Online Appendix for "Discriminatory Trade Promotions in Consumer Search Markets"

Maarten Janssen* and Edona Reshidi†

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In this online Appendix we consider several issues. First, we take up the issue of random independent trade promotions. Then we consider the case of two-part-tariffs. Afterwards, in Section 3, we provide additional numerical simulations for different search cost distributions, which reconfirm the paper's finding that the manufacturer earns higher profits under discriminatory trade promotions. In Section 4, we provide a more formal justification for using $d\overline{s}$, $d(1/g(0))$ and $d(1/g(\widehat{s}))$ interchangeably in the paper. Finally, in Section 5 we state and prove the proposition on retail equilibrium existence.

1 Random and independent trade promotions

One may wonder whether the analysis of the paper would simplify if the manufacturer gives independent random trade promotions instead of singling out some retailers to get a promotion. Independent random trade promotions would imply that unlike in the main body of the paper there is no consumer learning along the search process. In this Section we will show that such trade promotions indeed simplify the consumer search process, but they complicate the analysis regarding retail (and therefore manufacturer) behaviour. To make these points, we assume that the search cost is uniformly distributed on the interval $[0, 1]$.

For the benchmark case of uniform pricing, the analysis remains exactly as it is in the main paper, so let us explore the consequences of discriminatory trade promotions, where there is a probability q that a retailer gets a wholesale price w_H and a probability $1-q$ he gets w_L. Unlike in the main body of the paper, the probability that each retailer gets a high wholesale price is independent of whether or not other retailers get a high wholesale price.

A first question to address is whether or not retailers know the realization of the randomization or not. In most markets where a trade promotion is valid for some time, this is likely the more important case to consider. Essentially, if trade promotions are valid for some time retailers have a professional interest to get to know their competitors' prices and therefore will observe (some of) the retail prices of their competitors and for a retail equilibrium to exist one needs to ensure that

*Department of Economics, University of Vienna and CEPR; maarten.janssen@univie.ac.at.
†Bank of Canada, Banking and Payments Department; ereshidi@bank-banque-canada.ca.

no retailer wants to deviate given the other retail prices that are charged. This will be ensured in case retailers act as if they know the realization of the wholesale prices. If retailers do not know the the realization of the wholesale prices, but they observe the other retail prices, then they will want to adjust their prices once they observe their competitors' prices. This adjustment process may well result in an outcome that is *as if* retailers know wholesale prices.

If retailers know the realization of the randomization, then the retail problem becomes untractable, however, as one has to solve the problem for every possible realization of the randomization. With only three retailers that already implies that we have to solve for four cases, dependent on whether 0, 1, 2 or 3 retailers got the high wholesale price. Moreover, If retailers know the realization of the randomization, then there is an asymmetry between consumers and retailers creating further difficulties as there are no separate subgames.

If, on the other hand, retailers and consumers do not know the outcome of the randomization, then it is unclear how the manufacturer can commit to the randomization if no one observes the outcome. Note that these issue does not arise in the way we have set-up the problem as retailers would not like to change their behaviour even if they know who received the high or the low wholesale price.

In both cases, there is an additional complication when there is a finite number of retailers, namely that a pure strategy price equilibrium does not exist. The reason is that with a finite number of retailers, there is a positive probability that all retailers would receive the same high wholesale price and in that case consumers that continue to search all retailers in the hope of finding a retailer with a lower price will in the end know all prices and then buy from the retailer with the lowest price. This would give high cost retailers an incentive to undercut. Accordingly, a pure strategy price equilibrium for a high cost retailer does not exist. This would, however, then also create difficulties for the characterization of the low cost retailer's strategy. This last problem can only be overcome by assuming the existence of a continuum of retailers, which does not make the model more appealing.

If, however, one would side-step all these issues and assume that retailers will never be able to find out the prices charged by their competitors, but nevertheless the manufacturer can commit to wholesale prices and we focus on a continuum of retailers, then we can in principle analyze this second case. The low and high cost retailers are then expected to react to w_L and w_H by setting p_L^* and p_H^*, respectively. As consumers do not know which retailer faces the higher wholesale price, they do not know which retailer has the higher retail price. We will first show that for the behaviour of the high cost retailer nothing fundamental changes. The effect of discriminatory trade promotions on consumer search is that the low search cost consumers who happen to encounter the high cost retailer setting p_H^* will continue to search for lower retail prices. In particular, redefining $\widehat{s} = (1-q)\int_{p_L^*}^{p_H^*} D(p)dp$, all consumers who happen to observe p_H^* at their first search and have a search cost $s < \widehat{s}$ continue to search.

If the high-cost retailer sets the equilibrium price p_H^* his profit equals

$$\pi_r^{H*} = (1-\widehat{s})\, D(p_H^*)(p_H^* - w_H).$$

To determine the retail equilibrium prices p_L^* and p_H^*, we have to consider how a deviation price $\widetilde{p}$ affects demand. Consider first the determination of p_H^*. After observing a deviation price $\widetilde{p}$, consumers will continue to search if their search cost is such that

$$s < \widehat{s} + \int_{p_H^*}^{\widetilde{p}} D(p)dp.$$

Therefore, the profit of a retailer who has a wholesale price w_H and sets a price $\widetilde{p}$ in the neighbourhood of p_H^* will be:

$$\pi_r^H(\widetilde{p}, p_H^*; w_H^*) = \left(1 - \left(\widehat{s} + \int_{p_H^*}^{\widetilde{p}} D(p)dp\right)\right) D(\widetilde{p})(\widetilde{p} - w_H). \tag{1}$$

Taking the first-order condition of (1) with respect to $\widetilde{p}$ and substituting $\widetilde{p} = p_H^*$ yields

$$-\frac{D^2(p_H^*)(p_H^* - w_H)}{1 - \widehat{s}} + D^{'}(p_H^*)(p_H^* - w_H) + D(p_H^*) = 0. \tag{2}$$

Comparing this FOC condition with the FOC condition (**??**) in the paper reveals that *ceteris paribus* the only difference is that the first term is multiplied by $\frac{1}{1-\widehat{s}}$ instead of by 1 as in the main body of the paper.

