right] D(p_L^*)(p_L^* - w_L).$$

Importantly, each retailer gets $\frac{1}{N}$ of the consumers on their first search. Of these consumers that happen to visit the high cost retailer, a fraction $G(\hat{s})$ will continue to search and do not buy at that retailer, implying that all other retailers get a share $\frac{1}{N-1}$ of these consumers.

To determine the retail equilibrium prices p_L^* and p_H^* , we (again) have to consider how a deviation price \tilde{p} affects demand. As a consumer’s expected pay-off of continuing to search depends on whether he blames a low or high-cost retailer for the deviation, consumer beliefs regarding which retailer has deviated co-determine whether or not a deviation is profitable. For simplicity, in this section we assume that consumers believe that any deviation price is set by the retailer that received the high wholesale price. In the Appendix, we show that qualitatively our results continue to hold if consumers believe that local deviations around equilibrium prices come from retailers that deviate locally and we prove all our results, unless explicitly stated otherwise for these beliefs.¹⁶ Consider

¹⁶Probably the most reasonable assumption is that consumers blame a particular type of retailer for a local deviation from the equilibrium retail price. Because of the difference in cost, none of the retailers has an incentive to imitate the “other” retail price, and this extends to small neighbourhoods of these retail equilibrium prices. For simplicity, the exposition in this section has consumers always blaming high cost retailers.

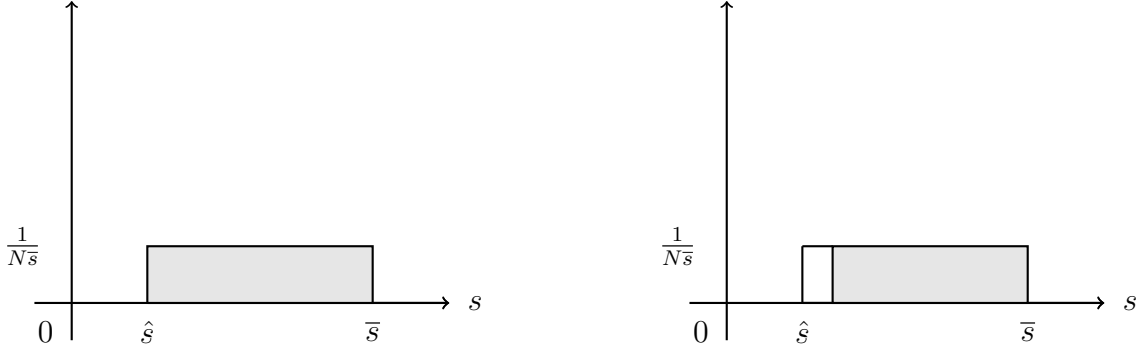


Figure 2: **Left:** Search cost compositions of demand for a high cost retailer. **Right:** Share of consumers that buy at the deviating high cost retailer.

first the determination of p_H^* . After observing a deviation price \tilde{p} , consumers will continue to search if their search cost is such that

$$s < \hat{s} + \int_{p_H^*}^{\tilde{p}} D(p) dp.$$

The left panel of Figure 2 illustrates the search cost composition of demand for the high cost retailer if he sets the equilibrium price: compared to the left side of Figure 1, it shows that the retailer has a smaller customer base as some low search cost consumers continue to search. The right panel of Figure 2 shows that some consumers with moderate search cost who would buy at p_H^* also continue to search for lower prices if the high cost retailer deviates to a higher price: only the ones in the grey area remain and buy from the deviating high cost retailer.

Therefore, the profit of a retailer who did not get a trade promotion and sets a price \tilde{p} in the neighbourhood of p_H^* will be:

$$\pi_r^H(\tilde{p}, p_H^*; w_H^*) = \frac{1}{N} \left(1 - G \left(\int_{p_L^*}^{p_H^*} D(p) dp \right) \right) D(p_H)(p_H - w_H). \quad (2)$$

Taking the first-order condition of (2) with respect to \tilde{p} and substituting $\tilde{p} = p_H^*$ yields that

$$-\frac{g(\hat{s})D^2(p_H^*)(p_H^* - w_H)}{1 - G(\hat{s})} + \left[D'(p_H^*)(p_H^* - w_H) + D(p_H^*) \right] = 0 \quad (3)$$

has to hold if there is a solution with $p_H^* > p_L^*$. In Section 6 we briefly comment on the situation where there is no solution with $p_H^* > p_L^*$ as this situation only arises if we consider alternative equilibrium selection rules.

Comparing this FOC condition with that in (1) reveals that *ceteris paribus* the only difference is that the first term is multiplied by the hazard rate $\frac{g(\hat{s})}{1 - G(\hat{s})}$ instead of $g(0)$. As this first term is negative, this implies that high cost retailers will have lower margins if $\frac{g(\hat{s})}{1 - G(\hat{s})} > g(0)$, which is the case as the search cost distribution has an increasing hazard rate.

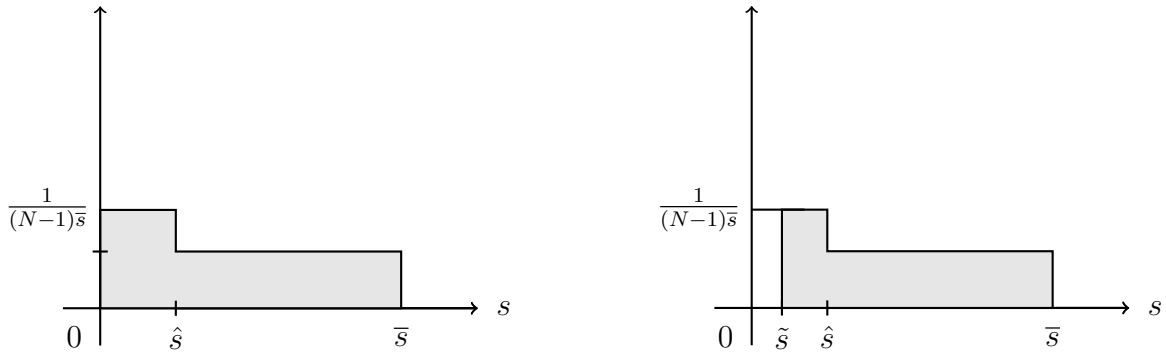


Figure 3: **Left:** Search cost compositions of demand for a low cost retailer. **Right:** Share of consumers that buy at the deviating low cost retailer.

Consider then a low cost retailer contemplating a deviation to a price $\tilde{p} > p_L^*$. Due to the fact that low search cost consumers continue to search if they observe p_H^* on their first search, low cost retailers will serve a disproportionately larger share of low search cost consumers. If there are only two retailers of which one gets a higher wholesale price than the other, as in Garcia and Janssen [2018], this larger share of low search cost consumers has little impact on the pricing behaviour of the low cost retailer as it has full market power up to the retail price of its high cost competitor. However, if there is another low cost retailer, then the strategic situation is very different. In that case, if they deviate and increase their prices, low-cost retailers are losing relatively many consumers to their low-cost competitor(s) as low search cost consumers are very price sensitive. For this competition effect to arise, there should be at least three retailers in total so that under discriminatory trade promotions, the manufacturer can give at least two retailers the promotion, each having a low-cost competitor.

The number of consumers a low cost retailer attracts is derived as follows. The left panel of Figure 3 illustrates the search cost composition of demand for the low cost retailer if he sets the equilibrium price. The low cost retailer serves consumers of all search cost types, namely $\frac{1}{N}$ of all consumers who visit him on their first search. In addition, his demand now consists of many more low search cost consumers as compared to uniform pricing: an additional fraction of $\frac{1}{N(N-1)}$ of consumers with a search cost smaller than $\int_{p_L^*}^{p_H^*} D(p)dp$ first visits the high-cost retailer and then visit the retailer on their second visit. Thus, in total a low cost retailer attracts a fraction $\frac{1}{N} + \frac{1}{N(N-1)} = \frac{1}{N-1}$ of the consumers with the lowest search cost. If a low-cost retailer deviates, consumers still buy from them if their search cost is larger than $\tilde{s} \equiv \int_{p_L^*}^{\tilde{p}} D(p)dp$. This is illustrated in the right panel of Figure 3. In the figure we can see that the low search cost consumers (area marked in white) will continue to search for lower prices if a low cost retailer deviates to a higher price: only the ones in the grey area remain.

Thus, when deviating to a price $\tilde{p} < p_H^*$, a low cost retailer's profits will be:

$$\frac{1}{N} \left[1 - G \left(\int_{p_L^*}^{\tilde{p}} D(p) dp \right) + \frac{G \left(\int_{p_L^*}^{p_H^*} D(p) dp \right) - G \left(\int_{p_L^*}^{\tilde{p}} D(p) dp \right)}{(N-1)} \right] D(\tilde{p})(\tilde{p} - w_L), \quad (4)$$

where the second term in the square brackets refers to the consumers who first visited the retailer and then continue to search, while the third term refers to the consumers who first visited the high cost retailer and buy from the deviating low-cost retailer even though he charges a higher price.

Taking the first-order condition of (4) with respect to \tilde{p} and evaluating it at the equilibrium value yields:

$$- \frac{Ng(0) D^2(p_L^*)(p_L^* - w_L)}{(N-1) + G(\hat{s})} + \left[D'(p_L^*)(p_L^* - w_L) + D(p_L^*) \right] \leq 0, \quad (5)$$

where for the same reason as in (1), the first-order condition formally holds with an inequality only; see Section 6 for further discussion.

Thus, as $\frac{Ng(0)}{(N-1)+G(\hat{s})} > g(0)$, also the low cost retailers have lower margins relative to the uniform pricing case. Intuitively, low cost retailers have a disproportionately large share of low search cost consumers who are price sensitive. In order to prevent them from continuing to search, retailers are willing to lower their margins. It is important to understand this in marginal terms: because each individual low cost retailer has $(1 + \frac{1}{N-1})$ times more consumers with very low search costs relative to $(1 + \frac{G(\hat{s})}{N-1})$ times more consumers in total, a low cost retailer loses relatively more consumers when he deviates and increases his price. In other words, his demand is now more price elastic, resulting in lower margins.

4 Profitability of Discriminatory Trade Promotions

To show that discriminatory trade promotions increase the manufacturer's profits, consider first the benchmark of uniform pricing. Here, the manufacturer chooses one wholesale price w (both on and off the equilibrium path). Normalizing production costs to zero, the profit of the manufacturer under uniform pricing is simply given by $wD(p(w))$ and the equilibrium uniform wholesale price w^* is set such that

$$\frac{\delta \Pi^M}{\delta w} = wD'(p^*(w)) \frac{\partial p^*}{\partial w} + D(p^*(w)) = 0, \quad (6)$$

where $\frac{\partial p^*}{\partial w}$ follows the comparative statics of the retail equilibrium condition (1).

