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INFORMATION ACQUISITION AND DIFFUSION IN MARKETS*

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Consumers acquire information through their own search efforts or through word-of-mouth communication within their social network. Information diffusion leads to free-riding and less active search. Free-riding consumers also create important positive externalities, however, as they are more likely to compare prices, imposing competitive pressure on firms. We show how market prices depend on network characteristics and search cost. For example, if search cost becomes small, price dispersion disappears, while prices converge to the monopoly level, with expected prices decreasing for small search cost. Prices are lower in societies with more connections, while price dispersion remains even in fully connected societies.

1. introduction

Decentralized markets rely on how information that is dispersed over many individuals is diffused (Hayek, 1945). Individual agents are, however, not endowed with a natural amount of information. Often, they have to spend resources, such as time, to search and acquire information. Accordingly, agents will only acquire information if the expected benefit exceeds the opportunity cost of doing so. This has led Grossman and Stiglitz (1980) to pose that efficient markets cannot exist if arbitrage is costly. Information can, however, also be acquired in less costly ways, namely through word-of-mouth (WOM) communication via friends (see, e.g., Ellison and Fudenberg, 1995, and Campbell, 2013).[1] WOM communication may come with a delay, however, as one has to wait for friends to communicate their information.

Costly information acquisition and diffusion (WOM communication) are clearly related. When few people acquire information themselves, little information will be diffused, whereas if information is disseminated efficiently, people may not have the incentive to spend resources to acquire information themselves. Thus, it is important to understand the interaction between the incentives to acquire information and the efficiency of the information diffusion process. This is especially so for online markets and online interaction through social networks, such as Facebook or LinkedIn. It is well documented[2] that online technologies have significantly reduced the search cost related to information acquisition and increased

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[1] Katz and Lazarsfeld (1955) is the classic study showing that information acquired through personal contacts is the prime reason why people buy a product.

[2] See, for example, Brown and Goolsbee (2002), Jensen (2007), Aker (2010), and Aker and Mbiti (2010). On WOM communication, see, for example, Godes and Mayzlin (2004), Chen et al. (2011), and Seiler et al. (2019).

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the possibilities of diffusion, and it is important to understand how these developments affect market outcomes.

In this article, we study the interaction between information acquisition, diffusion, and market power, and explain the impact of changes in the connectedness of people (impacting diffusion of information through WOM) and search costs on market outcomes. Our theoretical framework considers a homogeneous goods market where firms set prices and consumers engage in costly sequential search to acquire information about prices before buying one unit of the good (Stigler, 1961, and Diamond, 1971).[3] Consumers that have searched for prices spread this information through their network. The environment we study allows us to consider the impact of social network architecture, the speed of information transmission, and search costs on information acquisition and market power.

We find that independent of the acquisition (search) cost, there always exists a no-trade equilibrium that has the Diamond paradox at its origin: no one will acquire information if firms set very high prices, while setting high prices is (weakly) optimal if no one acquires information.[4] Importantly, WOM communication resolves the Diamond paradox, in that, it creates additional equilibria with positive sales. In all of these equilibria, searching consumers follow a reservation price strategy and firms do not price above this reservation price. With WOM communication, consumers determine whether or not to acquire information themselves. The possibility to get information through their social network implies that in any equilibrium, it cannot be the case that all consumers incur a search cost to acquire information themselves. If it were the case, prices would be equal to the willingness to pay and consumers would not be willing to spend a positive search cost, but instead free-ride on friends, as this also allows them to buy, possibly at the lowest price charged in the market without incurring the search cost. Thus, an endogenously determined fraction of consumers is informed through friends. By *free-riding on the information acquisition of friends*, the waiting consumers will be informed with positive probability about different prices, resolving the Diamond paradox.

Waiting consumers are the only ones who compare prices and they have a *positive effect* on market competition and consumer surplus. From a total surplus perspective, and under unit demand, this effect is unimportant, however, as long as there exists a positive fraction of waiting consumers: the increase in consumer surplus is offset by a decrease in profits. In our model, total surplus is affected by two factors: the search cost that is incurred by all consumers who search themselves and the cost due to some consumers delaying their purchases. We show that the market outcome is inefficient as there is insufficient search. In particular, if a small fraction of consumers who were not searching start to search, then this has a positive impact on total surplus as firms from which they buy sell without delay and, in addition, a larger fraction of consumers who delay their purchase will now be informed about prices by their friends.

In terms of the impact of the social network structure, we show that, contrary to what one may expect, even when the network gets very dense and many consumers have many connections, prices do *not* converge to the marginal cost of production and price dispersion remains. What matters for price dispersion and market power is the relative fraction of consumers that is informed of only one price. This fraction is endogenously determined and remains positive even in dense networks as information from friends comes with a delay creating an incentive to search and searching consumers buy immediately after acquiring information.

[3] Undoubtedly, consumers in social networks exchange a lot of information about product characteristics, such as quality, appearance, and/or convenience. Yet, casual evidence of reviews on platforms such as yelp or tripadvisor suggests that people also share information about prices. For example, a review of cafe Havelka in Vienna on yelp says “We didn’t have to wait long for service, and our orders came really quickly. 13.50 Euros for a sacher torte, an espresso, and a Melange.” In homogeneous goods markets, price communication is the only thing that matters. To study the impact of other types of information exchange, one would need to model differentiated goods and homophily in networks. The effect of exchanging price information in financial markets has been studied in Gray and Kern (2011) and in Han and Yang (2013).

[4] A no-trade equilibrium exists in many simultaneous and sequential search models where the first search is costly for all consumers (see, e.g., Diamond, 1971, and Burdett and Judd, 1983)

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The speed of information diffusion in networks is important in that it is a key determinant of the cost associated with waiting for information through WOM communication. We show that a higher speed of information diffusion has two opposing effects on market competition. First, it has a direct positive effect as more consumers do not search themselves, making price comparisons more likely. There is also an indirect effect, however, namely that as prices and price dispersion decline, consumers have more incentives to become active searchers themselves, especially when the speed of information diffusion is low to begin with. These two effects imply that both the fraction of active consumers and firms' profits have an inverted U-shape with respect to the speed of information diffusion: when this speed is low to begin with, firms have an incentive to increase it as this will also speed up their sales, but when the speed is already relatively large, the effect on expected price dominates and firms want to slow down information diffusion.