Consider now a low cost retailer. If he sets the equilibrium price, his profits would be equal to

$$\pi_r^L(p_L; p_L^*, p_H, w_L^*) = \left[1 + \frac{q}{1-q} G\left(\widehat{s}\right)\right] D(p_L^*)(p_L^* - w_L),$$

which accounts for the fact that a fraction $qG\left(\widehat{s}\right)$ of consumers first visits a high cost retailer and continues to search and they are spread over the fraction of $(1-q)$ firms with a low wholesale price.

There are two important differences with the case of uniform pricing in evaluating the profitability of a deviation by the low cost retailer. First, after observing a price $p_L < p_H^*$ consumers are less inclined to continue searching compared to the uniform pricing case as now there is a positive probability that they will encounter an even higher retail price, namely p_H^*, on their next search. We call this the *anti-competitive effect* of discriminatory trade promotions. As low search cost consumers will continue to search until they find the lowest expected price p_L^* in the market (or if they have exhausted searching all retailers, they buy at he lowest realized price in the market) the benefit of search equals $\int_{p_L^*}^{p_L} D(p)dp$, whereas the expected cost of search equals $(1-q)s + 2q(1-q)s + ... = \frac{s}{1-q}$. Thus, consumers encountering a price p_L on their first search will continue to search if their search cost is $s < (1-q)\int_{p_L^*}^{p_L} D(p)dp$.

For a low cost retailer contemplating a deviation to a price $p_L > p_L^*$ there is potentially a *pro-competitive effect* of discriminatory trade promotions on consumer search. Due to the fact that low search cost consumers continue to search if they observe p_H^* on their first search, low cost retailers will serve a disproportionately larger share of low search cost consumers. Therefore, they are losing relatively more consumers if they deviate and increase their prices. The number of additional consumers a low cost retailer attracts if it deviates is computed as follows. The fraction of consumers that first visits a high cost retailer and continues to search is equal to $q\widehat{s}$. Of these consumers a fraction $1/(1-q)$ visits the deviating firm on their second visit. From then on, because of independent wholesale (and retail) prices, their search is exactly equal to the search behaviour of consumers who encounter p_L on their first search and they thus do not continue to search and buy if their search cost is larger than $(1-q)\int_{p_L^*}^{p_L} D(p)dp$. Consumers that visit another low cost retailer on their second visit will not continue searching and thus not buy from the firm under consideration. Thus, when deviating to a price $\widetilde{p}$, with $p_L^* < \widetilde{p} < p_H^*$, a retailer's profit function will be:

$$\pi_r^L(p_L; p_L^*, p_H, w_L^*) = \left(1 + \frac{q\widehat{s}}{1-q}\right)\left[1 - G\left((1-q)\int_{p_L^*}^{\widetilde{p}} D(p)dp\right)\right] D(\widetilde{p})(\widetilde{p} - w_i). \tag{3}$$

In a symmetric retail equilibrium, we have to take the first-order condition of (3) with respect to $\widetilde{p}$ and evaluate it at the equilibrium values. This yields:

$$-(1-q)D^2(p_L^*)(p_L^* - w_L) + \left[D'(p_L^*)(p_L^* - w_L) + D(p_L^*)\right] \leq 0. \tag{4}$$

Comparing to condition (**??**) in the paper reveals that the negative, first term is now multiplied by $(1-q)$, which means that relative to uniform pricing, without consumer learning the low cost retailer has *higher* margins. This should not come as a surprise. All consumers that visit a deviating low cost retailer think the probability they visit another low cost retailer on their next search is smaller than 1 and are therefore less inclined to continue to search than under uniform pricing.

This implies that under independent discriminatory wholesale pricing, some retailers make lower margins, while others make higher margins. This implies that the analysis we used in the main paper to prove that the manufacturer can make more profit does not easily generalize. Consumer learning because wholesale prices are not independent of each other, while complicating the consumer search problem, makes the retail analysis and the manufacturer analysis easier to handle. Together with the conceptual issues outlined above, this made us decide to use the specification in the main body of the paper.

2 Two-part-tariffs

We now investigate how allowing the monopolist manufacturer to have the possibility of choosing two-part tariffs affects our results. Clearly, if the manufacturer has all the bargaining power, then he will set a wholesale price that induces the retailers to choose the integrated monopolist price and set a fixed fee equal to the retail profit. Discriminatory trade promotions do not add to the manufacturer's profit in this case. In most markets, however, the bargaining power is not exclusively with the manufacturer. In this section, we exogenously fix the relative bargaining power and denote by α the bargaining power of a given retailer, where α measures the share of the retail profit that the retailer can keep for himself. We show that our results continue to hold for any $\alpha > 0$. In an equilibrium under the uniform pricing scheme, an individual retailer's profit will be:

$$\pi_r^*(p^*) = \frac{\alpha}{N} D(p^*(w^*))(p^* - w^*).$$

Whereas, the monopolist manufacturer's profit in equilibrium is given by:

$$\pi(w^*) = w^* D(p^*(w^*)) + (1-\alpha)(p^* - w^*) D(p^*(w^*))$$

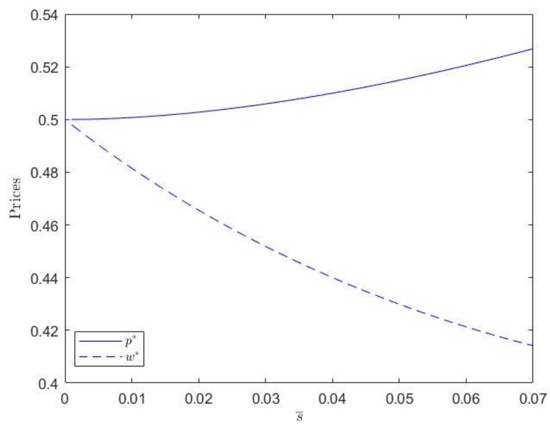
Thus, if $\alpha = 0$, the manufacturer extracts all profits from its retailers and if $\alpha = 1$, then the profits will be the same as in the paper. It is clear that with this formulation, the retailer's problem is identical to the one analysed in the paper and thus the equilibrium condition for the retail prices remains the same. On the other hand, the equilibrium condition for the uniform wholesale price changes since the manufacturer now directly maximizes:

$$\pi(w) = wD(p(w)) + (1-\alpha)(p - w)D(p(w))$$

Thus, with uniform pricing and two-part tariffs the wholesale price w is set such that:

$$wD'(p(w))\frac{\delta p^*}{\delta w} + D(p(w)) + (1-\alpha)\left[(p-w)D'(p(w))\frac{\delta p^*}{\delta w} + (\frac{\delta p^*}{\delta w} - 1)D(p(w))\right] = 0. \quad (5)$$

Figures 1 below, depicts retail and wholesale prices under uniform pricing for different values of $\overline{s}$, when $\alpha = 1$ and $\alpha = 0.1$ respectively.



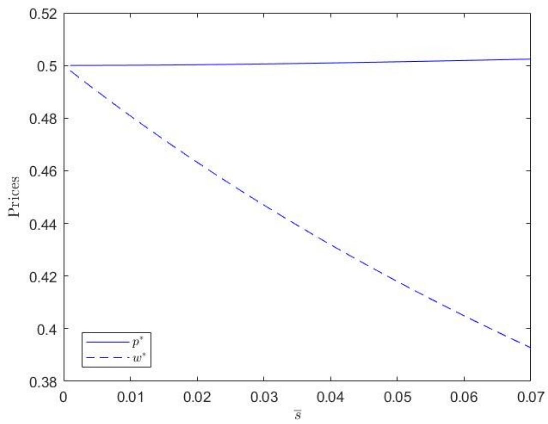


Figure 1: **Left:** Uniform retail and wholesale prices for different values of $\overline{s}$, when $\alpha = 1$. **Right:**Uniform retail and wholesale prices for different values of $\overline{s}$, when $\alpha = 0.1$.

Under discriminatory trade promotions with two-part tariffs, the manufacturer will chose two different wholesale prices, w_L and w_H, to directly maximize:

$$\begin{aligned} \pi(w_L, w_H) &= \frac{1}{N}\left[1 - G(\widehat{s})\right]\left[w_H D(p_H^*(w_H)) + (1-\alpha)(p_H^* - w_H)D(p_H^*(w_H))\right] \\ &+ \frac{N - 1 + G(\widehat{s})}{N}\left[w_L D(p_L^*(w_L)) + (1-\alpha)(p_L^* - w_L)D(p_L^*(w_L))\right] \end{aligned}$$

which yields the two following first-order conditions below:

$$0 = [w_H D(p_H^*) + (1-\alpha)(p_H^* - w_H)D(p_H^*) - w_L D(p_L^*) - (1-\alpha)(p_L^* - w_L)D(p_L^*)] \left(D(p_L^*)\frac{\partial p_L^*}{\partial w_H^*} - D(p_H^*)\frac{\partial p_H^*}{\partial w_H^*} \right)$$
$$+ \frac{N-1+G(\widehat{s})}{g(\widehat{s})} \left[w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_H^*} + (1-\alpha)\left(\frac{\partial p_L^*}{\partial w_H^*} D(p_L^*) + (p_L^* - w_L)D_L'^{*})\frac{\partial p_L^*}{\partial w_H^*} \right) \right]$$
$$+ \frac{[1-G(\widehat{s})]}{g(\widehat{s})} \left[D(p_H^*) + w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_H^*} + (1-\alpha)\left((\frac{\partial p_H^*}{\partial w_H^*} - 1)D(p_H^*) + (p_H^* - w_H)D_H'^{*})\frac{\partial p_H^*}{\partial w_H^*} \right) \right]$$

and

$$0 = [w_H D(p_H^*) + (1-\alpha)(p_H^* - w_H)D(p_H^*) - w_L D(p_L^*) - (1-\alpha)(p_L^* - w_L)D(p_L^*)] \left(D(p_L^*)\frac{\partial p_L^*}{\partial w_L^*} - D(p_H^*)\frac{\partial p_H^*}{\partial w_L^*} \right)$$
$$+ \frac{N-1+G(\widehat{s})}{g(\widehat{s})} \left[D(p_L^*) + w_L D'(p_L^*)\frac{\partial p_L^*}{\partial w_L^*} + (1-\alpha)\left((\frac{\partial p_L^*}{\partial w_L^*} - 1)D(p_L^*) + (p_L^* - w_L)D_L'^{*})\frac{\partial p_L^*}{\partial w_L^*} \right) \right]$$
$$+ \frac{[1-G(\widehat{s})]}{g(\widehat{s})} \left[w_H D'(p_H^*)\frac{\partial p_H^*}{\partial w_L^*} + (1-\alpha)\left(\frac{\partial p_H^*}{\partial w_L^*} D(p_H^*) + (p_H^* - w_H)D_H'^{*})\frac{\partial p_H^*}{\partial w_L^*} \right) \right]$$

The following three figures show that the profit of the manufacturer is indeed higher under discriminatory trade promotions compared to uniform pricing even under two-part tariffs. The figures show the manufacturer's profit functions under both setting for different values of α, first starting with the case of $\alpha = 1$ in Figure 2, which is what we have assumed in the paper, and then two other examples of smaller values of α that translate to cases where the retailer cannot keep all of his profit.