Consider next that the manufacturer charges one retailer a slightly higher price than the optimal wholesale contract under uniform pricing. Thus, denote by $w_L = w^*$ the price set to $N-1$ retailers and $w_H = w^* + \varepsilon$ the wholesale price set to one retailer. Retailers react by setting $p_L(w_L, w_H)$ and $p_H(w_L, w_H)$. The manufacturer's profit can then be written as

$$\frac{N-1 + G \left(\int_{p_L}^{p_H} D(p) dp \right)}{N} w_L D(p_L(w_L, w_H)) + \frac{1 - G \left(\int_{p_L}^{p_H} D(p) dp \right)}{N} w_H D(p_H(w_L, w_H)).$$

This expression can be understood as follows. Initially, each retailer receives $\frac{1}{N}$ of the consumers on their first search. Of the consumers who happen to visit the high cost retailer, the ones with a low enough search cost continue to search. As in Section 3, a fraction $G\left(\int_{p_L}^{p_H} D(p)dp\right)$ of consumers who first visit the high cost retailer continue to search and buy from a low cost retailer on their next search. The per consumer profit the manufacturer makes over each type of retailer is, of course, $w_i D(p_i(w_L, w_H))$, $i = L, H$.

We will argue that the first-order effect with respect to w_H is positive if evaluated at $w_L = w^*$ and $w_H = w^* + \varepsilon$, $\varepsilon > 0$. The first-order effect is given by

$$\begin{aligned} & \frac{N-1 + G\left(\int_{p_L(w^*, w^* + \varepsilon)}^{p_H(w^*, w^* + \varepsilon)} D(p)dp\right)}{N} w_L D(p_L(w^*, w^* + \varepsilon)) - \frac{N-1}{N} w^* D(p^*(w^*)) \\ & + \frac{1 - G\left(\int_{p_L(w^*, w^* + \varepsilon)}^{p_H(w^*, w^* + \varepsilon)} D(p)dp\right)}{N} w_H D(p_H(w^*, w^* + \varepsilon)) - \frac{1}{N} w^* D(p^*(w^*)) \\ = & \frac{N-1}{N} w^* [D(p_L(w^*, w^* + \varepsilon)) - D(p^*(w^*))] \\ & + \frac{G\left(\int_{p_L(w^*, w^* + \varepsilon)}^{p_H(w^*, w^* + \varepsilon)} D(p)dp\right)}{N} [w^* D(p_L(w^*, w^* + \varepsilon)) - (w^* + \varepsilon) D(p_H(w^*, w^* + \varepsilon))] \\ & + \frac{1}{N} [(w^* + \varepsilon) D(p_H(w^*, w^* + \varepsilon)) - w^* D(p^*(w^*))]. \end{aligned}$$

This expression contains three terms representing the manufacturer's gains from discriminatory trade promotions relative to uniform pricing. The first term represents the gains over all retailers that receive the low wholesale price under discriminatory trade promotions holding the number of consumers visiting a retailer constant, while the third term represents the same gains over the retailer receiving the high wholesale price. The middle term then represents the gain from a fraction of consumers who first visit the high cost retailer who continue searching and eventually buy from a low cost retailer.

It is clear that all three terms in the expression are positive if $D(p_L(w^*, w^* + \varepsilon)) > D(p_H(w^*, w^* + \varepsilon)) > D(p^*(w^*))$, which is the case if $p_L(w^*, w^* + \varepsilon) < p_H(w^*, w^* + \varepsilon) < p^*(w^*)$. In Section 3 we have shown that under discriminatory trade promotions low and high cost retailers have lower margins than under uniform pricing, but we have not considered in detail how retailers react to a small change in one of the wholesale prices. The following Lemma states that retail prices are discontinuous at $(w_L, w_H) = (w, w)$ and that the above inequalities hold.

Lemma 3 *For any $G(s)$ and for all (w_L, w_H) with $w_L = w$ and $w_H = w + \varepsilon$ (for ε small enough) such that a retail equilibrium exists, there exists a $k > 0$ such that $p_L^*(w_L, w_H) + k < p_H^*(w_L, w_H) + k < p^*(w)$.*

It is important to explain why the discontinuity arises. The equilibrium retail prices are determined by the fact that given the prices of the other retailers, each individual retailer loses so many consumers if he marginally increases his price that this decrease

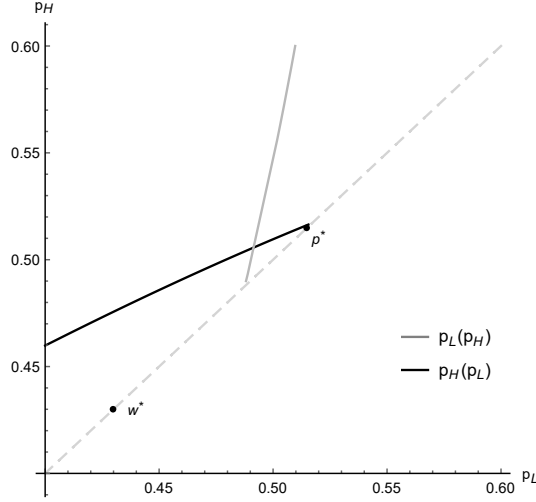


Figure 4: Retailers' behaviour for marginal deviations from uniform wholesale prices, when $\bar{s} = 0.05$, $w^* = 0.4299$ and $p^* = 0.5149$

just offsets the increase in margin over all consumers who stay. For any $\hat{s} > 0$ retailers with a trade promotion get more than the average number of low search cost consumers and these consumers are very price sensitive and the first to continue to search. Thus, relative to uniform pricing, of all consumers who consider buying from a retailer with a trade promotion, more consumers are leaving if the retailer marginally increases his price. In order to prevent them from continuing to search, retailers who got a trade promotion are willing to accept lower margins. Note also that this effect is discontinuous relative to uniform pricing because each retailer that got a trade promotion gets $\frac{1}{N-1}$ more low search cost consumers for every $\hat{s} > 0$.

The discontinuity is illustrated in Figure 4. The two dots on the 45 degree line represent the equilibrium retail price $p^*(w)$ and the equilibrium wholesale price w^* under uniform pricing. The two “reaction curves”¹⁷ represent (3) and (5) for $w_L = w$ and $w_H = w + \varepsilon$, where $\varepsilon = 0.001$. It is clear that the prices are strategic complements. For $p_L = p^*(w)$ the reaction curve for the high cost retailer shows that the optimal reaction to $w_H = w + \varepsilon$ is a slight increase in p_H . As some low search cost consumers that first visit the high cost retailer will now continue to search for low prices, the low cost retailers with $w_L = w$ now charge a p_L that is strictly (and discontinuously) smaller than $p^*(w)$. Because of strategic complementarity, the intersection point of the “two reaction curves” has both prices strictly smaller than $p^*(w)$.

Thus, we have our main result:

Theorem 4 *If a retail equilibrium exists, the manufacturer makes strictly more profit under discriminatory trade promotions compared to uniform pricing.*

This is a strong result as it is independent of the specific shape of the demand curve

¹⁷Note that these are not real reaction curves as we have imposed the equilibrium condition that in equilibrium the low cost retailers should set the same price.

or the shape of the search cost distribution. The result does not specify, however, the optimal form of discriminatory trade promotions. In the next section, we will characterize optimal discriminatory trade promotions for the special case where $N = 3$.

In the online Appendix, we show that the result continues to be true if there are initial cost differences between retailers and consumers do not know which retailer has a higher cost. Thus, and unlike the literature on wholesale price discrimination that shows that a manufacturer price discriminates to *reduce* the natural cost differences between retailers (Katz [1987], DeGraba [1990] and Yoshida [2000]), in our case a manufacturer may further exacerbate initial cost differences between retailers shifting more demand to the inefficient retailer.

5 Optimal Wholesale Contracts with Three Retailers

We now analyse optimal wholesale contracts and the implications for consumer welfare and retail profits. It should be clear that it is not optimal for the manufacturer to induce a retail equilibrium where $\hat{s} \geq \bar{s}$. If that would be an equilibrium, retailers receiving a high wholesale offer would be effectively foreclosed from the market, putting the remaining retailers in a similar position as under uniform pricing with the exception that the remaining effective retailers will charge higher margins as consumers (not knowing which retailer got a high wholesale price) are less inclined to continue searching if they observe an off-equilibrium retail price. As this can never be optimal for the manufacturer, it must be the case that $0 \leq \hat{s} \leq \bar{s}$.

To characterize the optimal wholesale contract for an arbitrary number of retailers is beyond the scope of this paper as it is difficult to characterize the optimal search rule in general and how many retailers should get identical wholesale prices. This is different in case there are three retailers. In that case, there are only a limited number of treatments: either two retailers get a trade promotion, while one gets an unfavourable treatment, or only one retailer gets a trade promotion, while the other two get an unfavourable treatment, or all get different wholesale prices. Intuitively, from the manufacturer's perspective the last two treatments have two disadvantages compared to the first treatment and the one we have assumed so far. First, if the manufacturer only gives one retailer a lower wholesale price, then this retailer has full market power up to the retail price of the second-lowest retail price leading to unnecessary high margins for this low-cost retailer and therefore to lower profits for the manufacturer. Second, consumers will be less inclined to continue to search if they have first visited one of the higher-priced retailers, because it may take them more than one search to find the lowest-priced retailer and in that case they have to pay the search cost twice. Again, this will give high cost retailers more incentives to increase their margins compared to the high cost retailer in a market where all other retailers charge lower retail prices as they have lower wholesale prices.

For search cost distributions that are sufficiently concentrated around 0, meaning that

$g(0)$ is sufficiently large, we can formalize these intuitions and also characterize the retail margins and the fraction of consumers that continue searching if they first visit the high cost retailer.

We state the result for the uniform distribution where $g(0) = 1/\bar{s}$ and the fraction of searching consumers equals \hat{s}/\bar{s} .

Proposition 5 *For $N = 3$ and search cost distributions that are sufficiently concentrated around 0, it is optimal for the manufacturer to offer w_L to 2 retailers and $w_H > w_L$ to one retailer. If $\bar{s} \rightarrow 0$, the wholesale prices w_L^* and w_H^* and retail prices p_L^* and p_H^* converge to w^* and p^* , with $p^* = w^*$ solving $w^*D'(w^*) + D(w^*) = 0$, while $\hat{s}/\bar{s} = \frac{37}{340}$ and the retail margins are given by:*

$$\begin{aligned}\frac{d(p_L^* - w_L^*)}{d\bar{s}} &= \left(\frac{3}{140} + \frac{15}{17} \right) \frac{1}{D(p_L^*)} < \\ \frac{d(p_H^* - w_H^*)}{d\bar{s}} &= \left(-\frac{1}{20} + \frac{16}{17} \right) \frac{1}{D(p_L^*)} < \frac{1}{D(p_L^*)} = \frac{d(p^* - w^*)}{d\bar{s}}.\end{aligned}$$

Thus, the result confirms that relative to uniform pricing both retailers make lower margins under discriminatory trade promotions. The result also shows that around 11% of consumers who first visit the high cost retailer continue to search.