The impact of search cost is best illustrated by considering the case where the search cost becomes arbitrarily small. We show that in this case price dispersion disappears and almost all consumers become active themselves and buy immediately after they have searched themselves. As almost no consumer makes price comparisons, prices converge to monopoly levels. When the search cost increases, more consumers wait to get information through WOM communication so that a larger fraction of consumers make price comparisons, resulting in lower prices. Thus, and contrary to common wisdom, prices may be decreasing in search cost when WOM communication plays a role.

Our article provides a new argument to overcome the Diamond paradox.[5] Wolinsky (1986) resolves the Diamond paradox by having firms produce heterogeneous products and consumers searching not only for price but also for a good product match. Varian (1980) and Stahl (1989) impose search cost heterogeneity among consumers, where some exogenously determined fraction of "shoppers" have zero search cost and compare all prices before buying. Unlike these two papers, we endow consumers with the possibility to acquire information through WOM communication in addition to their own information acquisition by means of search. In this way, we endogenize the fraction of price comparing consumers and resolve the Diamond paradox.

Galeotti (2010) also combines WOM communication and consumer search. There are two main modeling differences between his paper and ours. First, in Galeotti (2010), consumers search for prices in a nonsequential fashion, whereas we have a sequential search framework. In most consumer retail markets, consumers observe the price at a firm before they decide whether to search another firm, making the sequential search paradigm more relevant. Second, Galeotti (2010) assumes the first search is free so that all consumers know at least one price, whereas we have truly costly search. These differences in modeling lead to pronounced differences in our understanding of information acquisition and diffusion in markets. First, in our setting, prices are increasing in search cost and tend to the monopoly price when search cost tends to zero, whereas in Galeotti (2010) prices converge to their competitive levels. Sequential search is the main reason for this difference. Second, if people get better connected and the network becomes dense, we show that price dispersion and market power remain, whereas in Galeotti (2010), prices converge to marginal cost. Costly first search is responsible for this important difference.[6] Third, our result that free-riding has a positive impact on prices is not present in Galeotti (2010) as under nonsequential search with the first search being free, searching consumers are always better informed than waiting consumers.[7] Fourth, our

[5] If individual consumers have downward sloping demand (or the first search is somehow free), then the Diamond paradox takes on a somewhat different form, namely, that all firms charge the monopoly price.

[6] Janssen et al. (2005) were the first to study the impact of the first search being costly on the participation of consumers in the marketplace. In their setting (and in contrast to ours), search is, however, the only source of information acquisition.

[7] Miegielsen (2014) adopts a sequential search framework but considers a model where somehow consumers possess information about prices before engaging in search and the amount of information that consumers have (and share with each other) is given exogenously.

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modeling of search and communication through a network allows us to consider the impact of the speed with which communication travels through the network. Finally, we offer a new resolution to the Diamond paradox.

There is also a growing literature on how WOM communication affects the pricing and advertising policy of firms in the market. Earlier papers in this literature (see, e.g., Biyalogorsky et al., 2001; Jun and Kim, 2008; Galeotti and Goyal, 2009; Kornish and Li, 2010; Campbell, 2013; Chuhay, 2015; Arbatskaya and Konishi, 2016; Bloch, 2016; Fainmesser and Galeotti, 2016) consider how a monopoly firm can introduce its product optimally through a network assuming that consumers passively wait until they receive an advertisement from the firm, or they are informed through their network. Instead, we allow consumers to actively reach out and search for information and we study markets where firms compete in prices. A more recent literature on networks concerns the strategic interaction of competitors on networks. Chen et al. (2018) and Fainmesser and Galeotti (2020) study how firms price discriminate between different individuals depending on whether or not an individual influences many consumers. Campbell (2019) studies how in markets with product differentiation awareness of the available products is communicated through a social network and how this affects firms' prices and the efficiency of market outcomes. Campbell et al. (2020) consider how information about product quality of an experience good flows through the network and how this affects the quality provision by firms. None of these papers studies, however, the interaction between the incentives of consumers to acquire their own information through search and the diffusion of information through the social network and their impact on firms' prices and market outcomes.

The rest of the article is organized as follows. The next section lays out a simple model of sequential search and information diffusion. It also discusses standard consumer search models and how our article relates to them. Section 3 proves that with WOM communication, active markets exist and that in equilibrium there is insufficient search. Section 4 then presents the comparative statics analysis, while Section 5 shows how the equilibrium outcome is qualitatively similar if consumers have different number of links. We conclude with a discussion.

2. A MODEL AND PRELIMINARY RESULTS

Throughout this article, we consider a duopoly[8] market for a homogeneous good where firms compete in prices. The unit cost of production is constant and normalized to zero. As firms may choose mixed strategies, we denote the strategy of a firm i by $F_i(p)$, representing the probability that a firm charges a price not larger than p. The support of the price distribution is determined endogenously with $\underline{p}$ and $\overline{p}$ being the lower and upper bound of the support, with the possibility of some prices in the interior of the interval not being chosen. On the demand side of the market, there is a unit mass of identical *consumer networks*, on which we expatiate in the next paragraph. A consumer in each network demands a unit of the product and has a willingness to pay equal to $v > 0$. If a consumer does not consume, she receives a payoff of zero. To buy a product, consumers have to be informed about their prices. To get this information, consumers can either search themselves or get informed via their network of friends through WOM communication.

The model we present and analyze in this and the next section is a stylized model of how search and WOM communication interact. The main results we derive continue to hold in a more general model that is analyzed in a working paper version (see, Atayev and Janssen, 2023). In our model, each network consists of the same finite number of consumers. Consumers within a network have the same number of $k \geq 2$ links[9] and consumers indi-

[8] With more than two firms, the characterization of the mixed strategy distribution in prices is more complicated and in this case, it is difficult to analyze the gains of search versus the gains of free riding. Galeotti (2010) also considers duopoly markets.

[9] If consumers have only one link, then there is no consumer who will be informed about both prices and the Diamond paradox results. The reason is (as it will become clear through the analysis) that our model shares a feature of

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vidually and simultaneously decide whether to be active and search themselves or to be passive and wait for information from friends. Active consumers can only acquire information through their own search effort, whereas passive consumers can only buy when they get information from their friends. Active consumers search sequentially (see, e.g., Kohn and Shavell, 1974, and Stahl, 1989): after randomly choosing the first firm and obtaining its price quote at a search cost s, with $0 < s < v$, they optimally decide whether to buy the product or continue to search. If they search the second firm, they buy at the lowest-priced firm as long as that price is smaller than v. Consumers who searched share all the information they have acquired with their immediate friends.[10] Consumers who decided to wait observe the information their immediate friends have shared with them and buy at the lowest of the prices they are informed of. As they have to wait for their friends to share information, their payoff is discounted by a factor δ, $0 < \delta < 1$.