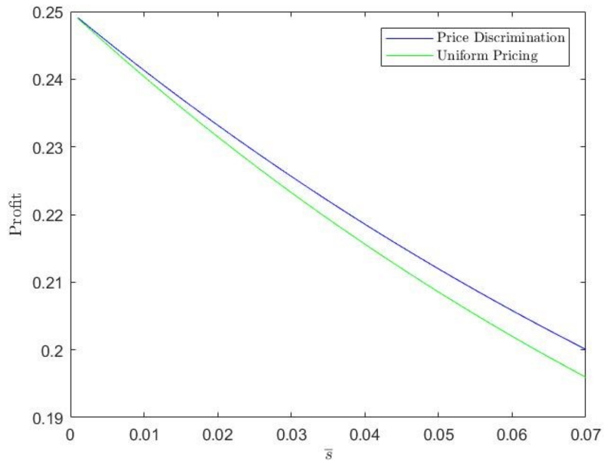


Figure 2: Manufacturer's Profit for different values of $\overline{s}$ and $\alpha = 1$

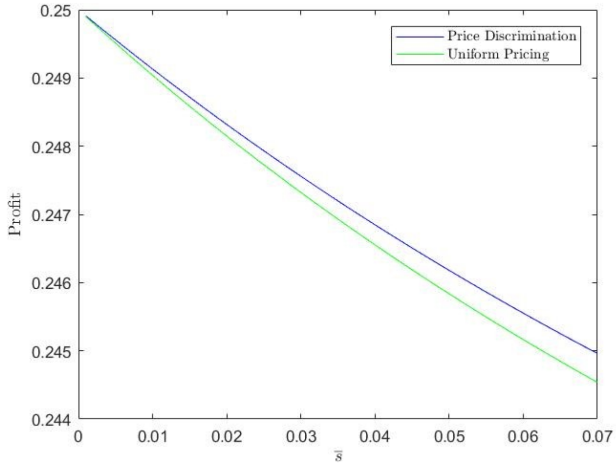


Figure 3: Manufacturer's Profit for different values of $\overline{s}$ and $\alpha = 0.1$

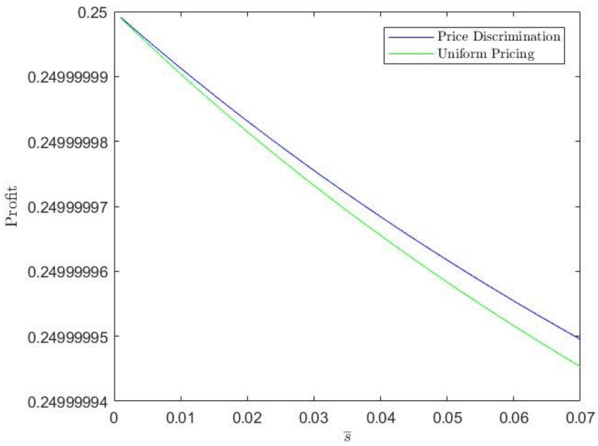


Figure 4: Manufacturer's Profit for different values of $\overline{s}$ and $\alpha = 0.000001$

3 Different search cost distributions

In this Section, we show that the numerical results regarding the retail and wholesale prices, consumer surplus and manufacturer's profit, obtained in the paper for the case where consumers' search costs were uniformly distributed on $[0, \overline{s}]$, are robust also under different search cost distributions. First, we present the case of the Exponential distribution and then also the results from a special case of the Kumaraswamy distribution.

3.1 Exponential Distribution

Here we have assumed that the consumers' search cost follow the Exponential distribution and thus we have that $G(s) = 1 - e^{-\lambda s}$. Figure 5(Left) shows that for the case of linear demand and an exponential search cost distribution that the average consumer surplus, including the first costly search, is higher under discriminatory trade promotions. Similarly, as for the case of the uniform distribution, Figure 5(Right) shows that retail prices, depending on the support of the search cost distribution, can either all be lower than the uniform retail price, or that the high retail price can be higher than what retailers would charge under uniform pricing. Figure 6(Left) shows that the manufacturer makes higher profits under discriminatory trade promotions compared to uniform pricing, even if search costs are exponentially distributed. Furthermore, in Figure 6(Right),we see that, just as we had in the paper under uniform search costs, wholesale prices are non-monotonic in the support of the search cost distribution even for the case of Exponential distribution. Therefore, the wholesale prices can be lower or higher under discrimination compared to uniform pricing, but this will still lead to higher manufacturer profits. The intuition behind the non-monotonicity of wholesale prices is described in the paper.