If the manufacturer offers two retailers a lower wholesale price than the third retailer, she will choose wholesale prices w_L and w_H to maximize the following profit function:

$$\Pi^M(w_L, w_H) = \frac{\bar{s} - \hat{s}}{3\bar{s}} w_H D(p_H^*(w_L, w_H)) + \frac{2\bar{s} + \hat{s}}{3\bar{s}} w_L D(p_L^*(w_L, w_H)). \quad (7)$$

By setting w_L and w_H optimally, the manufacturer takes into account how both retail prices change in reaction to changes in w_L and w_H . In general, it is not optimal to set w_H marginally higher than w_L , as we considered in the previous section. The reason is that if she would do so, the manufacturer gains more profit per transaction over the retailers receiving the trade promotions and then it is optimal to increase the fraction of consumers who buy at the lowest price by making more consumer search by increasing the size of the trade promotion.

Optimality requires that the first-order conditions with respect to w_L and w_H are satisfied:

$$\begin{aligned}0 &= (w_L D(p_L^*) - w_H D(p_H^*)) \left(D(p_H^*) \frac{\partial p_H^*}{\partial w_H^*} - D(p_L^*) \frac{\partial p_L^*}{\partial w_H^*} \right) \\ &\quad + (\bar{s} - \hat{s}) \left[D(p_H^*) + w_H D'(p_H^*) \frac{\partial p_H^*}{\partial w_H^*} \right] + (2\bar{s} + \hat{s}) w_L D'(p_L^*) \frac{\partial p_L^*}{\partial w_H^*},\end{aligned} \quad (8)$$

and

$$\begin{aligned}0 &= (w_L D(p_L^*) - w_H D(p_H^*)) \left(D(p_H^*) \frac{\partial p_H^*}{\partial w_L^*} - D(p_L^*) \frac{\partial p_L^*}{\partial w_L^*} \right) \\ &\quad + (2\bar{s} + \hat{s}) \left[D(p_L^*) + w_L D'(p_L^*) \frac{\partial p_L^*}{\partial w_L^*} \right] + (\bar{s} - \hat{s}) w_H D'(p_H^*) \frac{\partial p_H^*}{\partial w_L^*}.\end{aligned} \quad (9)$$

Thus, under discriminatory trade promotions there are three effects of a change in a wholesale price. First, there is the direct effect that a change in either w_L or w_H has on the profit a manufacturer makes over the retailer in question. This effect is represented by the second term in the above equations. Importantly, however, there are also two indirect effects. A second (indirect) effect, represented by the third term, is that an increase in w_L , respectively w_H leads to an increase in the retail prices of the other type of retailer, indirectly lowering the manufacturer profits. For example, an increase in w_L raises p_L^* and thereby decreases the incentives of consumers that first visit the high cost retailer to continue searching. This increases the market power of the high cost retailer and the price he charges. Similarly, an increase in w_H increases the incentives of consumers that first visit the high cost retailer to continue searching, but as these are not the marginal consumers that would continue searching if the low cost retailer deviates, it increases the market power of the low cost retailers and the retail prices they set. Third, (and the second indirect effect), represented by the first term, the profit per consumer the manufacturer makes over the different retailers may not be equal, with the manufacturer generally making more profit over the low cost retailers than over the high cost retailers. Thus, by choosing optimal wholesale prices and the size of the trade promotion, the manufacturer must take into account how many consumers will continue to search and buy at the respective prices.

Figure 5, below, illustrates a typical example of the different manufacturer profit functions in case of optimal wholesale contracts. The lower solid curve illustrates, as a benchmark, the profit function $wD(p(w))$ under uniform wholesale pricing, where w^* directly maximizes this expression. The other two curves represent the per consumer profit the manufacturer makes over the low and the high cost retailer, $w_LD(p_L^*(w_L, w_H^*))$, respectively $w_HD(p_H^*(w_L^*, w_H))$. A first notable aspect of Figure 5 is that (in line with Proposition 4) the manufacturer makes more per consumer profit over both types of retailers than under uniform pricing. A second important aspect is that the choices of w_L^* and w_H^* do not directly maximize the per consumer profits $w_LD(p_L^*(w_L, w_H^*))$, respectively $w_HD(p_H^*(w_L^*, w_H))$. In particular, the equilibrium levels of w_L^* and w_H^* are at the point of the curves where $w_LD(p_L^*(w_L, w_H^*))$ and $w_HD(p_H^*(w_L^*, w_H))$ are increasing. A final aspect is that $w_L^*D(p_L^*(w_L^*, w_H^*)) > w_H^*D(p_H^*(w_L^*, w_H^*))$.

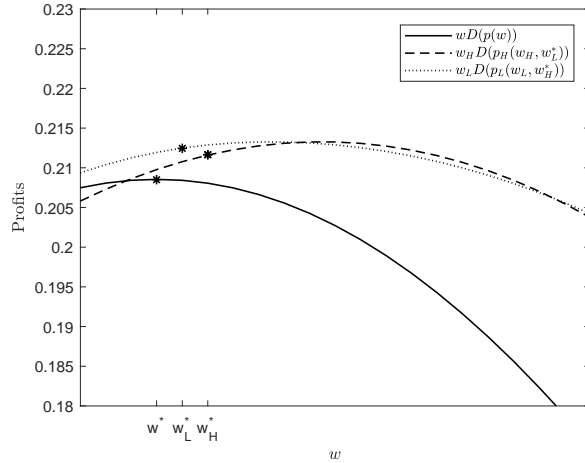


Figure 5: Manufacturer's Profit for $\bar{s} = 0.05$, when $w^* = 0.4299$ and $p^* = 0.5149$

The interaction of these three effects implies that it is difficult to analytically characterize the equilibrium with discriminatory trade promotions beyond stating the F.O.C.s that need to be satisfied.

Next, we show that in the special case of a uniform search cost distribution and linear demand, we can actually go further and claim that giving two retailers a lower and one a higher wholesale price is always optimal, even if the search cost distribution is not concentrated around 0.

Proposition 6 *If the search cost distribution is uniform and demand is linear, the manufacturer's optimal pricing policy is to give two retailers the lowest wholesale price and one retailer a higher wholesale price.*

The proof we develop in the Appendix builds on the intuition provided in the beginning of this section. We compare the manufacturer's profit under uniform pricing and the profit when giving two retailers a higher wholesale price. We argue that the latter can only be higher than the former if the retailer with the lowest wholesale price sets the retail monopoly price as otherwise he charges the same price as the other two retailers and their margins would be identical to the retail margins under uniform pricing (while the retail margin of the retailer with the lowest wholesale price is larger). As a consequence, the only way uniform pricing may result in lower prices is, if the manufacturer makes more profit per consumer over the two retailers with the higher wholesale price. We use this to create an artificial upper bound on the manufacturer's profit in case he sets a higher wholesale price, namely one where no consumer would continue to search for lower retail prices. For any uniform search cost distribution, we can compute the manufacturer's profit under uniform pricing and the profit at this upper bound and show that the former is larger than the latter. As we know that trade promotions where two retailers get a lower wholesale price yield higher manufacturer profits than uniform pricing, the result

follows. As the argument is not very tight and uses several approximations, it is clear that a similar result should also hold for many other search cost distributions and demand functions. However, it is difficult to prove this analytically.

Given the above Proposition, we can now numerically analyse the consequences of optimal trade promotions. Figure 6(Left) shows that depending on how concentrated the search cost distribution is, the manufacturer can increase profits by up to 2.6% if it engages in discriminatory trade promotions. To obtain maximal profits, the figure on the right shows that both wholesale prices under discriminatory trade promotions can both be lower or higher than under uniform prices. Figure 6(Right) also shows that all wholesale prices are non-monotonic in the search cost parameter. This is quite intuitive: if the search cost distribution is concentrated around 0, the retail market is very competitive and the manufacturer wants to set the monopoly price of the integrated monopolist. On the other hand, when search costs can also be very large, retailers almost have monopoly power and under linear demand, the optimal reaction of the manufacturer is to set the same wholesale price.

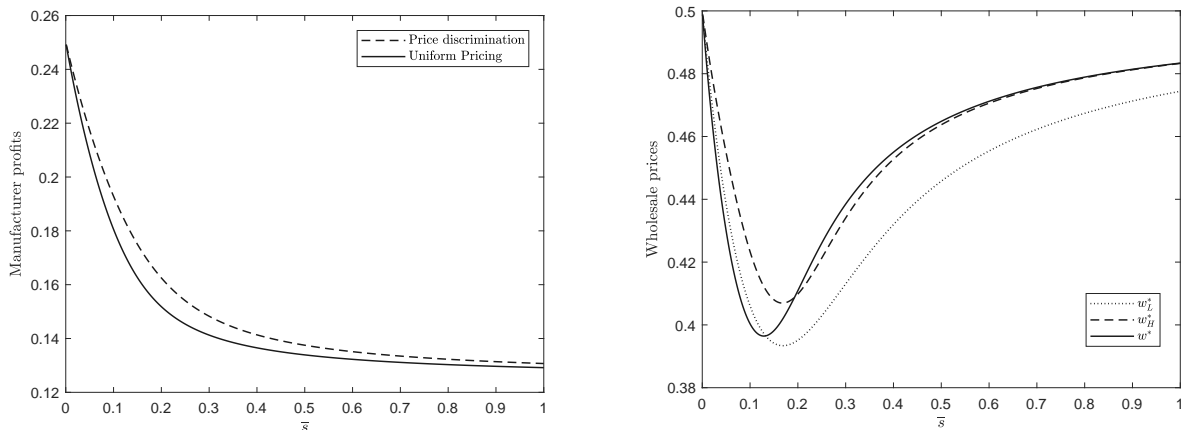


Figure 6: $N = 3$ and $D(p) = 1 - p$. **Left:** Manufacturer's profit for different values of \bar{s} . **Right:** Wholesale prices for different values of \bar{s} .

Figure 7 depicts the profits of retailers under discriminatory trade promotions relative to uniform pricing. Note that discriminatory trade promotions have two different effects on the low cost retailers' profits. First, given that low cost retailers sell to more price sensitive consumers, their margins are lower, which pushes their profits down. Second, low cost retailers sell to more consumers compared to uniform pricing, which pushes their profits up. The figure shows that the latter effect may dominate when consumers' search costs are non-negligible. In such instances, the low cost retailers make higher profits under discriminatory trade promotions and thus would have no incentive to advertise that they have the lowest prices in the market. On the other hand, high cost retailers do not have an incentive to advertise their prices either as consumers would simply divert their search to other retailers.

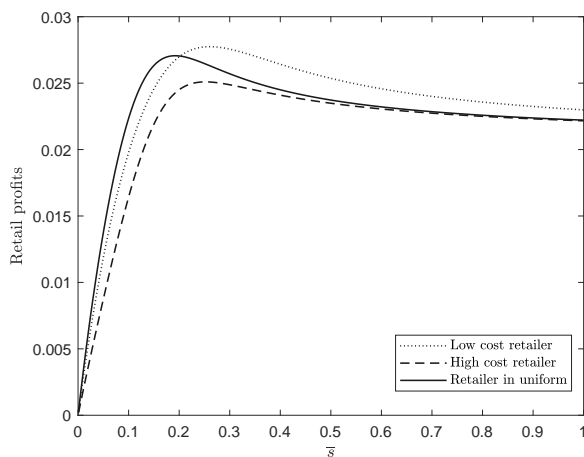


Figure 7: Retailer Profits for different values of \bar{s} , when $N = 3$ and $D(p) = 1 - p$.