The timing of the game, players' strategies, and payoffs are as follows. Firms and consumers simultaneously choose prices and whether to actively search, respectively. Having searched one firm, active consumers decide whether or not to search the second firm and whether or not to buy. This all happens in period 1, i.e., an active consumer who searches both firms still buys in period 1. The only event happening in period 2 is that consumers who decided to be passive in the first period get information from their immediate friends and decide whether or not to buy. A firm i's strategy is its price $p_i, i = 1, 2$, whereas a consumer has to decide on whether or not to search and if to search whether to search the second firm. Thus, the consumer strategy can be denoted by $(q, \sigma(p))$, where $q \in [0, 1]$ is the search probability and $\sigma(p)$ represents the search strategy of an active consumer that specifies at which price the consumer stops searching and buys and when to continue searching. The strategy of a waiting consumer is not explicitly taken into account in this notation as it is clearly optimal for them to buy at the lowest price they received, given this price is smaller than v. We normalize total market demand to 1. The search strategy of consumers and the firms' prices lead to a demand, $d_{it}(p_i, p_j)$, $i \neq j, i, j = 1, 2$ for firm i in period t. As the number of consumers is simply a multiplication factor in a firm's payoff expression, we simplify notation and work with a firm's average payoff, which is given by $p_i(d_{i1}(p_i, p_j) + \delta d_{i2}(p_i, p_j))$, where $d_{it}(p_i, p_j)$ is the fraction of the total demand that is served by firm i. Active consumers who buy at price p receive a payoff of $v - p - s$, if they search one time and a payoff of $v - p - 2s$ if they search twice, whereas passive consumers get a payoff of $\delta(v - p)$ if they buy at price p. Any consumer who has decided not to search buys from neither firm and obtains zero if he has no neighbor who searched.

We use symmetric perfect Bayesian equilibria (PBE) with passive beliefs as solution concept. A PBE is described by a set of prices for both firms, p_1 and p_2, and consumers' strategies $(q, \sigma(p))$ such that they are optimal given beliefs and the strategies of the others.

2.1. *Relation to the Consumer Search Literature.* We now present a canonical consumer search model to be able to highlight our contribution to the search literature and show three preliminary results. Consider the classic model of sales by Varian (1980) where there is a fraction $\lambda \in (0,1)$ of consumers that is informed about both prices and a fraction $1 - \lambda$ of uninformed consumers that only knows one price, and a maximum price $\overline{p} > 0$, which for the time

the well-known Stahl (1989) model of sequential search that firms will only consider setting prices that searching consumers will immediately accept. Thus, only waiting consumers are in the position to compare prices and if consumers have only one link, they will not be informed about both prices either.

[10] We use the term "immediate friends" for people in a social network a consumer has a direct connection with. Consumers can also be indirectly linked to other consumers, namely, if they are connected only via a mutual friend. These consumers are not referred to as "immediate friends."

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being can be thought of as being exogenously given.[11] Varian (1980) shows that the profit of firm i is given by

$$\Pi_i(p_i) = \left(\frac{1-\lambda}{2} + \lambda(1 - F_j(p_i))\right)p_i,$$

and that the unique equilibrium is in mixed strategies where firms have to be indifferent between any price in the support of the mixed strategy distribution. He also shows that the upper bound of the support is equal to $\overline{p}$. As a firm charging $\overline{p}$ only sells to half of the uninformed consumers, $\Pi_i(p_i) = (1-\lambda)\overline{p}/2$, so that one can solve for the symmetric equilibrium mixed strategy distribution as

$$F(p) = 1 - \frac{1-\lambda}{2\lambda}\frac{\overline{p} - p}{p}.$$

Given $F(p)$, the expected price $E[p]$ and the expected minimum of two prices $E[\min\{p_1, p_2\}]$ can be determined, where both depend on $\overline{p}$ and λ, with

$$E[p] = \frac{1-\lambda}{2\lambda}\overline{p}\ln\left(1 + \frac{2\lambda}{1-\lambda}\right),$$
$$E[\min\{p_1, p_2\}] = \frac{1-\lambda}{\lambda}\overline{p}\left(\frac{1-\lambda}{2\lambda}\ln\left(1 + \frac{2\lambda}{1-\lambda}\right) - 1\right).$$

Stahl (1989) endogenizes $\overline{p}$ by allowing uninformed consumers (who in his model get to know one price for free) to pay a search cost s to observe more prices. In a duopoly model, this essentially means that these consumers get fully informed about both prices by paying an additional s. He characterizes a so-called reservation price equilibrium (RPE), where consumers buy immediately if, and only if, the price p they observe is smaller than or equal to this reservation price r. At r consumers are indifferent between buying immediately and continuing to search. As consumers' payoffs of buying and continuing to search after observing any price $\widetilde{p}$ are given by $v - \widetilde{p}$ and $v - (1 - F(\widetilde{p}))\widetilde{p} - F(\widetilde{p})E(p|p < \widetilde{p}) - s$, respectively, it is easy to see that r is implicitly defined by

$$F(r)\int_{\underline{p}}^{r}(r - p)dF(p) = s. \tag{1}$$

Stahl (1989) shows that if there is a solution to (1), it must be unique for any nondegenerate $F(p)$. In addition, all equilibria have the property that $\overline{p} = \min\{r, v\}$ so that consumers decide to buy immediately after receiving some information, that is, $F(r) = 1$.[12] Thus, (1) reduces to $v - r = v - E[p] - s$, yielding the solution

$$r = \frac{s}{1 - \frac{1-\lambda}{2\lambda}\ln\left(1 + \frac{2\lambda}{1-\lambda}\right)},$$

where the reservation price depends on s and λ. Thus, in the Stahl (1989) model, both $E[p]$ and $E[\min\{p_1, p_2\}]$ depend on the exogenous parameters s and λ (and on v if $r > v$).