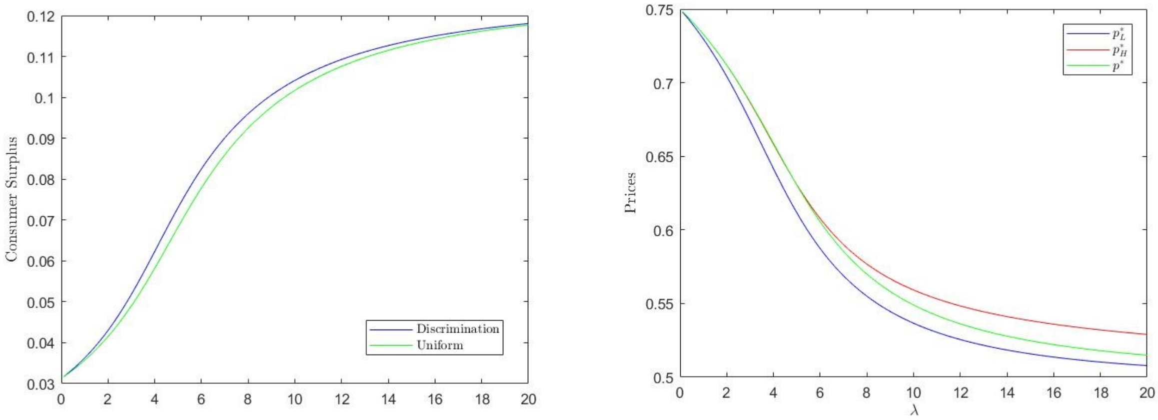


Figure 5: **Left:** Consumer surplus for different values of λ. **Right:** Retail prices for different λ.

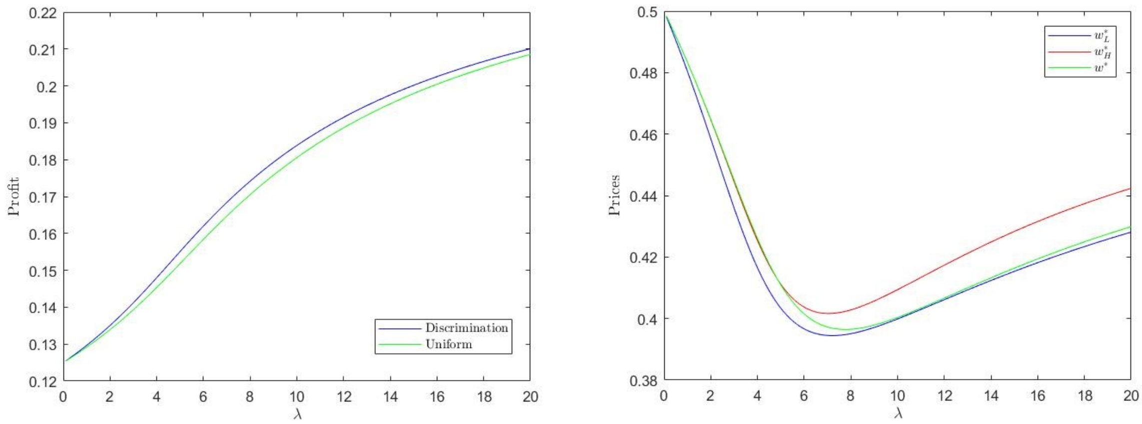


Figure 6: **Left:** Manufacturer's profit for different values of λ. **Right:** Wholesale prices for different values of λ.

3.2 Kumaraswamy Distribution

Below, we present the results of numerical analysis obtained for the case when consumers search costs follow a special case of the Kumaraswamy distribution, where the shape parameter a is restricted to 1 and the shape parameter b is unrestricted. Therefore, we have that $G(s) = 1 - (1-s)^b$. Figure 7(Left), shows that even for the Kumaraswamy distribution, consumer surplus is higher under discriminatory trade promotions. More specifically, for the case of $b = 8$, consumer surplus is almost 4% higher than under uniform pricing. In Figure 7(Right), we also see that the behaviour of retail prices depends on the shape parameter b. The high retail price can sometimes either be higher or lower than the uniform retail price. Finally, Figure 8(Left) confirms that the manufacturer is better off under discrimination even for this given search cost distribution. The profit increase can be as high as 2.5% under discriminatory trade promotions.

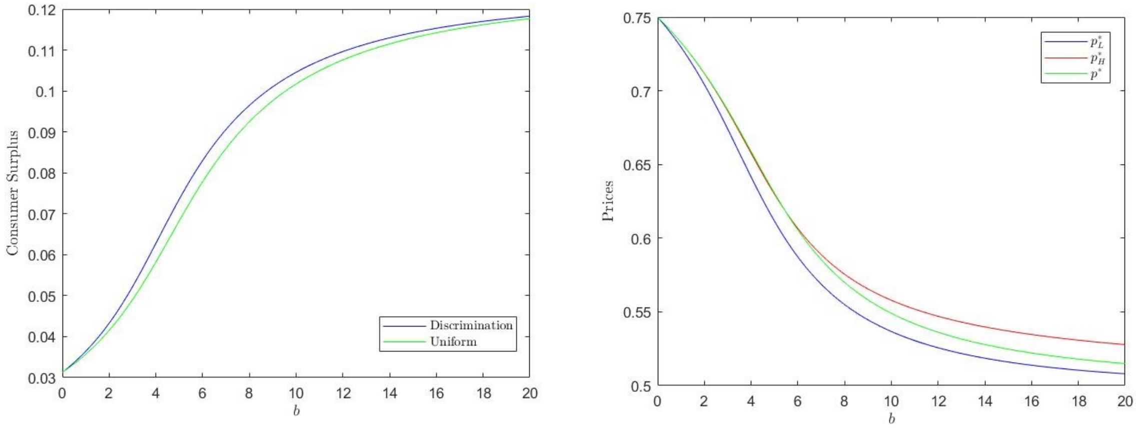


Figure 7: **Left:** Consumer surplus for different values of b. **Right:** Retail prices for different b.