We next analyse the effect on consumer surplus. Figure 8(Left) shows that the average consumer surplus including search cost is higher under discriminatory trade promotions relative to uniform pricing.¹⁸ The difference can be approximately 8% (for $\bar{s} \approx 0.2$). Figure 8(Right) shows that, depending on the support of the search cost distribution, all consumers are better off (as all retail prices are lower) under discriminatory trade promotions, or that only consumers that buy at the high retail price are worse off.

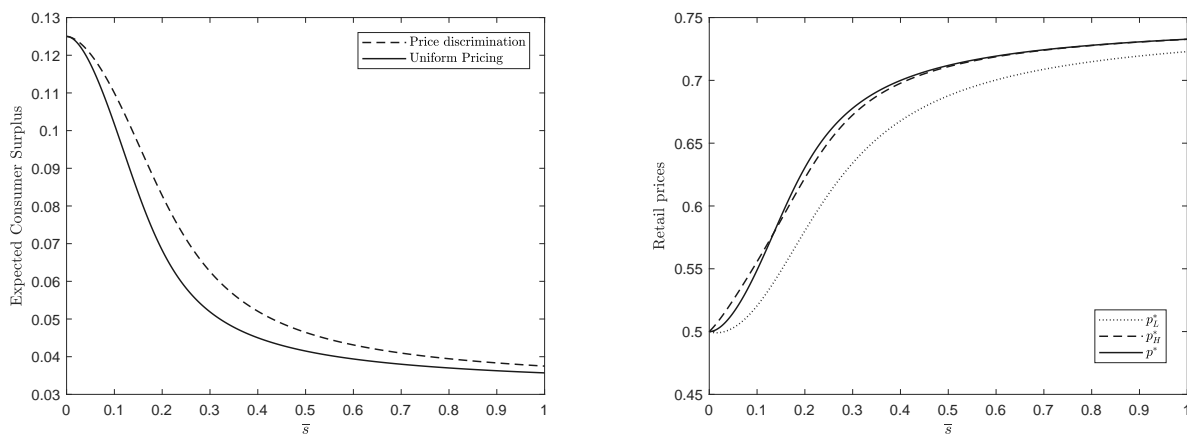


Figure 8: $N = 3$ and $D(p) = 1 - p$. **Left:** Expected Consumer Surplus for different values of \bar{s} . **Right:** Retail Prices for different values of \bar{s} .

¹⁸A numerical analysis for other search cost distributions (such as the exponential distribution and the Kumaraswamy distribution) is provided in an online Appendix.

6 Equilibrium Selection

In this section, we briefly discuss the issue of multiplicity of equilibria in the retail market and the equilibrium selection rule we apply. We first do so by considering the retail equilibrium price under uniform wholesale pricing. If we consider w to be exogenously given, then (1) determines a range of equilibrium retail prices $p^*(w)$ with $p^M(w)$ as the highest possible equilibrium retail price and the lowest possible equilibrium price, the price at which (1) holds with equality. The equilibrium retail price results from a combination of retailers' incentives and consumer beliefs. In particular, for any retail price consumers expect retailers to charge, they will always buy immediately from the first retailer they visit if this firm charges a price that is not higher than this expected retail price. This implies that up to the expected retail price, each retailer is a monopolist. For instance, if for a given w consumers expect all retail prices to be at least equal to $p^M(w)$, then it is optimal for retailers to charge $p^M(w)$. However, if consumers expect all retail prices to be such that (1) holds with equality, then retailers have an incentive to charge a retail price that makes this true. It is clear that for a given w the manufacturer makes more profit, the lower the retail equilibrium price is, and the lowest possible equilibrium price is the one where (1) holds with equality.

Similarly, considering discriminatory trade promotions, (5) may hold with strict inequality as retailers cannot attract more consumers by lowering their prices. Again this implies that there is a continuum of possible retail prices for the low-cost retailers to charge with $p^M(w_L)$ their highest possible equilibrium retail price and the price at which (5) holds with equality as their lowest possible equilibrium price. On the other hand, for any $p_H > p_L$ (3) has to hold with equality as the high cost retailer can increase the number of consumers that buy by lowering the retail price.

In the previous sections, for both uniform wholesale pricing and discriminatory trade promotions we have considered the retail equilibrium that yields the highest profit for the manufacturer, i.e., the equilibria where the respective FOCs hold with equality. As the focus of the paper is on the manufacturer's profits, this is the most interesting comparison to make. The results of the previous sections imply that the maximal profit a manufacturer can make under discriminatory trade promotions is larger than the maximal profit under uniform wholesale pricing.

We now show, however, that a similar result holds true if we would consider the retail equilibrium that yields the highest profits for the retailers and the lowest profit for the manufacturer for both uniform wholesale pricing and discriminatory trade promotions.¹⁹ If under uniform pricing, retailers choose $p^M(w)$ for any w , then the manufacturer chooses w such that we get double marginalization with $p^M(w^M)$ as the resulting retail price. In

¹⁹The considerations below reveal some complications that arise when considering other possible retail equilibria as, for example, we need to take into account that in an equilibrium $p_H \geq p_L$. This is automatically satisfied when we consider that the FOCs in the retail market hold with equality and $w_L < w_H$.

this equilibrium, the last two terms of (1), which describe the standard monopoly condition for the retailers, are equal to 0 and because the first term is negative, the whole expression is strictly negative. Consider then discriminatory trade promotions with $w_L = w^M$ and $w_H = w^M + \varepsilon$ for some small enough $\varepsilon > 0$ and the low-cost retailers also coordinating on $p^M(w^M)$. As under uniform pricing, (5) holds with strict inequality as the last two terms are equal to 0 and the first term is negative. The important consideration now is that it follows from (3) that for any small enough $\varepsilon > 0$ the high cost retailer will not choose $p^M(w^M + \varepsilon)$, and in fact will not choose a price larger than $p^M(w^M)$. The reason is that the first term in (3) is strictly smaller than 0, while for any small enough $\varepsilon > 0$ and any $p > p^M(w^M)$ the last two terms of (3) are either non-positive (for $p \geq p^M(w^M + \varepsilon)$) or an arbitrary small, positive number (for $p^M(w^M) < p < p^M(w^M + \varepsilon)$).²⁰ It then follows that if the manufacturer chooses $w_H = w^M + \varepsilon$ for any small enough $\varepsilon > 0$ the high cost retailer will not want to raise the retail price above $p^M(w^M)$ as for any $p > p^M(w^M)$ (3) is strictly negative. On the other hand, the high-cost retailer will not set a retail price $p_H < p^M(w^M)$ as he will not attract more consumers by doing so. It follows that it is optimal for the high cost retailer to set the same retail price as the other retailers, *i.e.*, $p_H(w^M + \varepsilon) = p^M(w^M)$. But this then implies that the manufacturer makes more profit by discriminating between retailers as she can make sure that retail prices (and thus demand) are identical under uniform and discriminatory wholesale pricing, while one of her wholesale prices is strictly larger.

The above argument shows that for any search cost distribution and for any demand function, the manufacturer obtains more profit by discriminating between retailers even if we consider the equilibrium where retailers coordinate on the retail equilibrium where the manufacturer makes the lowest profit. Without addressing the optimal scheme to discriminate between retailers in this case, it may be illuminating to investigate by how much the manufacturer could increase w_H to keep the high-cost retailer charging $p^M(w^M)$. We do so by considering the linear demand case and the uniform search cost distribution we considered in the previous section. Under uniform pricing, we have $w^M = 1/2$ and $p^M(w^M) = 3/4$ so that the manufacturer's profit equals $1/8$. For linear demand and a uniform search cost distribution, (3) becomes

$$-(1 - p_H)^2(p_H - w_H) + (1 - 2p_H + w_H)(\bar{s} - \hat{s}) = 0.$$

To see how far the manufacturer could increase w_H to keep the high-cost retailer from raising the retail price above $3/4$, we can substitute $p_H = 3/4$ and $\hat{s} = 0$ to get $w_H = \frac{8\bar{s} + \frac{3}{4}}{16\bar{s} + 1}$. This expression ranges from $1/2$ to $3/4$ when \bar{s} varies from ∞ to 0 . Thus, depending on how concentrated the search cost distribution is, the manufacturer can squeeze the high cost retailer quite substantially and increase profits. For example, when \bar{s} is close to 0 , by keeping the same retail prices manufacturer profit can be as large as $7/48$ which is a 16.6% increase compared to the profit under uniform pricing.

²⁰The last point follows from the standard assumption that $D''(p)(p - w) + 2D'(p) < 0$.

Note that in the above considerations, consumers do not search beyond the first firm. Where in the previous sections, the manufacturer uses consumer search to reduce the margin of the high cost retailer, in this section it is the *threat of consumer search* that makes that the high cost retailer does not want to increase retail prices.

7 Discussion and Conclusion

The focus of this paper is on the interaction between wholesale and retail markets where consumers in the retail market have heterogeneous search cost. We have shown that the manufacturer is able to increase profits by setting discriminatory trade promotions. Such promotions induce lower retail margins, while consumers on average are better off. Setting different wholesale prices to different retailers stimulates consumers to search for lower prices.

The vast bulk of the price discrimination literature focuses on firms differentiating between consumers with different valuations. In this paper, we have focussed on a very different function of price discrimination, namely to indirectly screen consumers with different search costs. In our story, it is essential that (i) some consumers *believe* that some retailers have lower prices than others because they contract at a lower wholesale price, but do not know which retailer has which wholesale (or retail) price, and that (ii) consumers differ in their search cost. For (i) to be true, it must be that either retailers cannot effectively advertise their prices to a majority of consumers (for example because consumers do not read these advertisements), or that a minimum advertised price (MAP) is in place forbidding retailers to advertise low retail prices (see Asker and Bar-Isaac [2020]). Our analysis shows that manufacturers may have an incentive to impose a MAP.

We have focussed on a specific form of discriminatory trade promotions where the manufacturer sets linear prices to all of her retailers. In an online Appendix, we show that our main result continues to hold if the manufacturer extracts some, but not the full, retail profits in terms of a fixed fee. The mechanism that is at the core of this paper, namely that the manufacturer can create a more competitive retail market by treating retailers asymmetrically, may also affect other non-price aspects of the vertical relationship between manufacturers and retailers and we think that it is worthwhile in future research to see on which issues that are governed in contractual arrangements, manufacturers may induce asymmetries between retailers to induce more retail competition and when this may benefit or harm consumers.

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8 Appendix A: A closer look at the retail market and consumer beliefs

In the main body of the paper, we have assumed for simplicity that consumers always blame the high cost retailer for a deviation. In this appendix we will look more closely at the implications of this assumption and its alternatives. We will first argue that a retail equilibrium only exists if consumers hold high cost retailers responsible for deviations in the neighbourhood of p_H^* . The reason is as follows. If the high-cost retailer sets the equilibrium price p_H^* his profit equals

$$\pi_r^{H*} = \frac{1}{N} (1 - G(\hat{s})) D(p_H^*)(p_H^* - w_H).$$

If consumers attribute a deviation in the neighbourhood of p_H^* to a low cost retailer, then after observing a price $p_H \neq p_H^*$ they become more pessimistic about finding lower prices on their next search than after observing p_H^* . The main reason is that initially expecting k (with k possibly equal to 1) out of N retailers to have a high price, after observing p_H^* they expect that $N - k$ out of the remaining $N - 1$ retailers have a low price. If after observing a marginally different price consumers suddenly expect only $N - k - 1$ out of the remaining $N - 1$ retailers to have a low price, then fewer consumers continue to search if they observe such a deviation price than after observing p_H^* , making it profitable for a high cost retailer to deviate.²¹ Thus, the profit of a retailer who has a wholesale price w_H are as in Section 3.