Janssen et al. (2005) add to the Stahl (1989) model the idea that uninformed consumers should also pay a search cost s for the first price quote. In that case, the search strategy should also consider whether or not to search in the first place. As the consumers' outside option in

[11] In Varian (1980), $\overline{p} = v$.

[12] This is also true in our model and the intuitive argument in our context is given just above Lemma 3.

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traditional search models is not to buy and get a payoff of 0, this yields the additional condition that $v - E[p] - s \geq 0$, resulting in a fraction $q(s, \lambda) \in (0, 1]$ of consumers becoming active.

Relative to this consumer search literature, our article endogenizes the fraction λ of fully informed consumers via the determination of q. Importantly, as in Janssen et al. (2005), consumers' search strategy should not only determine when to stop searching and buy, but also whether or not to start searching in the first place. In contrast to Janssen et al. (2005) , the "outside option" is not exogenous (and equal to 0) in our model, but equal to the payoff of not searching, which is endogenous and depending on how other consumers are searching and how information is transmitted. Consumers may not only obtain information through their own search effort, but also by relying on friends. From an ex ante perspective, consumers that do not search and acquire information themselves can either buy at $E[p]$ (if they are only informed about one price) or at $E[\min\{p_1, p_2\}]$ (if they are informed about two prices). The fraction q will be endogenously determined by these considerations. In this regard, the relation with Burdett and Judd (1983) is also important. They study a simultaneous search model and show that an equilibrium exists where consumers randomize between searching once and twice, and in that equilibrium, the indifference condition for consumers is given by $v - E[p] - s = v - E[\min\{p_1, p_2\}] - 2s$: some consumers are willing to incur the search cost twice as there is a chance they buy at a lower price. Consumers who observe both prices are crucial in generating price dispersion in Burdett and Judd (1983). In our article, it is the passive consumers that are crucial in generating price dispersion: they incur a cost due to the delay with which they observe prices, but they save on search costs and may also buy at the lowest price.

2.2. *Preliminary Results.* Having described how our model fits into the consumer search literature, we now present some preliminary results for our model that are familiar to the search literature. A first result is that there always exists a trivial "no trade" equilibrium with $q = 0$.

LEMMA 1. *For any $s > 0$, there exists an equilibrium without sales where $q = 0$.*

As the first search is costly, this result should not come as a surprise: knowing they will not sell to anyone, firms may set prices larger than $v - s$ and this pricing behavior rationalizes consumers' beliefs that it is not rational to search. If no one searches, no information is shared and consumers cannot buy.

A second preliminary result is that there does not exist an equilibrium with $q = 1$. The reason is similar to the Diamond paradox (Diamond, 1971): if everyone searches themselves, they all buy immediately without comparing prices. If no one is informed about both prices, we have the equivalent of λ being equal to 0 in the Varian (1980) model described above, and the price distribution becomes degenerate. But this would imply that firms have an incentive to price at v and consumers do not have an incentive to search themselves.

LEMMA 2. *For any $s > 0$, there does not exist an equilibrium where $q = 1$.*

Thus, in any active market with positive sales, it must be that $0 < q < 1$.

Our final preliminary result is that $\overline{p}$ cannot be larger than r (as in Stahl, 1989). To understand why, note first that $\overline{p}$ cannot have a mass point as (with $0 < q < 1$) there will be a positive fraction of passive consumers who compare prices, making it optimal to undercut. Second, active searchers have the same options as in Stahl (1989) so that (1) also characterizes their optimal stopping rule. As a consequence, active consumers do not buy at a price $\overline{p}$ if it is larger than r. Therefore, and third, waiting consumers will either not be informed of any price at all or if $\overline{p} > r$, they will always be informed of a price lower than $\overline{p}$ as searching consumers will always continue to search and find a lower price if they observe $\overline{p} > r$. Thus, a firm will

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not sell to any consumer if it charges $\overline{p} > r$. As it is clear that in addition no firm would set a price larger than v, we have the following result:

LEMMA 3. *Any equilibrium where goods are bought has* $F(\min\{r, v\}) = 1$.

3. ANALYSIS

We now construct an equilibrium where some consumers buy. Note first that in any such equilibrium we should have that $r < v$. This follows from the following three facts: (i) markets can only be active if consumers are indifferent, that is, $0 < q < 1$; (ii) the expected payoff of waiting is strictly positive implying that the expected payoff of searching must be positive, that is, $v - E[p] - s > 0$; and (iii) the reservation price is determined by $r = E[p] + s$ (as $F(r) = 1$).

The expected payoff of waiting consumers equals:

$$\delta\big(1-(1-q)^k\big)v - 2\delta\left[\left(1-\frac{q}{2}\right)^k - (1-q)^k\right]E[p] - \delta\left[1+(1-q)^k - 2\left(1-\frac{q}{2}\right)^k\right]E[\min\{p_1, p_2\}]$$

$$= \delta\big(1-(1-q)^k\big)(v - E[p]) + \delta\left[1+(1-q)^k - 2\left(1-\frac{q}{2}\right)^k\right](E[p] - E[\min\{p_1, p_2\}]).$$

This expression can be understood as follows. There is a probability $(1-q)^k$ that none of a consumer's friends is active. With the remaining probability of $1-(1-q)^k$, they buy and receive the value v of consuming the product. From an ex ante perspective, they either pay the expected price $E[p]$ or the expected minimum price $E[\min\{p_1, p_2\}]$. The probability that all of a consumer's friends receive the price of one particular firm, say firm 1, is given by $(1-q/2)^k - (1-q)^k$, where the first term is the probability that none of the friends observes the price of the other firm (firm 2). Thus, the probability that all of a consumer's friends observe one and the same price that is set by either firm 1 or firm 2 is given by $2[(1-q/2)^k - (1-q)^k]$. The probability that a consumer's friends receive information about both prices is then given by the complement of not getting any price or all getting the same price, that is, $1-(1-q)^k - 2[(1-q/2)^k - (1-q)^k]$, which reduces to the above expression involving $E[\min\{p_1, p_2\}]$.