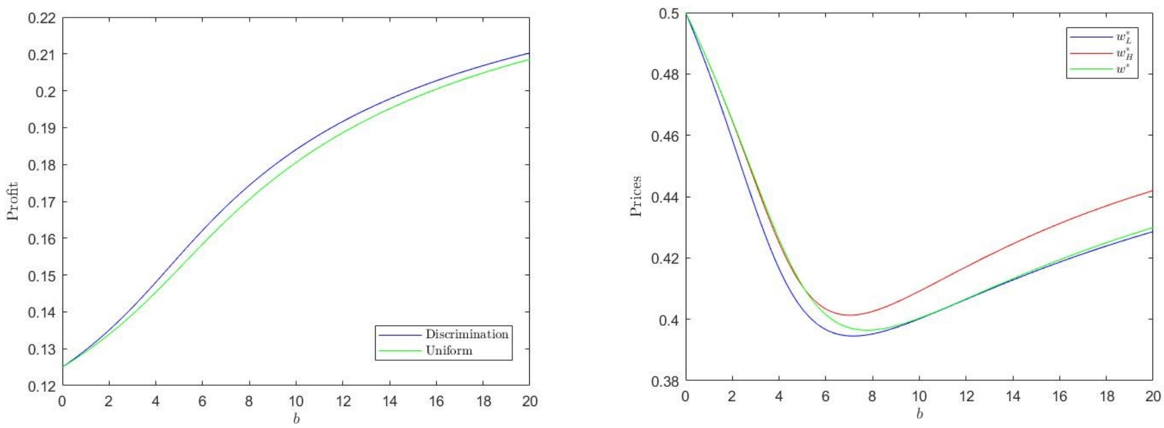


Figure 8: **Left:** Manufacturer's profit for different values of b. **Right:** Wholesale prices for different values of b.

4 $\overline{s}$ approximation

In this Section, we argue more formally that in a neighbourhood of $\overline{s} = 0$ we can approximate $\overline{s}$ with $1/g(0)$ or $1/g(\widehat{s})$ providing a formal justification for using $d\overline{s}$, $d(1/g(0))$ and $d(1/g(\widehat{s}))$ interchangeably.

Take a sequence of upper bounds of the search cost distribution $\{\overline{s}_n\}_{n=1}^{\infty} \to 0$ and define $g_n(0) = \lim_{\Delta s \downarrow 0} g_n(\Delta s)$.

From the fact that we have assumed that there exists an $M < \infty$ such that $-M < g_n^{'}(s) < M$ it follows that

$$1 = \int_0^{\overline{s}_n} g_n(s) \leq [g_n(0) + M\overline{s}_n]\, \overline{s}_n.$$

This implies that $1/\overline{s}_n - g_n(0) \leq M\overline{s}_n$ so that

$$\lim_{n\to\infty} (1/\overline{s}_n - g_n(0)) \leq 0 = M \lim_{n\to\infty} \overline{s}_n.$$

Similarly,

$$1 = \int_0^{\overline{s}_n} g_n(s) \geq [g_n(0) - M\overline{s}_n]\, \overline{s}_n,$$

which implies that $1/\overline{s}_n - g_n(0) \geq M\overline{s}_n$ so that

$$\lim_{n\to\infty} (1/\overline{s}_n - g_n(0)) \geq 0 = M \lim_{n\to\infty} \overline{s}_n.$$

So, $\lim_{n\to\infty} \overline{s}_n = \lim_{n\to\infty} 1/g_n(0)$.

Extending this argument we now show that $\lim_{n\to\infty} \overline{s}_n = \lim_{n\to\infty} 1/g_n(\widehat{s}_n)$ for every $\widehat{s}_n = f_n(\overline{s}_n)$ with $\lim_{n\to\infty} f_n(\overline{s}_n) = 0$. In particular, as for every $\widehat{s}_n < \overline{s}_n$ we can write $\widehat{s}_n = f_n(\overline{s}_n)$ and provide upper and lower bounds of $g_n(\widehat{s}_n)$ as $g_n(0) - f_n(\overline{s}_n)M \leq g_n(\widehat{s}_n) \leq g_n(0) + f_n(\overline{s}_n)M$, we have, for example, that

$$1 = \int_0^{\overline{s}_n} g_n(s) \leq [g_n(0) + M\overline{s}_n]\, \overline{s}_n \leq [g_n(\widehat{s}_n) + Mf_n(\overline{s}_n) + M\overline{s}_n]\, \overline{s}_n.$$

This implies that $1/\overline{s}_n - g_n(\widehat{s}_n) \leq M\overline{s}_n + +Mf_n(\overline{s}_n)$ so that

$$\lim_{n\to\infty} (1/\overline{s}_n - g_n(\widehat{s}_n)) \leq 0 = M \lim_{n\to\infty} (\overline{s}_n + f_n(\overline{s}_n)).$$

In the same way we can establish that 0 is the lower bound of $\lim_{n\to\infty}(1/\overline{s}_n - g_n(\widehat{s}_n))$.

5 Existence

As stated in the paper, we provide here the proposition providing sufficient conditions that the retail market equilibrium as characterized by either (**??**) or (**??**) and (**??**) exists. All the equation numbers in this part of the online appendix refer to the main paper.

Proposition 1 *If $\overline{s}$ is close enough to 0 and $g(s)$ is large, a retail price equilibrium exists for any $w_H > w_L$, where the retail prices are given by $p_L^*(w_L, w_H)$ and $p_H^*(w_L, w_H)$ as the solutions to (3) and (11). In addition, the retail equilibrium prices of the whole game, $p_L^*(w_L^*, w_H^*)$ and $p_H^*(w_L^*, w_H^*)$, are uniquely defined for any (w_L, w_H) with $w_H \geq w_L$.*

Proof. We prove that the "best response" curves (3) and (11) have an intersection point. To prove this we show that If the search cost distribution is sufficiently concentrated around 0 (in the sense that $g(0)$ is large enough), the second-order derivative of retailers' profit function is negative if the first-order condition holds (and thus that the profit function is quasi-concave). This implies that these "best responses" are continuous functions.