For deviations in the neighbourhood of p_L^* , we are free to specify which retailer consumers blame for such a deviation, but the equilibrium price itself depends on how we specify these beliefs. If consumers attribute these deviations to a low cost retailer, there is one important difference compared to the analysis of Section 3. Consumers are now less inclined to continue searching if they observe a deviation price on their first visit as there is a positive probability that they will encounter an even higher retail price on their next search. As low search cost consumers will continue to search until they find the lowest expected price p_L^* in the market, the expected cost of search equals $\frac{N-2}{N-1}s + \frac{1}{N-1}2s = \frac{N}{N-1}s$ as they know there is a possibility they have to search twice before observing the lowest

²¹In the current setup, there is some learning by consumers along the search process and in this sense, the analysis is related to earlier papers by Benabou and Gertner [1993], Dana [1994] and later work by Janssen et al. [2017]. An alternative way of modelling would have the manufacturer giving every retailer one of two wholesale prices with a certain probability that is identical for every retailer, which would eliminate the learning aspect. In the online Appendix, we discuss that the pros and cons of doing so.

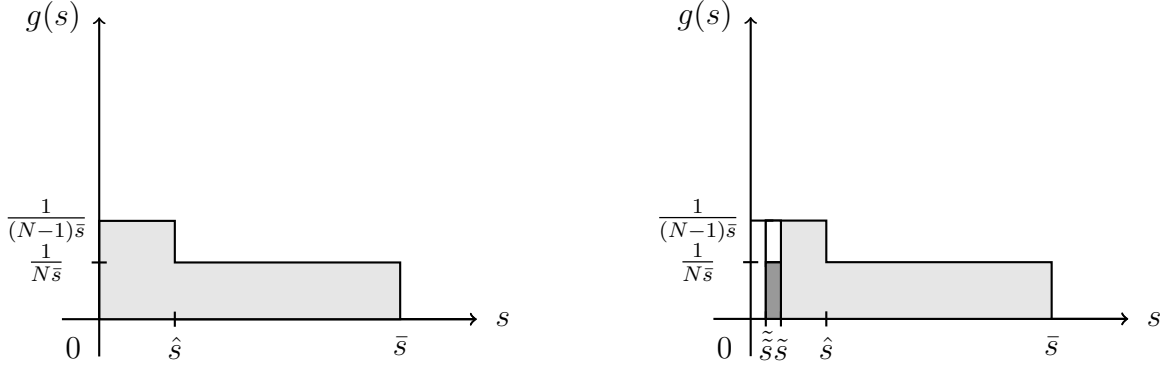


Figure 9: **Left:** Search cost compositions of demand for a low cost retailer. **Right:** Share of consumers that buy at the deviating low cost retailer; where $s \sim U[0, \bar{s}]$.

expected price p_L^* . Thus, first time consumers encountering a price p_L will continue to search if their search cost is $s < \frac{N-1}{N} \int_{p_L^*}^{\tilde{p}} D(p) dp \equiv \tilde{s}$.

Together with the effect we encountered before that the low cost retailers get a disproportionately large share of low search cost consumers, a low cost retailer's profit function when deviating to a price p_L , with $p_L^* < p_L < p_H^*$, will be:

$$\frac{1}{N} \left[1 - G \left(\frac{N-1}{N} \int_{p_L^*}^{\tilde{p}} D(p) dp \right) + \frac{G \left(\int_{\tilde{p}}^{p_H^*} D(p) dp \right)}{(N-1)} \right] D(\tilde{p})(\tilde{p} - w_L). \quad (10)$$

Compared to (4) the main new effect of these alternative beliefs is the term $\frac{N-1}{N}$ reflecting that first-time searchers are less inclined to continue searching. This is illustrated in Figure 9, where the left panel is identical to that of Figure 3, but in the right panel we can see that when consumers blame the low cost retailer for the deviation, then the deviating retailer loses fewer low search cost consumers that came to the retailer on their first search. The darker grey area shows the consumers that after a deviation leave the firm under more optimistic beliefs, but now stay and buy.

In a symmetric retail equilibrium, we again have to take the first-order condition of (10) with respect to p_L and evaluate it at the equilibrium value. This yields:

$$- \frac{\left(\frac{(N-1)^2}{N} + 1 \right) g(0) D^2(p_L^*)(p_L^* - w_L)}{(N-1) + G(\hat{s})} + D'(p_L^*)(p_L^* - w_L) + D(p_L^*) = 0. \quad (11)$$

Comparing this F.O.C. with that in (1) reveals that *ceteris paribus* the only difference is that the first term is multiplied by $\frac{(N-1)^2 + 1}{(N-1) + G(\hat{s})} g(0)$ instead of $\frac{1}{\bar{s}}$, which in the general model would be equal to $g(0)$. It is easy to see that the term in (11) is larger than $g(0)$ if, and only if, $G(\hat{s}) \leq 1/N$. If \hat{s} is small, this condition is always satisfied. On the other hand, comparing the F.O.C. for these more pessimistic beliefs, with the F.O.C. of the low-cost retailer developed in Section 3 given in (5), reveals that the effect of the first term is now smaller because of the more pessimistic beliefs.

In the online appendix, we provide sufficient conditions for a unique retail equilibrium to exist under discriminatory trade promotions.

9 Appendix B: Proofs

Proof of Lemma 2. Suppose not and that $w_j \geq w_i$, and $p_j < p_i$. Write $\tilde{D}(p)$ for the total demand of a firm charging price p . As $D(p)$ is decreasing and a firm cannot get more consumers buying if it charges a lower price it must be that $\tilde{D}(p)$ is decreasing. As p_j and p_i are the optimal retail prices for the two retailers to charge, we should have $(p_j - w_j)\tilde{D}(p_j) \geq (p_i - w_j)\tilde{D}(p_i)$ and $(p_j - w_i)\tilde{D}(p_j) \leq (p_i - w_i)\tilde{D}(p_i)$. These inequalities can be rewritten as $w_i \left(\tilde{D}(p_j) - \tilde{D}(p_i) \right) \geq p_j \tilde{D}(p_j) - p_i \tilde{D}(p_i) \geq w_j \left(\tilde{D}(p_j) - \tilde{D}(p_i) \right)$. But as $\left(\tilde{D}(p_i) - \tilde{D}(p_j) \right) > 0$ this cannot be as $w_j \left(\tilde{D}(p_i) - \tilde{D}(p_j) \right) \geq w_i \left(\tilde{D}(p_j) - \tilde{D}(p_i) \right)$.

Proof of Lemma 3. The proof is given for the assumptions regarding consumer beliefs that are given in the online Appendix, but also hold for the beliefs considered in the main text. Consider for an arbitrarily small ε a set of wholesale prices (w_L, w_H) with $w_L = w$ and $w_H = w + \varepsilon$. From the first-order conditions it is easy to see that $\partial p_H^*(w, w + \varepsilon)/\partial \varepsilon > 0$. As $\frac{\left(\frac{(N-1)^2}{N} + 1\right)g(0)}{(N-1)+G(\hat{s})} > g(0)$ for $G(\hat{s}) \approx 0$ it follows that there exists a $k_1 > 0$ such that the $p_L^*(w_L, p_H)$ that solves (11) for $w_L = w$ and $p_H = p^* + o(\varepsilon)$ is such that $p_L^*(w_L, p_H) < p^* - k_1$. That is, the “best response” of the low cost retailers is discontinuous at $(w_L, w_H) = (w, w)$. Because of the strategic complementarity it follows that $p_L^*(w, w + \varepsilon)$ and $p_H^*(w, w + \varepsilon)$ are discontinuous at $\varepsilon = 0$ and that $p_L^*(w, w + \varepsilon)$ and $p_H^*(w, w + \varepsilon)$ are both strictly smaller than $p^*(w) - k$ for some k .

Finally, the claim that $p_L^*(w, w + \varepsilon) < p_H^*(w, w + \varepsilon) - k$ for some $k > 0$ follows from the fact that $\hat{s} = 0$ if $p_L^*(w, w + \varepsilon) = p_H^*(w, w + \varepsilon)$ and that in that case (3) reduces to (1) implying that $p_H^*(w, w + \varepsilon) > p^*(w)$, whereas from (11) it would follow that $p_L^*(w, w + \varepsilon) < p_H^*(w, w + \varepsilon)$. The continuity of (3) and (11) at $(w_L, w_H) = (w, w + \varepsilon)$ implies that $p_L^*(w, w + \varepsilon) < p_H^*(w, w + \varepsilon)$.

Proof of Proposition 5.

We start arguing that the optimal wholesale contract must have two retailers getting the lowest wholesale price. To do so, we argue that the alternative ways to discriminate between retailers give the manufacturer lower profit than uniform pricing. In both alternative ways to discriminate (either two retailers get the higher wholesale price or they all get different wholesale prices) between retailers only one retailer gets the low wholesale price w_L . As retail profits $(p - w)D(p)$ is assumed to be concave, this retailer will either (i) choose the retail monopoly price $p^M(w_L)$ or (ii) choose $p(w_L)$ to be equal to the retail price chosen by the retailer with the second-lowest wholesale price. In the first case, from the single-peakedness of the profit function $pD(p)$ it follows that the manufacturer profit cannot be larger than $\frac{1}{3}w_L D(p^M(w_L)) + \frac{2}{3}p^M(w_L)D(p^M(w_L))$, which results if the

manufacturer could fully squeeze the two retailers that do not get the lowest wholesale price. On the other hand, under uniform pricing, the manufacturer chooses w to maximize $wD(p^*(w))$, where $p^*(w)$ is given implicitly by (1). We know that $p^*(w) < p^M(w)$, that $\partial p^*(w)/\partial g(0) < 0$ and that $p^*(w) \rightarrow w$ if $g(0) \rightarrow \infty$. It follows that there must be a critical value of $\widehat{g}(0)$ such that for all $g(0) > \widehat{g}(0)$ uniform pricing yields higher profits than $\frac{1}{3}w_L D(p^M(w_L)) + \frac{2}{3}p^M(w_L)D(p^M(w_L))$. As we know from Theorem 3 that giving two retailers a low wholesale price w_L and one retailer a higher wholesale price w_H yields more profits than uniform pricing, it must be optimal to do so.