The probability q that a consumer is active is such that the consumer is indifferent between being active and passive and, therefore, in equilibrium we should have:

$$\begin{aligned} v - E[p] - s &= \delta\big(1-(1-q)^k\big)(v - E[p]) \\ &\quad + \delta\left(1+(1-q)^k - 2\left(1-\frac{q}{2}\right)^k\right)(E[p] - E[\min\{p_1, p_2\}]), \end{aligned} \tag{2}$$

which can be rewritten as

$$\frac{s}{v} = \big(1-\delta+\delta(1-q)^k\big)\frac{v - E[p]}{v} - \delta\left(1+(1-q)^k - 2\left(1-\frac{q}{2}\right)^k\right)\frac{E[p] - E[\min\{p_1, p_2\}]}{v}, \tag{3}$$

where the left-hand side (LHS) and right-hand side (RHS), respectively, represent the relative cost and the relative benefit of being active.

We now turn to the determination of the equilibrium pricing strategy of the firms. As in canonical models of consumer search, an individual firm faces a trade-off between extracting surplus from consumers who observe only its price and competing for price-comparing consumers. If we let μ_1 denote the fraction of consumers waiting and seeing only this firm's price

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and μ_2 the fraction of consumers waiting and seeing both prices, an individual firm's expected profit setting price $p \le \min\{r, v\}$ is given by

$$\Pi(p) = \Big(\frac{q}{2} + \delta\mu_1 + \delta\mu_2(1 - F(p))\Big)p.$$

Clearly, in the first period, a firm only sells to consumers that search themselves and randomly choose the firm in question, which happens with probability $q/2$. In the second period, a firm only sells to consumers that have not searched themselves. Some of these consumers only receive the price of this particular firm and buy from that firm without comparing prices. As explained above, this happens with probability $(1 - q/2)^k - (1 - q)^k$. Thus, the probability a consumer buys in period 2 without comparing prices is given by $\mu_1 = (1 - q)[(1 - q/2)^k - (1 - q)^k]$. Other waiting consumers observe the price of both firms, and from the above considerations it follows that the probability a consumer buys in period 2 while comparing prices is given by $\mu_2 = (1 - q)[1 + (1 - q)^k - 2(1 - q/2)^k]$. A firm will only sell to these consumers if it has the lowest price, which happens with probability $(1 - F(p))$. Note that both μ_1 and μ_2 are continuous in q and for any $0 < q < 1$, it must be that $\mu_1, \mu_2 > 0$ and $\mu_1 + \mu_2 < 1$.

Equating these expected profits with the profit of setting a price equal to the upper bound of the distribution gives the equilibrium price distribution as

$$F(p) = 1 + \eta - \eta\frac{\overline{p}}{p}, \text{ with support } [\underline{p}, \overline{p}], \tag{4}$$

where $\eta = (q/2 + \delta\mu_1)/(\delta\mu_2)$ and $\underline{p} = \overline{p}\eta/(1 + \eta)$ solves $F(\underline{p}) = 0$, whereas $\overline{p} = \min\{r, v\}$. The fraction η is the ratio of consumers who do not compare prices to those that do compare prices (as in the traditional models of Varian, 1980, and Stahl, 1989). Here, the fraction of consumers who are informed about only one price consists of the fraction of active consumers and those passive consumers who receive only one price quotation from friends, whereas the fraction of consumers who are informed about both prices consists of only passive consumers who receive information about the offerings of both firms through their social network.

It is easy to see that for any $0 < q < 1$, $\eta > 0$, so that $F(p)$ is well defined. It is clear that as q approaches 0 or 1, η becomes infinitely large[13] and price dispersion disappears. Also, as η is finite for any intermediate value of q, it is clear that the relation between η and q is nonmonotonic. When q increases, there are two effects that create this nonmonotonicity: the direct effect is that a larger fraction of consumers is active and, in equilibrium, they buy immediately after observing their first price offer, while the indirect effect is that the relative fraction of passive consumers that are informed about multiple prices changes.

Having explained the different conditions that should hold in an RPE with positive sales, we are now able to provide the main result of this section.

THEOREM 1. *There exists an* $\bar{s} \le v$ *such that an RPE exists if and only if* $s \le \bar{s}$. *If an RPE exists it is determined by the probability* q^* *that solves (2) and by* r *and* $F(p)$ *as given by (1) and (4), with* $0 < q^* < 1$. *Furthermore, as* $\lim_{s\to 0} q^* = 1$ *and price dispersion disappears, that is,*

$$\lim_{s\to 0}\overline{p} = \lim_{s\to 0}\underline{p} = v. \tag{5}$$

[13] Although the latter limited result is straightforward, the former is not. Yet, one can show that

$$\lim_{q\downarrow 0}\frac{\frac{q}{2\delta} + (1-q)\Big(\big(1-\frac{q}{2}\big)^k - (1-q)^k\Big)}{(1-q)\Big(1 + (1-q)^k - 2\big(1-\frac{q}{2}\big)^k\Big)} = \lim_{q\downarrow 0}\frac{\frac{1}{2\delta} - \big(1-\frac{q}{2}\big)^k + (1-q)^k - k(1-q)\Big(\frac{(1-\frac{q}{2})^{k-1}}{2} - (1-q)^{k-1}\Big)}{k(1-q)\Big(\big(1-\frac{q}{2}\big)^{k-1} - (1-q)^{k-1}\Big) - 1 - (1-q)^k + 2\big(1-\frac{q}{2}\big)^k}$$

$$= \infty,$$

where we applied l'Hopital's rule to obtain the first equality. The second equality follows from the facts that the numerator converges to $(1 + \delta k)/2$, whereas the denominator goes to 0.

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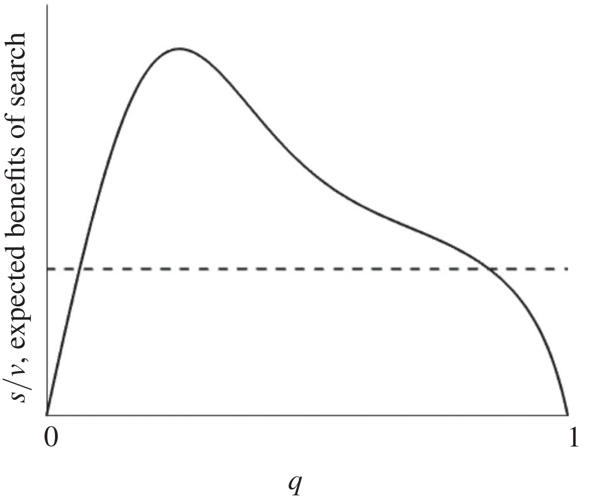