To prove the existence of a retail equilibrium we need to additionally verify that (3) and (11) indeed define "best responses". The point is that (3) and (11) assume certain out-of-equilibrium beliefs in the neighbourhood of the respective equilibrium prices. We need to make sure that a high (low) cost retailer has an incentive to imitate the equilibrium price of the low (high) cost retailer and that we can find out-of-equilibrium beliefs that are such that no retailer wants to deviate from the reaction defined by (3) and (11).

Note that if the other low cost retailers set p_L^* (and the high cost retailer sets p_H^*) the FOC for a low cost retailer is such that $p_L \geq p_L^*$ should solve[1]

$$\begin{aligned}
&-\left(\frac{N-1}{N}g\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)+\frac{1}{(N-1)}g\left(\int_{p_L^*}^{p_L} D(p)dp\right)\right)D^2(p_L)(p_L-w_L)\\
&+\left[1-G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)+\frac{G\left(\int_{p_L^*}^{p_H^*} D(p)dp\right)-G\left(\int_{p_L^*}^{p_L} D(p)dp\right)}{(N-1)}\right]\\
&\cdot\left[D^{'}(p_L)(p_L-w_L)+D(p_L)\right]=0.
\end{aligned}$$

The second-order derivative of the profit function of a low cost retailer is then given by

$$\begin{aligned}
&-\left(\left(\frac{N-1}{N}\right)^2 g'\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)+\frac{1}{(N-1)}g'\left(\int_{p_L^*}^{p_L} D(p)dp\right)\right)D^3(p_L)(p_L-w_L) \qquad (6)\\
&-\left(\frac{N-1}{N}g\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)+\frac{1}{(N-1)}g\left(\int_{p_L^*}^{p_L} D(p)dp\right)\right)D(p_L)\left[2D^{'}(p_L)(p_L-w_L)+D(p_L)\right]\\
&-\left(\frac{N-1}{N}g\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)+\frac{1}{(N-1)}g\left(\int_{p_L^*}^{p_L} D(p)dp\right)\right)\left[D^{'}(p_L)(p_L-w_L)+D(p_L)\right]\\
&+\left[1-G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)+\frac{G\left(\int_{p_L^*}^{p_H^*} D(p)dp\right)-G\left(\int_{p_L^*}^{p_L} D(p)dp\right)}{(N-1)}\right]\left[D''(p_L)(p_L-w_L)+2D'(p_L)\right].
\end{aligned}$$

The last line is clearly negative as $D^{''}(p_L)(p_L-w_L)+2D'(p_L)<0$ by assumption. The first and the third line together are also clearly negative as from the FOC it follows that $D^2(p_L)(p_L-w_L)$ is close to 0 and $D^{'}(p_L)(p_L-w_L)+D(p_L)>0$, while $g'(\cdot)/g(\cdot)$ and $D(p_L)$ are bounded. The terms in the second line are also negative if $2D^{'}(p_L)(p_L-w_L)+D(p_L)>0$. Using the FOC, $2D^{'}(p_L)(p_L-w_L)+D(p_L)$ has the sign of

$$D^{'}(p_L)+\frac{\left(\frac{N-1}{N}g\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)+\frac{1}{(N-1)}g\left(\int_{p_L^*}^{p_L} D(p)dp\right)\right)D^2(p_L)}{1-G\left(\frac{N-1}{N}\int_{p_L^*}^{p_L} D(p)dp\right)+\frac{G\left(\int_{p_L^*}^{p_H^*} D(p)dp\right)-G\left(\int_{p_L^*}^{p_L} D(p)dp\right)}{(N-1)}},$$

[1]Only if $p_L = p_L^*$ can the FOC hold with inequality.

which is clearly positive as $g(\cdot)$ is unbounded for all relevant p_L. Thus, as all four terms are negative, the whole expression is clearly negative.

The second-order derivative of the profit function of the high cost retailer is easier to obtain (as there is only one high cost retailer);

$$\begin{aligned} &-\frac{g(\widehat{s})D(p_H^*)\left[2D^{'}(p_H^*)(p_H^*-w_H)+D(p_H^*)\right]}{1-G\left(\widehat{s}\right)} \\ &-\left(\frac{g^{'}\left(\widehat{s}\right)\left(1-G\left(\widehat{s}\right)\right)+g^2(\widehat{s})}{\left(1-G\left(\widehat{s}\right)\right)^2}\right)D^3(p_H^*)(p_H^*-w_H)+2D^{''}(p_H^*)+D^{'}(p_H^*)(p_H^*-w_H), \end{aligned}$$

which, using the FOC, can be rewritten as

$$\begin{aligned} &-\frac{g(\widehat{s})D(p_H^*)(p_H^*-w_H)\left[D^{'}(p_H^*)+\frac{g(\widehat{s})D^2(p_H^*)}{1-G(\widehat{s})}\right]}{1-G\left(\widehat{s}\right)} \\ &-\left(\frac{g^{'}\left(\widehat{s}\right)\left(1-G\left(\widehat{s}\right)\right)+g^2(\widehat{s})}{\left(1-G\left(\widehat{s}\right)\right)^2}\right)D^3(p_H^*)(p_H^*-w_H)+2D^{''}(p_H^*)+D^{'}(p_H^*)(p_H^*-w_H). \end{aligned} \tag{7}$$

As the search cost distribution has an increasing hazard rate $\frac{g(\widehat{s})}{1-G(\widehat{s})}>g(0)$. It follows that if $g(0)$ is sufficiently large, the term in the square brackets is positive so that the whole expression is negative.