We now turn to the comparative statics results. Even though the Proposition is for $N = 3$, we provide the expressions for general N and set w_L to $N - 1$ retailers and w_H to 1 retailer. The result for $N = 3$ simply follows by substitution. The total differential of the FOCs (8) and (9) with respect to w_H and w_L in the neighbourhood of $\frac{1}{g(0)} = 0$ where $wD'(p) \approx -D(p)$ and $D(p_L^*) \approx D(p_H^*)$ can be written as

$$0 = (dp_H - dw_H - (dp_L - dw_L)) \left(\frac{\partial p_H^*}{\partial w_H} - \frac{\partial p_L^*}{\partial w_H} \right) - (dp_H - dp_L) \left(1 - \left(\frac{\partial p_H^*}{\partial w_H} - \frac{\partial p_L^*}{\partial w_H} \right) \right) \quad (12)$$

$$+ \left([1 - G(\widehat{s})] \left[1 - \frac{\partial p_H^*}{\partial w_H} \right] - [N - (1 - G(\widehat{s}))] \frac{\partial p_L^*}{\partial w_H} \right) d \frac{1}{D(p_H^*)g(\widehat{s})}$$

and

$$0 = (dp_H - dw_H - (dp_L - dw_L)) \left(\frac{\partial p_H^*}{\partial w_L} - \frac{\partial p_L^*}{\partial w_L} \right) + (dp_H - dp_L) \left(1 + \left(\frac{\partial p_H^*}{\partial w_L} - \frac{\partial p_L^*}{\partial w_L} \right) \right) \quad (13)$$

$$+ \left([N - (1 - G(\widehat{s}))] \left[1 - \frac{\partial p_L^*}{\partial w_L} \right] - [1 - G(\widehat{s})] \frac{\partial p_H^*}{\partial w_L} \right) d \frac{1}{D(p_L^*)g(\widehat{s})}.$$

We now derive how equilibrium retail prices react to changes in wholesale prices. Rewriting the retail first-order conditions (3) and (11) as

$$-D^2(p_H^*)(p_H^* - w_H) + \frac{(1 - G(\widehat{s}))}{g(\widehat{s})} \left[D'(p_H^*)(p_H^* - w_H) + D(p_H^*) \right] = 0, \quad (14)$$

and

$$- \left(\frac{(N-1)^2}{N} + 1 \right) D^2(p_L^*)(p_L^* - w_L) + \frac{((N-1) + G(\widehat{s}))}{g(0)} \left[D'(p_L^*)(p_L^* - w_L) + D(p_L^*) \right] = 0, \quad (15)$$

and using the fact that in the neighbourhood of $\bar{s} = \frac{1}{g(0)} = 0$ we have that $p_H^* \approx w_H$ and $D(p_H^*) \approx D(p_L^*)$,²² the total differential of (14) approximately yields $-D^2(p_H^*)(dp_H^* - dw_H) -$

²²Note that the discontinuity in the retail equilibrium at $(w, w + \varepsilon)$ described in Proposition 5, becomes arbitrarily small in the neighbourhood of $\bar{s} = \frac{1}{g(0)} = 0$.

$D(p_H^*) (D(p_H^*)dp_H^* - D(p_L^*)dp_L^*) + D(p_H^*)d\frac{1-G(\hat{s})}{g(\hat{s})} = 0$, or

$$-2dp_H^* + dw_H + dp_L^* + d\frac{1-G(\hat{s})}{D(p_H^*)g(\hat{s})} \approx 0.$$

Taking the total differential of (15) and leaving out “irrelevant” terms we obtain $-\left(\frac{(N-1)^2}{N} + 1\right) D^2(p_L^*) (dp_L^* - dw_L) + \frac{g(\hat{s})}{g(0)} D(p_L^*) (D(p_H^*)dp_H^* - D(p_L^*)dp_L^*) + D(p_L^*)d\frac{N-(1-G(\hat{s}))}{g(0)} \approx 0$. As $g'(s)$ is bounded and $g(0)$ is very large, it must be that $\frac{g(\hat{s})}{g(0)} \approx 1$ so that we can rewrite this condition as

$$-\frac{N^2+1}{N} dp_L^* + \frac{N^2-N+1}{N} dw_L + dp_H^* + d\frac{N-(1-G(\hat{s}))}{D(p_L^*)g(0)} \approx 0.$$

Thus, the total effects of w_L and w_H on retail prices can be calculated by substituting these two equations into each other:

$$-\frac{2N^2-N+2}{2N} dp_L^* + \frac{N^2-N+1}{N} dw_L + \frac{1}{2} dw_H^* + d\frac{2N-(1-G(\hat{s}))}{2D(p_L^*)g(\hat{s})} = 0.$$

or

$$(2N^2 - N + 2) dp_L^* = 2(N^2 - N + 1) dw_L + N dw_H + Nd\frac{2N-(1-G(\hat{s}))}{D(p_L^*)g(\hat{s})}, \quad (16)$$

and

$$\frac{N^2+1}{N} \left(-2dp_H^* + dw_H + d\frac{1-G(\hat{s})}{D(p_H^*)g(\hat{s})} \right) + \frac{N^2-N+1}{N} dw_L + dp_H^* + d\frac{N-(1-G(\hat{s}))}{D(p_L^*)g(0)} = 0$$

or

$$(2N^2 - N + 2) dp_H^* = (N^2 + 1) dw_H + (N^2 - N + 1) dw_L + d\frac{N^2 + (N^2 - N + 1)(1 - G(\hat{s}))}{D(p_H^*)g(\hat{s})}. \quad (17)$$

Thus, we have that

$$\begin{aligned} \frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*} &= \frac{(N^2 - N + 1) - 2(N^2 - N + 1)}{2N^2 - N + 2} = \frac{-N^2 + N - 1}{2N^2 - N + 2} < 0, \\ \frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*} &= \frac{(N^2 - N + 1) - N}{2N^2 - N + 2} = \frac{N^2 - N + 1}{2N^2 - N + 2} > 0. \end{aligned}$$

Using these expressions, we are now able to further evaluate (12) and (13). First, note that in their first-order approximation, (12) and (13) are identical. To see that, note that adding (12) and (13) gives

$$\begin{aligned} 0 &= \left(2dp_H - dw_H - (2dp_L - dw_L) - [1 - G(\hat{s})] d\frac{1}{g(\hat{s})} \right) \left(\left(\frac{\partial p_H^*}{\partial w_H} - \frac{\partial p_L^*}{\partial w_H} \right) + \left(\frac{\partial p_H^*}{\partial w_L} - \frac{\partial p_L^*}{\partial w_L} \right) \right) \\ N \left[1 - \frac{\partial p_L^*}{\partial w_H} - \frac{\partial p_L^*}{\partial w_L} \right] & d\frac{1}{D(p_L^*)g(\hat{s})}, \end{aligned} \quad (18)$$

where $\left(\frac{\partial p_H^*}{\partial w_H} - \frac{\partial p_L^*}{\partial w_H} \right) = -\left(\frac{\partial p_H^*}{\partial w_L} - \frac{\partial p_L^*}{\partial w_L} \right)$ and $\frac{\partial p_L^*}{\partial w_H} + \frac{\partial p_L^*}{\partial w_L} = 1$.

The total differential of the first first-order condition in the neighbourhood of $\bar{s} = \frac{1}{g(0)} = 0$ where $wD'(p) \approx -D(p)$ and $D(p_L^*) \approx D(p_H^*)$ can be written as

$$\begin{aligned} & \left(D(p_L^*) + w_L D'(p_L^*) \frac{\partial p_L^*}{\partial w_L^*} - w_H D'(p_H^*) \frac{\partial p_H^*}{\partial w_L^*} \right) \left(D(p_H^*) \frac{\partial p_H^*}{\partial w_H^*} - D(p_L^*) \frac{\partial p_L^*}{\partial w_H^*} \right) dw_L - \\ & \left(D(p_H^*) - w_L D'(p_L^*) \frac{\partial p_L^*}{\partial w_H^*} + w_H D'(p_H^*) \frac{\partial p_H^*}{\partial w_H^*} \right) \left(D(p_H^*) \frac{\partial p_H^*}{\partial w_H^*} - D(p_L^*) \frac{\partial p_L^*}{\partial w_H^*} \right) dw_H - \\ & \left(D(p_H^*) + w_H D'(p_H^*) \frac{\partial p_H^*}{\partial w_H^*} - w_H D'(p_H^*) \frac{\partial p_L^*}{\partial w_H^*} \right) D(p_L^*) \left(\left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*} \right) dw_H + \left(\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*} \right) dw_L \right) \\ & + \left(\left[D(p_H^*) + w_H D'(p_H^*) \frac{\partial p_H^*}{\partial w_H^*} \right] + [N-1] w_L D'(p_L^*) \frac{\partial p_L^*}{\partial w_H^*} \right) d \frac{1}{g(\hat{s})} = 0, \end{aligned}$$

or

$$\begin{aligned} & \left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*} \right) \left(\left(1 + \frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*} \right) dw_L - \left(1 - \frac{\partial p_H^*}{\partial w_H^*} + \frac{\partial p_L^*}{\partial w_H^*} \right) dw_H \right) \quad (19) \\ & - \left(1 - \left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*} \right) \right) \left(\left(\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*} \right) dw_H + \left(\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*} \right) dw_L \right) \\ & + \left(\left[1 - \frac{\partial p_H^*}{\partial w_H^*} \right] - [N-1] \frac{\partial p_L^*}{\partial w_H^*} \right) d \frac{1}{D(p_H^*)g(\hat{s})} = 0. \end{aligned}$$

Using the expressions for $\frac{\partial p_H^*}{\partial w_H^*} - \frac{\partial p_L^*}{\partial w_H^*}$ and $\frac{\partial p_H^*}{\partial w_L^*} - \frac{\partial p_L^*}{\partial w_L^*}$ (19) can be simplified as

$$\begin{aligned} & \frac{N^2 - N + 1}{(2N^2 - N + 2)} \left(\left(1 + \frac{-N^2 + N - 1}{(2N^2 - N + 2)} \right) dw_L - \left(1 - \frac{(N^2 - N + 1)}{(2N^2 - N + 2)} \right) dw_H \right) \\ & - \left(1 - \frac{(N^2 - N + 1)}{(2N^2 - N + 2)} \right) \left(\frac{(N^2 - N + 1)}{(2N^2 - N + 2)} dw_H + \frac{-N^2 + N - 1}{(2N^2 - N + 2)} dw_L \right) \\ & + \left(\left[1 - \frac{N^2 + 1}{2N^2 - N + 2} \right] - [N-1] \frac{N}{2N^2 - N + 2} \right) d \frac{1}{D(p_H^*)g(\hat{s})} = 0, \end{aligned}$$

or

$$2 \frac{N^2 - N + 1}{2N^2 - N + 2} (N^2 + 1) (dw_L - dw_H) + d \frac{1}{D(p_H^*)g(\hat{s})} = 0,$$

Substituting this into (16) and (17) yields

$$dp_L^* - dw_L^* = \left(\frac{N}{2(N^2 + 1)(N^2 - N + 1)} + \frac{N(2N - 1)}{(2N^2 - N + 2)} \right) d \frac{1}{D(p_L^*)g(\hat{s})}, \quad (20)$$

and

$$dp_H^* - dw_H^* = \left(-\frac{1}{2(N^2 + 1)} + \frac{2N^2 - N + 1}{2N^2 - N + 2} \right) d \frac{1}{D(p_L^*)g(\hat{s})}. \quad (21)$$

Substituting $N = 3$ yields the result stated in the proposition.