FIGURE 1

ILLUSTRATION OF EXISTENCE OF AN RPE FOR $v = 1, s = 0.05, k = 5, \delta = 0.9$

The equilibrium endogeneously determines $q^* \in (0, 1)$. It follows that even though ex ante all consumers are identical, in equilibrium the population is endogeneously split into two parts. One part of the population is active and acquires information themselves through costly search, whereas the other part is passive and only relies on information diffused by friends. As active consumers buy at the first price they discover, while with positive probability passive consumers buy at the lowest of both prices, price dispersion endogeneously arises in equilibrium. The limiting result can be explained by delving deeper into the consumer indifference equation (2). Note that if $s = 0$, then there is no reason for a consumer to be passive and we have that $q = 1$. Moreover, we have that $E[p] = E[\min\{p_1, p_2\}]$ as price dispersion disappears as active consumers will continue to search if there is a possibility to obtain a lower price. Thus, as the RHS of (2) converges to 0, the LHS should also converge to 0, implying that $E[p] = v$.[14]

Figure 1 provides an illustration of the theorem. The horizontal axis represents the fraction q of searching consumers, while the cost and the expected benefits of search are presented on the vertical axis. The solid curve represents the expected benefits, whereas the dashed horizontal line represents the cost of search. In terms of Equation (3), the solid curve represents the RHS of the equation and the dashed line represents the LHS. In the proof, we show what the figure presents, namely, that when q approaches 0 or 1, the expected benefit of search approaches 0. This is quite intuitive: if q approaches 0 or 1, there are very few consumers who compare prices either because there are almost no nonsearching consumers (q close to 1) or because there is almost no information that is diffused in the system (q close to 0). Thus, firms exercise their market power, prices get close to v, and individually consumers have no incentive to search. As for interior values of q the expected benefits are positive and continuous in q, it must be the case that for small enough values of s an equilibrium exists.

The figure shows that for small enough values of s there are two intersection points and, hence, two equilibrium values of q^* where the market is active. One may argue, however, as in other search models (see, e.g. Burdett and Judd, 1983, Fershtman and Fishman, 1992, Janssen and Moraga-Gonzalez, 2004 and Honda, 2015) that the equilibrium corresponding to the higher search probability can be called a "stable" equilibrium in the sense that if q falls

[14] Note that for $\delta = 1$, the limiting result may be different as in this case the LHS of (2) is always equal to 0. This is the main reason why we included a cost of waiting into the simple model of this section.

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(slightly) short of the equilibrium value the expected benefit of search exceeds the cost so that consumers have an incentive to search more intensively. The comparative static analysis focuses on this "stable" equilibrium.

We finally consider the welfare properties of the equilibrium. There are generally three causes for a welfare loss in our model: the search cost that is incurred by everybody who searches, the cost due to some consumers delaying their purchases, and a "loss" owing to some consumers waiting, not being informed about prices, and dropping out of the market altogether. We show that the market outcome is inefficient: either the market is inactive which, as $s < v$, is clearly inefficient, or (if the search cost is small enough) the market generates positive sales, but also in that case, there is too little search from a social welfare perspective. To see the latter, consider that consumers search optimally from a social perspective and denote by $\widetilde{q}$ the socially optimal search probability. In a social optimum the marginal benefit of an individual searching has to be equal to the (social) cost of searching, which is s:

$$\delta(1-\widetilde{q})^k v + (1-\delta)v + \delta k(1-\widetilde{q})^k v = s. \tag{6}$$

The first term on the LHS reflects the fact that if a consumer waits he may not have received information from friends, which happens with probability $(1-\widetilde{q})^k$, and he drops out of the market. The advantage of searching is that a consumer buys for sure. The second term reflects that if an individual is active instead of passive, he does not wait to buy. Finally, the third term on the LHS reflects the fact that by searching an individual's neighbors may now buy instead of not buying. This happens if a neighbor himself did not search, which happens with $(1-\widetilde{q})$, and did not get information from all of his other neighbors, which happens with probability $(1-\widetilde{q})^{k-1}$. As this applies to all the k neighbors an individual has, this additional benefit arises with probability $k(1-\widetilde{q})^k$.

To understand whether there is inefficient search, we have to compare this to the individual incentives to search as given in (2), and to do so, it is convenient to rewrite the LHS of (6) as $((1-\delta)+\delta(1-\widetilde{q})^k)v + \delta k(1-\widetilde{q})^k v$. The first term in both equations differs to the extent that an individual consumer is only interested in his own improvement in surplus when searching, represented by $v - E[p]$, whereas social surplus increases by v as firms also benefit from the consumer searching. The second term of the social benefit of additional search does not have a counterpart in (2). This second term reflects the fact that waiting consumers have a higher probability of receiving any information and buying the good, which is a positive externality that individual consumers do not take into account when they make their individual search decision. Finally, the second term on the RHS of (2) reflects the fact that individual consumers may benefit by waiting as they can buy at the lowest of two prices if they are informed of both of them. This term is absent from (6), however, as firms' profits are to the same extent negatively affected. [15]

All these three differences point in the same direction: there are social benefits to search that are not taken into account by an individual consumer deciding on whether or not to actively search. Moreover, there is an individual benefit of waiting that does not exist at the social level. Thus, we have the following proposition:

Proposition 1. *In any RPE, the equilibrium level of search is smaller than the social optimum level, that is,* $q^* < \widetilde{q}$.

4. COMPARATIVE STATICS

Using the equilibrium characterization, we now analyze how market outcomes depend on exogenous parameters. We will first focus on the impact of network structure on equilibrium

[15] Thus, even though the consumers that do not search provide a positive externality to other consumers, in that, by comparing prices they put competitive pressure on firms, this effect does not play a role in the overall welfare analysis in terms of total surplus as long as markets are active.

prices, before concentrating on the speed of communication in the network and the cost of searching. A social network like Facebook has significantly increased the number of connections people have (although there remain a nonnegligible fraction of consumers who do not use Facebook or other social networks) and the speed of information diffusion through the network. Online markets have also significantly reduced search cost s. In this section, we discuss the implications of these changes on market outcomes. Unless explicitly discussed otherwise, the changes in firms' profits is perfectly in line with expected price. In an RPE, the expected market price is proportional to s and given by $E[p] = s\eta \ln(1 + 1/\eta)/(1 - \eta \ln(1 + 1/\eta))$, which is increasing in η.