We now show that neither type of retailer has an incentive to imitate the equilibrium price of the other type of retailer and that we can find out-of-equilibrium beliefs that are such that no retailer wants to deviate from the reaction defined by (3) and (11). For a low cost retailer this is obvious. The above shows that a low cost retailer does not want to deviate if consumers believe that non-equilibrium prices are set by low cost retailers, i.e., if consumers have pessimistic views about the chance of finding a lower price on their next search. As fewer consumers buy after a deviation if they believe that non-equilibrium prices are set by a high cost retailer (i.e., they have more optimistic beliefs about finding a lower price on their next search), the deviation profits will be even lower under alternative beliefs.

To show that a high cost retailer does not want to deviate for alternative beliefs, a slightly more involved argument is needed. Note that from (11) it follows that if $g(0)$ is large enough p_L^* arbitrarily is close to w_L. Thus, $p_L^*<w_H$ for every $w_L<w_H$ if $g(0)$ is large enough. The high-cost retailer does not want to imitate the retail equilibrium price of the low cost retailer or to set a price in the neighbourhood of p_H^*. We also need to show that for any $p\in(p_L^*,p_H^*)$ we can find out-of-equilibrium beliefs about who has deviated such that the high cost retailer does not have an incentive to deviate to prices outside the neighbourhood of p_H^*. For any $p\in(p_L^*+\varepsilon,p_H^*-\varepsilon)$ (that is outside the immediate neighbourhoods of the equilibrium prices) we can write $p=\alpha p_L^*+(1-\alpha)p_H^*$ for some $\alpha\in(0,1)$ and choose a function $f(\cdot)$ such that the consumer out-of-equilibrium belief Pr(low cost retailer has deviated to price p) $=f(\alpha)$. Given that the profit function of the high cost retailer (assuming any deviation is attributed to a high cost retailer) is quasi-concave and that the high cost retailer does not have an incentive to deviate to prices in the

neighbourhood of p_L^* it follows that there exists a a continuous function $f(\alpha)$ such that the high cost retailer does not want to deviate to prices $p \in (p_L^* + \varepsilon, p_H^* - \varepsilon)$. If consumers blame high cost retailers for deviations to prices $p > p_H^*$, it is clear that these retailers also do not want to deviate upwards.

We conclude that the equations (11) and (3) define real "best response" for a set of out-of-equilibrium beliefs. As the profit functions are quasi-concave, the "best responses" are continuous.

Finally, we have to show uniqueness. We do this by proving that the slopes of "best response" functions are positive and smaller than 1. Rewriting the FOC of the high cost retailer (3) as $F_H(p_H^*; p_L^*, w_H) = 0$, the total differential is

$$\frac{\partial F_H}{\partial p_H^*} dp_H^* + \frac{\partial F_H}{\partial p_L^*} dp_L^* + \frac{\partial F_H}{\partial w_H^*} dw_H^* = 0,$$

where $\frac{\partial F_H}{\partial p_H^*}$ is given by (13) and thus negative. Also, $\frac{\partial F_H}{\partial p_L^*} = \left(\frac{g^{'}(\widehat{s})(1-G(\widehat{s}))+g^2(\widehat{s})}{(1-G(\widehat{s}))^2} \right) D(p_L^*) D^2(p_H^*)(p_H^* - w_H) > 0$ if $g^{'}(\widehat{s}) > -\frac{g^2(\widehat{s})}{(1-G(\widehat{s}))}$, which is the case if the search cost distribution has an increasing hazard rate. Thus, $\frac{\partial p_H^*}{\partial p_L^*} = -\frac{\partial F_H}{\partial p_L^*} / \frac{\partial F_H}{\partial p_H^*} < 1$.

Similarly, we can rewrite the FOC for the low cost retailers as $F_L(p_L^*; p_H^*, w_L) = 0$ so that $\frac{\partial p_L^*}{\partial p_H^*} = -\frac{\partial F_L}{\partial p_H^*} / \frac{\partial F_L}{\partial p_L^*}$. As $\frac{\partial F_L}{\partial p_L^*}$ is given by the second-order derivative evaluated at the equilibrium value p_L^*, we get it by substituting $p_L = p_L^*$ into (12) to obtain

$$\begin{aligned} & -\frac{\left(\frac{(N-1)^2}{N}+1\right) g(0) D(p_L^*)(p_L^*-w_L) \left[D^{'}(p_L^*) + \frac{\left(\frac{(N-1)^2}{N}+1\right) g(0) D^2(p_L^*)}{(N-1)+G(\widehat{s})} \right]}{(N-1)+G(\widehat{s})} \\ & -\left(\frac{g(\widehat{s})\left(\frac{(N-1)^2}{N}+1\right)}{((N-1)+G(\widehat{s}))^2} \right) g(0) D^3(p_L^*)(p_L^*-w_L) + D^{''}(p_L^*)(p_L^*-w_L) + 2D^{'}(p_L^*). \end{aligned} \tag{8}$$

Moreover, as $\frac{\partial F_L}{\partial p_H^*} = \left(\frac{g(\widehat{s})}{((N-1)+G(\widehat{s}))^2} \right) \left(\frac{(N-1)^2}{N} + 1 \right) g\,(0)\, D(p_H^*) D^2(p_L^*)(p_L^* - w_L^*)$ is positive and smaller in absolute value than $\frac{\partial F_L}{\partial p_L^*}$ it follows that $\frac{\partial p_L^*}{\partial p_H^*} = -\frac{\partial F_L}{\partial p_H^*} / \frac{\partial F_L}{\partial p_L^*} < 1$.