Also, we can approximate the fraction of consumers that continue to search after visiting the high cost retailer, $G(\hat{s})$, by

$$D(p_H^*) \frac{dp_H^* - dp_L^*}{d\left(\frac{1}{g(0)}\right)} = -\frac{N^2 - N + 1}{2N^2 - N + 2} D(p_H^*) \frac{dw_L^* - dw_H^*}{d\left(\frac{1}{g(0)}\right)} + d \frac{1}{g(\hat{s})} \frac{1}{(2N^2 - N + 2)},$$

which can be rewritten as

$$D(p_H^*) \left(\frac{dp_H^*}{d\left(\frac{1}{g(0)}\right)} - \frac{dp_L^*}{d\left(\frac{1}{g(0)}\right)} \right) = \frac{4N^2 - N + 4}{2(N^2 + 1)(2N^2 - N + 2)} < \frac{1}{N}.$$

Finally, we need to show that for any $p \in (p_L^* + \varepsilon, p_H^* - \varepsilon)$ we can find out-of-equilibrium beliefs about who has deviated such that the high cost retailer does not have an incentive to deviate to prices outside the neighbourhood of p_H^* . If he would deviate and set p_L^* his profits will be equal to $(p_L^* - w_H^*)D(p_L^*)$ and we first show that in a neighbourhood of $\bar{s} = 0$ this is strictly smaller than his equilibrium profits $(1 - G(\hat{s}))(p_H^* - w_H^*)D(p_H^*)$. This is the case if, and only if,

$$\frac{dp_L^* - dp_H^*}{d(\frac{1}{g(0)})} + \frac{dp_H^* - dw_H^*}{d(\frac{1}{g(0)})} < \frac{dp_H^* - dw_H^*}{d(\frac{1}{g(0)})}(1 - G(\hat{s})),$$

or $G(\hat{s})\frac{dp_H^* - dw_H^*}{d(\frac{1}{g(0)})} < \frac{dp_H^* - dp_L^*}{d(\frac{1}{g(0)})}$, or $-\frac{1}{2(N^2+1)} + \frac{2N^2-N+1}{2N^2-N+2} < 1$. This is certainly the case. By the same token, a deviation to a price p in the neighbourhood of p_L^* or any price smaller than p_L^* is not optimal. For any $p \in (p_L^* + \varepsilon, p_H^* - \varepsilon)$ (that is outside the immediate neighbourhoods of the equilibrium prices) we can write $p = \alpha p_L^* + (1 - \alpha)p_H^*$ for some $\alpha \in (0, 1)$ and define the following consumer out-of-equilibrium belief $\Pr(\text{low cost retailer has deviated to price } p) = \alpha$. Given that the profit function of the high cost retailer (assuming any deviation is attributed to a high cost retailer) is concave and that the high cost retailer does not have an incentive to deviate to prices in the neighbourhood of p_L^* it follows that given these beliefs, the high cost retailer does not want to deviate to prices $p \in (p_L^* + \varepsilon, p_H^* - \varepsilon)$. If consumers blame high cost retailers for deviations to prices $p > p_H^*$, it is clear that these retailers also do not want to deviate upwards.

Proof of Proposition 6.

We compare the manufacturer profit under uniform pricing and when he sets a low wholesale price to one retailer and a higher wholesale price to two retailers. In the proof, for brevity the latter we call the 1-2 case. we show that for any \bar{s} the manufacturer profit under uniform pricing is larger than for the 1-2 case. At the end of the proof we also show that giving all retailers a different wholesale price cannot be optimal. As Theorem 1 shows that the profit of giving two retailers a low wholesale price is larger than the profit under uniform pricing, it follows that giving two retailers a low wholesale price is optimal.

Under uniform wholesale prices, the retail equilibrium condition (1) yields

$$-(1 - p^*)^2(p^* - w) - \bar{s}(p^* - w) + \bar{s}(1 - p^*) = 0,$$

which can be rewritten as

$$w = p - \frac{\bar{s}(1 - p)}{(1 - p)^2 + \bar{s}}.$$

One can redefine the manufacturer's problem as choosing the retail price to maximize profit, taking this relationship between wholesale and retail prices into account. Thus, the manufacturer chooses p to maximize

$$\left(p - \frac{\bar{s}(1 - p)}{(1 - p)^2 + \bar{s}}\right)(1 - p) = p(1 - p) - \bar{s} + \frac{\bar{s}^2}{(1 - p)^2 + \bar{s}},$$

which yields as a FOC

$$1 - 2p + \bar{s}^2 \frac{2(1-p)}{((1-p)^2 + \bar{s})^2} = 0.$$

As this is a higher-order polynomial, we solve this equation indirectly. Define a parameter α such that $\bar{s} = \alpha(1-p)^2$. This allows us to rewrite the FOC as $1 - 2p + \frac{2\alpha^2(1-p)}{(1+\alpha)^2} = 0$ or,

$$p = \frac{1 + 2\alpha + 3\alpha^2}{2 + 4\alpha + 4\alpha^2} = 1 - \frac{(1 + \alpha)^2}{2 + 4\alpha + 4\alpha^2},$$

while

$$\bar{s} = \alpha \left(\frac{(1 + \alpha)^2}{2 + 4\alpha + 4\alpha^2} \right)^2, \quad (22)$$

which is a monotone function of α and equals 0 at $\alpha = 0$. The maximal manufacturer profit can then be written as

$$\begin{aligned} & \left(1 - \frac{(1 + \alpha)^2}{2 + 4\alpha + 4\alpha^2} - \frac{\alpha \left(\frac{(1+\alpha)^2}{2+4\alpha+4\alpha^2} \right)^2 \frac{(1+\alpha)^2}{2+4\alpha+4\alpha^2}}{\left(\frac{(1+\alpha)^2}{2+4\alpha+4\alpha^2} \right)^2 + \alpha \left(\frac{(1+\alpha)^2}{2+4\alpha+4\alpha^2} \right)^2} \right) \frac{(1 + \alpha)^2}{2 + 4\alpha + 4\alpha^2} \quad (23) \\ &= \left(1 - \frac{(1 + 2\alpha) \frac{(1+\alpha)^2}{2+4\alpha+4\alpha^2}}{1 + \alpha} \right) \frac{(1 + \alpha)^2}{2 + 4\alpha + 4\alpha^2} \\ &= \frac{1 + \alpha + 2\alpha^2}{2 + 4\alpha + 4\alpha^2} \frac{(1 + \alpha)^2}{2 + 4\alpha + 4\alpha^2} \\ &= \left(\frac{1}{2} - \frac{\alpha}{2 + 4\alpha + 4\alpha^2} \right) \left(\frac{1}{4} + \frac{\frac{1}{2}(1 + 2\alpha)}{2 + 4\alpha + 4\alpha^2} \right) \\ &= \frac{1}{8} + \frac{2 + 4\alpha + 4\alpha^2 + 4\alpha^3}{4(2 + 4\alpha + 4\alpha^2)^2}. \end{aligned}$$

Next, consider the manufacturer's profit for the 1-2 case. It is clear that that the manufacturer would *not* find it optimal to set $w_L < w_H$ such that the retail equilibrium prices are such that $p_L^* = p_H^*$, as in that case p_H^* would be set according to the retail equilibrium condition under uniform pricing (1) and the manufacturer profit would be $(\frac{2}{3}w_H + \frac{1}{3}w_L) D(p_H^*)$ which would be smaller than the profit $w^* D(p^*(w^*))$ under uniform pricing. So, if the 1-2 case would generate higher profits it must be that have $p_L^* < p_H^*$.

Three things follow. First, as there is nothing to constrain the low cost retailer's equilibrium price it must be the case that $p_L^* = p^M(w_L)$. Second, as the manufacturer profit in this 1-2 case equals

$$\frac{2(\bar{s} - \hat{s})}{3\bar{s}} w_H D(p_H^*(w_L, w_H)) + \frac{\bar{s} + 2\hat{s}}{3\bar{s}} w_L D(p_L^*(w_L, w_H))$$

and $w_L D(p_L^*(w_L, w_H)) = w_L D(p^M(w_L))$ and this is smaller than the manufacturer profit $w^* D(p^*(w^*))$ under uniform pricing, the 1-2 case can only yield more profit than under uniform pricing if $w_H D(p_H^*(w_L, w_H)) > w_L D(p_L^*(w_L, w_H))$. As in that case the manufacturer profit is strictly smaller than the profit if $\hat{s} = 0$, uniform pricing certainly will give higher profits if

$$w^* D(p^*(w^*)) > \frac{2}{3} w_H D(p_H^*(w_L, w_H)) + \frac{1}{3} w_L D(p^M(w_L)). \quad (24)$$

Third, the high cost retailers' FOC is

$$-g\left(\frac{2}{3}\widehat{s}\right)D^2(p_H^*)(p_H^* - w_H) + \left(1 - G\left(\frac{2}{3}\widehat{s}\right)\right) \left[D'(p_H^*)(p_H^* - w_H) + D(p_H^*)\right] = 0, \quad (25)$$

where the term $\frac{2}{3}\widehat{s}$ comes from the fact that a searching consumer on average needs to pay a search cost of $3s/2$ to find the lowest priced retailer and his gain in that case equals $\widehat{s} = \int_{p_L^*}^{p_H^*} D(p)dp$. For linear demand and a uniform search cost distribution, this yields

$$-(1 - p_H^*)^2(p_H^* - w_H) + \left(\bar{s} + \frac{1}{3}(1 - p_H^*)^2 - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2\right) (1 - 2p_H^* + w_H) = 0, \quad (26)$$

which can be rewritten as

$$\begin{aligned} w_H &= p_H^* - \frac{\left(\bar{s} + \frac{1}{3}(1 - p_H^*)^2 - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2\right) (1 - p_H^*)}{\frac{4}{3}(1 - p_H^*)^2 + \left(\bar{s} - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2\right)} \\ &= 2p_H^* - 1 + \frac{(1 - p_H^*)^3}{\frac{4}{3}(1 - p_H^*)^2 + \left(\bar{s} - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2\right)}. \end{aligned}$$

Substituting this and $p^M(w_L) = \frac{1 - w_L}{2}$ into the RHS of (24) we have that if the 1-2 case could yield more profit than under uniform pricing, the maximal profit equals

$$\frac{1}{3}w_L \left(\frac{1 - w_L}{2}\right) + \frac{2}{3} \left(2p_H^* - 1 + \frac{(1 - p_H^*)^3}{\frac{4}{3}(1 - p_H^*)^2 + \left(\bar{s} - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2\right)}\right) (1 - p_H^*).$$

Maximizing this upper bound of profit by assuming that the manufacturer is directly in control of p_H^* and w_L . Taking the derivative with respect to p_H^* yields

$$(3 - 4p_H^*) - \frac{\frac{8}{3}(1 - p_H^*)^5 + 4(1 - p_H^*)^3 \left(\bar{s} - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2\right)}{\left(\frac{4}{3}(1 - p_H^*)^2 + \left(\bar{s} - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2\right)\right)^2} = 0.$$