4.1. *The Impact of Denser Networks.* We first investigate the limiting behavior when all consumers tend to have many links. One may think that if all consumers potentially get information from many friends, competition would prevail and prices converge to marginal cost with price dispersion being eliminated. Surprisingly, however, as the next result shows, price dispersion remains an essential feature of any RPE, and even in the limit when all consumers have infinitely many links, price dispersion remains and prices are bounded away from 0.

PROPOSITION 2. *For any k, any RPE with positive sales is characterized by price dispersion, that is, $\underline{p} < \overline{p}$. Moreover, $\lim_{k\to\infty} \underline{p} < \lim_{k\to\infty} \overline{p}$. Finally, for any $s < (1 - \delta)v$, there exists a $\widehat{k}(s)$ such that for $k > \widehat{k}(s)$, an RPE with positive sales exists.*

The reason that price dispersion remains if the network gets denser is as follows. For any given $0 < q < 1$, almost all consumers who wait get informed about both prices as k gets very large. Thus, $\mu_1 \approx 0$, while $\mu_2 \approx 1 - q$, so that that $\eta \approx q/(2\delta(1 - q))$. This expression is easily interpreted: if k is very large, only searching consumers observe one price and of them only half visits a given firm, whereas all the waiting consumers observe both prices. Like in the models of Varian (1980) and Stahl (1989), this ratio determines the way prices are dispersed. The factor δ appears as the profit a firm gets over waiting consumers is discounted. For any given $0 < q < 1$, η is finite and if s is small enough, there is a benefit to search. A consequence of price dispersion is that prices do not converge to marginal cost. The proof that an RPE with positive sales exist for small enough values of s essentially shows that if $k \to \infty$, the benefit of search continues to have a shape similar to the one depicted in Figure 1.

It is important to understand that for this result to hold the first search should be costly. If this were not the case, the Diamond paradox applies with $p^* = v$ as all consumers would obtain one price quote and as this price is smaller than r, they would buy immediately, with no consumer waiting to get information from friends.[16] This also explains why in Galeotti (2010) prices do converge to marginal cost if the network of consumers gets dense: it is the consequence of the first search being assumed to be free in his model. Similarly, if there are other ways to introduce price information into the network without some consumer having to incur a cost for it (e.g., by firms truthfully advertising prices), then prices would converge to marginal cost as this information gets diffused through the network.

It is difficult to analytically analyze the effect of increasing k when k is finite. Using numerical simulations, Figures 2 and 3 show the impact by gradually increasing k from 2 to 25 for $v = 1$, $s = 0.05$, and $\delta = 0.93$. Figure 3 depicts the impact of network density on the expected price. It shows that the expected price is decreasing in k and that certainly when k is small, this impact is relatively strong. The main, direct impact can be understood by noting that for given fraction q of consumers that search the term μ_1, which is proportional to $(1 - q/2)^k - (1 - q)^k$, is decreasing in k, whereas μ_2, which is proportional to $(1 + (1 - q)^k - 2(1 - q/2)^k)$, is increasing in k. It follows that for given q, η is decreasing in k.

[16] Note that even if $\delta = 1$, price dispersion remains in the limit when $k \to \infty$. In this case, $q \to 0$ in such a way that $(1 - q)^k$ remains strictly positive.

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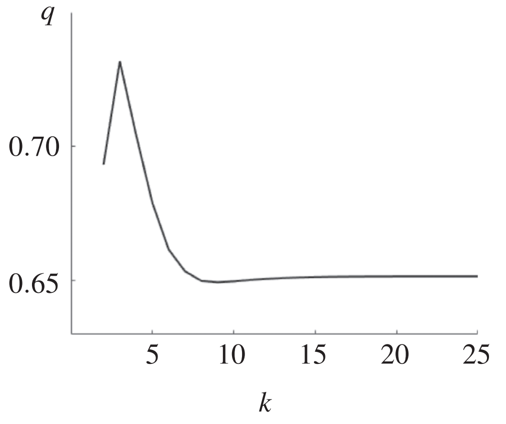

FIGURE 2

IMPACT OF k ON THE SHARE OF ACTIVE CONSUMERS

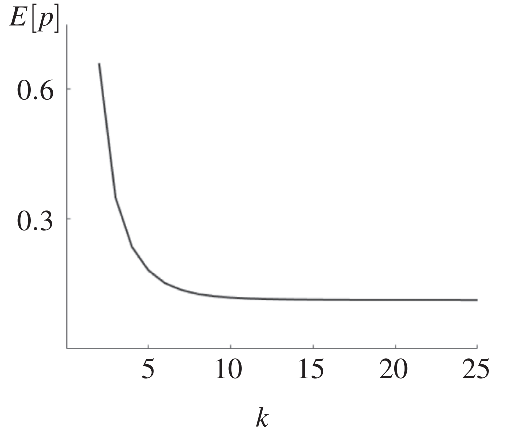

FIGURE 3

IMPACT OF k ON EXPECTED PRICE

There is, however, also an indirect effect through the fraction q of consumers that searches. The effect of k on q is depicted in Figure 2 and the effect is nonmonotonic. Two opposing forces contribute to this nonmonotonic effect. First, as k increases, waiting consumers are more likely to get information from friends and make price comparisons, making it more attractive to wait. Second, the opposite effect is that as k increases, the level of price dispersion falls so that the main benefit of waiting (which is to be able to buy at the lowest of two prices) becomes less important, making waiting less attractive. For small k, the latter effect dominates, whereas for large k the first effect dominates. The indirect effect of k through q is, however, not as strong as the direct effect and thus, the overall impact on expected price is decreasing.

Note that the rise in the fraction of consumers actively acquiring information as the network becomes denser is strikingly different from Galeotti (2010) and Galeotti and Goyal (2010) where an agent's probability of actively acquiring information negatively correlates with the number of links they have. The combined price effect through a lower expected price and lower price dispersion as measured by $E[p] - E[\min\{p_1, p_2\}]$ is at the heart of this.