Taking the derivative with respect to w_L yields

$$0 = \left(\frac{1 - 2w_L}{2}\right) - 2 \left(\frac{\frac{1 - w_L}{6}(1 - p_H^*)^4}{\left(\frac{4}{3}(1 - p_H^*)^2 + \left(\bar{s} - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2\right)\right)^2}\right).$$

Defining a parameter β such that $\bar{s} - \frac{1}{3}\left(\frac{1 - w_L}{2}\right)^2 = \beta(1 - p_H^*)^2$, these equations can be rewritten as

$$3 - 4p_H^* - \frac{\left(\frac{8}{3} + 4\beta\right)(1 - p_H^*)}{\left(\frac{4}{3} + \beta\right)^2} = 0$$

and

$$0 = \frac{1 - 2w_L}{2} - 2 \left(\frac{\frac{1-w_L}{6}}{\left(\frac{4}{3} + \beta\right)^2} \right)$$

so that

$$\left(\frac{4}{3} + \beta\right)^2 (3 - 4p_H^*) - \left(\frac{8}{3} + 4\beta\right)(1 - p_H^*) = 0$$

$$0 = \left(\frac{4}{3} + \beta\right)^2 \left(\frac{1 - 2w_L}{2}\right) - \frac{1 - w_L}{3}$$

or

$$p_H^* = \frac{\frac{8}{3} + 4\beta + 3\beta^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} = 1 - \frac{\left(\frac{4}{3} + \beta\right)^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2}$$

$$w_L = \frac{\frac{1}{2}\left(\frac{4}{3} + \beta\right)^2 - \frac{1}{3}}{\left(\frac{4}{3} + \beta\right)^2 - \frac{1}{3}} = \frac{1}{2} - \frac{3}{2(4 + 3\beta)^2 - 6}$$

and

$$\bar{s} = \beta \left(\frac{\left(\frac{4}{3} + \beta\right)^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} \right)^2 + \frac{1}{12} \left(\frac{1}{2} + \frac{3}{2(4 + 3\beta)^2 - 6} \right)^2. \quad (27)$$

It is clear that \bar{s} is monotonically increasing in β and equals 0 at $\beta = -\frac{1}{3}$.

Substituting these expressions into the manufacturer profit function for the 1-2 case yields

$$\begin{aligned} & \frac{1}{3} \left(\frac{1}{2} - \frac{3}{2(4 + 3\beta)^2 - 6} \right) \left(\frac{\frac{1}{2} + \frac{3}{2(4 + 3\beta)^2 - 6}}{2} \right) + \\ & \frac{2}{3} \left(1 - \frac{2\left(\frac{4}{3} + \beta\right)^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} + \frac{\left(\frac{4}{3} + \beta\right)}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} \right) \frac{\left(\frac{4}{3} + \beta\right)^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} \\ &= \frac{1}{24} - \frac{1}{6} \left(\frac{3}{2(4 + 3\beta)^2 - 6} \right)^2 + \frac{2}{3} \left(1 - \frac{\frac{20}{9} + \frac{13}{3}\beta + 2\beta^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} \right) \frac{\left(\frac{4}{3} + \beta\right)^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} \\ &= \frac{1}{24} - \frac{1}{6} \left(\frac{3}{2(4 + 3\beta)^2 - 6} \right)^2 + \frac{2}{3} \left(\frac{1}{2} - \frac{\beta}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} \right) \left(\frac{1}{4} + \frac{\frac{2}{3} + \beta}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} \right) \\ &= \frac{1}{8} - \frac{1}{6} \left(\frac{3}{2(4 + 3\beta)^2 - 6} \right)^2 + \frac{2}{3} \frac{\frac{40}{27} + \frac{8}{3}\beta + 2\beta^2 + \beta^3}{\left(\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2\right)^2} \end{aligned} \quad (28)$$

Write $\pi_U(\bar{s})$ and $\pi_D(\bar{s})$ for the manufacturer profit under uniform pricing and the upper bound profit for the 1-2 case. We need to show that $\pi_U(\bar{s}) > \pi_{12}(\bar{s})$ for any \bar{s} . Above we have expressed these profits as functions of α and β , respectively: $\pi_U(\alpha)$ and $\pi_{12}(\beta)$ and we also have expressed the search cost as a function of the same two parameters, i.e., $s_U(\alpha)$ and $s_{12}(\beta)$. These functions are given by the equations (23), (28), (22) and (27),

respectively. As both profit functions are decreasing and both search cost functions are increasing functions in α , resp. β , $\pi_U(\bar{s}) > \pi_{12}(\bar{s})$ is true for any \bar{s} if for every $\beta \geq -\frac{1}{3}$ we can find a value of α such that $s_U(\alpha) > s_{12}(\beta)$ and $\pi_U(\alpha) > \pi_{12}(\beta)$. Below we show that both these inequalities are satisfied if we choose $a = \beta + \frac{1}{2}$.

We first prove that $\pi_U(\beta + \frac{1}{2}) > \pi_{12}(\beta)$. From (23) and (28) it follows that this is certainly the case if

$$\frac{2 + 4(\beta + \frac{1}{2}) + 4(\beta + \frac{1}{2})^2 + 4(\beta + \frac{1}{2})^3}{4\left(2 + 4(\alpha + \frac{1}{2}) + 4(\beta + \frac{1}{2})^2\right)^2} > \frac{2\frac{40}{27} + \frac{8}{3}\beta + 2\beta^2 + \beta^3}{3\left(\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2\right)^2},$$

which is equivalent to $\frac{7}{2}\frac{\frac{2}{7}(5\frac{1}{2} + 11\alpha + 7\alpha^2 + 4\alpha^3)}{4(5 + 8\alpha + 4\alpha^2)^2} > \frac{2\frac{40}{27} + \frac{8}{3}\alpha + 2\alpha^2 + \alpha^3}{3\left(\frac{40}{9} + \frac{20}{3}\alpha + 4\alpha^2\right)^2}$. As the numerator of the LHS is larger than that of the RHS, the inequality certainly holds if $\left(\frac{40}{9} + \frac{20}{3}\alpha + 4\alpha^2\right)^2 > \frac{16}{21}(5 + 8\alpha + 4\alpha^2)^2$ or $\frac{40}{9} + \frac{20}{3}\alpha + 4\alpha^2 > \frac{4}{\sqrt{21}}(5 + 8\alpha + 4\alpha^2)$, which is always the case.

We next prove that $s_U(\beta + \frac{1}{2}) > s_{12}(\beta)$. From (22) and (27) it follows that this is the case if

$$\beta \left(\frac{\left(\frac{4}{3} + \beta\right)^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} \right)^2 + \frac{1}{12} \left(\frac{1}{2} + \frac{3}{2(4 + 3\beta)^2 - 6} \right)^2 < \left(\beta + \frac{1}{2}\right) \left(\frac{(1 + (\beta + \frac{1}{2}))^2}{2 + 4(\beta + \frac{1}{2}) + 4(\beta + \frac{1}{2})^2} \right)^2. \quad (29)$$

Consider first that $\beta > 0$. In this case the inequality holds if the following two conditions hold: (i) $\frac{\left(\frac{4}{3} + \beta\right)^2}{\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2} < \frac{(1 + (\beta + \frac{1}{2}))^2}{2 + 4(\beta + \frac{1}{2}) + 4(\beta + \frac{1}{2})^2}$ and (ii) $\frac{1}{\sqrt{6}} \left(\frac{1}{2} + \frac{3}{2(4 + 3\beta)^2 - 6} \right) - \frac{(1 + (\beta + \frac{1}{2}))^2}{2 + 4(\beta + \frac{1}{2}) + 4(\beta + \frac{1}{2})^2} < 0$. Condition (i) holds if $\left(\frac{16}{9} + \frac{8}{3}\beta + \beta^2\right)(5 + 8\beta + 4\beta^2) < (\beta^2 + 3\beta + \frac{9}{4})\left(\frac{40}{9} + \frac{20}{3}\beta + 4\beta^2\right)$, which is equivalent to $-\frac{10}{9} + \left(\frac{20}{3} + \frac{8}{9} - 15\right)\beta < 0$, which is certainly true for all $\beta > 0$. Condition (ii) is certainly true if $\frac{1}{\sqrt{6}} \left(\frac{1}{2} + \frac{3}{12}\right) \left(2 + 4(\beta + \frac{1}{2}) + 4(\beta + \frac{1}{2})^2\right) < \beta^2 + 3\beta + \frac{9}{4}$, or $\frac{1}{\sqrt{6}} \left(\frac{1}{2} + \frac{3}{12}\right) (5 + 8\beta + 4\beta^2) < \beta^2 + 3\beta + \frac{9}{4}$, which is true for $0 < \beta < 1$. For all $\beta \geq 1$ (ii) is true if $\frac{1}{\sqrt{6}} \left(\frac{1}{2} + \frac{3}{92}\right) (5 + 8\beta + 4\beta^2) < \beta^2 + 3\beta + \frac{9}{4}$, which is always true. Consider next that $-\frac{1}{3} \leq \beta < 0$, as both sides of (29) are decreasing in β the inequality certainly holds for all $-\frac{1}{3} \leq \beta < 0$ if it holds if we substitute $\beta = 0$ in the LHS and $\beta = -\frac{1}{3}$ in the RHS, which is the case as $\frac{1}{12} \left(\frac{1}{2} + \frac{3}{26}\right)^2 - \frac{1}{6} \left(\frac{(1 + \frac{1}{6})^2}{2 + \frac{4}{6} + 4(\frac{1}{6})^2}\right)^2 < 0$.

To conclude the proof, we should also consider the case where the manufacturer sets different wholesale prices $w_L < w_M < w_H$ to different retailers. As in the 1-2 case, this could only result in higher manufacturer profits than the 2-1 case (giving two retailers the lower wholesale price) if the low cost retailer sets the retail monopoly price that is strictly smaller than the retail price of the w_M retailer. For the retailer with the highest wholesale price, we should distinguish two cases.

First, it could be that the wholesale prices are such that all the consumers that first visit the high cost retailer and that would find it optimal to continue searching, only stop searching when they have found the lowest wholesale price. This would be the case if w_M and w_H are close to each other. In that case, the considerations to determine the optimal

retail price for the w_M and w_H retailers are the same as in (25) as it takes consumers in expectation 1.5 visits to find the lowest retail price. In that case, it is optimal, however, to set $w_M = w_H$. We would effectively have the 1-2 case then and above we have shown that this is not optimal.

Second, it could also be that the wholesale prices are such that some consumers that first visit the high cost retailer that find it optimal to continue searching, stop searching on their second visit independent of whether they have found the lowest wholesale price. This would be the case if the difference between w_M and w_H is relatively large. The considerations to determine the optimal retail price for the high cost retailer are then similar to the one for the high cost retailer in the 2-1 case, i.e., (3) as the indifferent consumer will only search once more. The w_L and the w_M retailers would set higher margins than the low cost retailers in the 2-1 case, however, as the low cost retailer sets the monopoly retail price and the w_M retailer gets disproportionately many consumers from the high cost retailer that anyway will not continue to search. Thus, in this case, the manufacturer profit can definitely not be larger than in the 2-1 case.