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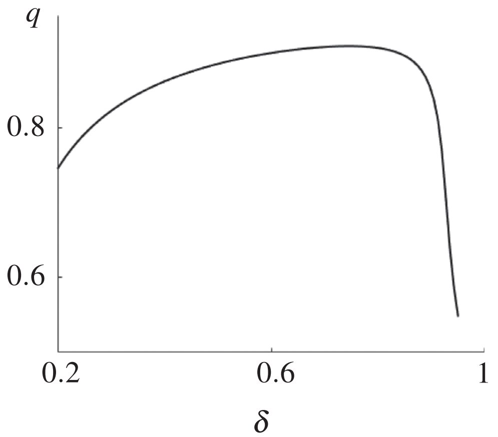

FIGURE 4

IMPACT OF δ ON THE SHARE OF ACTIVE CONSUMERS

4.2. *The Speed of Information Diffusion.* Social media have significantly increased the speed with which consumers may share information. In our model, speed is only relevant in terms of the delay with which passive consumers buy and this is measured by δ. A higher δ, resulting in faster information transmission, permits waiting consumers to access information provided by friends more quickly. The effect on market outcomes are more subtle, however, as δ affects the incentives to search by making it less costly to wait. Thus, the net effect on market outcomes is not clear. As with the comparative statics with respect to the density of the network, there is a direct and an indirect effect of δ on prices. The direct effect can be seen by taking the partial derivative of η with respect to δ. As this derivative is negative, an increase in the speed of information processing in the population increases the share of price comparing consumers, putting downward pressure on prices. This should not come as a surprise: the larger δ, the more attractive it is to wait for information from friends. There is also an indirect effect via q, however. For small values of δ, the indirect effect is very similar to the indirect effect we mentioned in relation to the impact of k, namely that as prices and price dispersion decline, consumers have more incentives to become active searchers themselves. As δ becomes large, there is almost no downside to waiting anymore and consumers massively change their behavior and do not search.

Numerical simulations in Figures 4 and 5 show the effect of an increase in δ on the share of searching consumers and the expected price when we change the value of δ (here from 0.2 to 0.95) and keep the other parameter values at $v = 1$, $s = 0.05$, and $k = 5$. Figure 5 shows that even though the indirect effect is nonmonotonic, the direct effect dominates and that the total effect of δ on the expected price is negative.

Interestingly, the speed of information diffusion also has a nonmonotonic effect on firms' profits as shown in Figure 6. When the speed of information diffusion is relatively low, firms prefer a large share of searching consumers as these consumers buy immediately, whereas the sales to waiting consumers are heavily discounted. Thus, even though expected price is monotonically decreasing in the speed of information diffusion, the fact that the fraction of searching consumers is increasing offsets the decrease in revenue per consumer. On the other hand, when the speed of information diffusion is already relatively high to begin with, firms would not want to increase it further. In the Online Appendix, we show that the effect for large values of δ remains true if firms do not discount future profits, or if we reformulate firms' profits as $(p_i d_{i1}(p_i, p_j) + \delta p_i d_{i2}(p_i, p_j))/(1 + \delta)$. The strong negative impact of δ on expected price is responsible for this. However, the inverted U-shape feature of profits becomes either less pronounced or disappears and the negative impact of δ on profits dominates.

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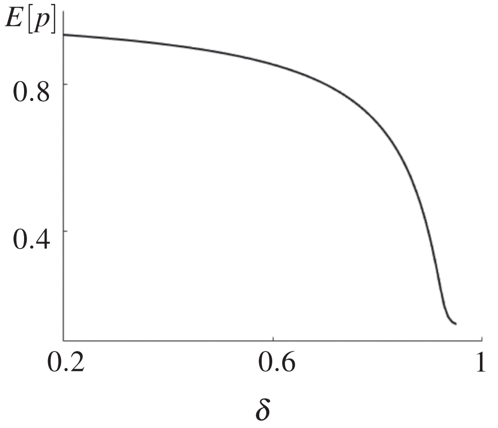

FIGURE 5

IMPACT OF δ ON EXPECTED PRICE

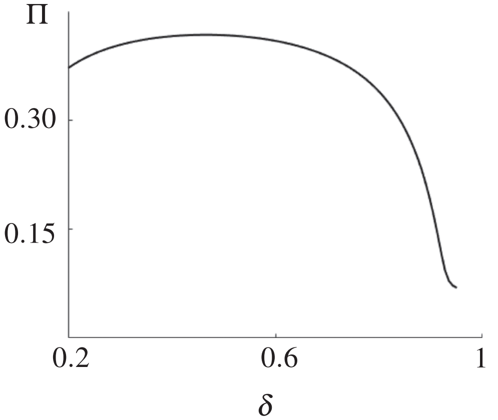

FIGURE 6

IMPACT OF δ ON FIRM PROFIT

4.3. *The Effects of Changes in Search Cost.* Finally, we study the impact of a change in s. We already know that the optimal search probability goes to 1 in the limit when s goes to zero, suggesting that as s starts increasing from 0, the optimal search probability decreases. The next proposition shows that the underlying effects also hold true outside the region where s is close to 0. The reason is simple: in any stable equilibrium the benefit of search is decreasing in the share of active consumers (cf., Figure 1). Thus, if the search cost s is increasing, to satisfy the consumer indifference Equation (2), the equilibrium share of active consumers is decreasing.

PROPOSITION 3. *In any stable RPE, the equilibrium share of active consumers q^* is decreasing in s.*

The effect of s on the expected equilibrium price is more subtle, however. We know from Theorem 1 that in the limit when s is arbitrarily small, prices converge to the monopoly price v: when s is arbitrarily small, almost all consumers are actively searching themselves and very few consumers make price comparisons. This immediately implies that for small enough search cost, the expected price must be decreasing in s : as more consumers make price comparisons when s increases, there will be more competition between firms. We know, however,

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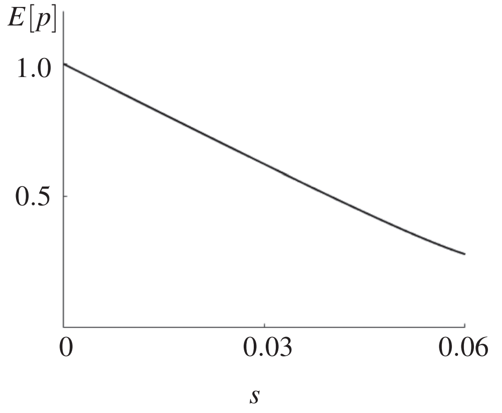

FIGURE 7

IMPACT OF s ON EXPECTED PRICE

that the relation between η and q is nonmonotonic and that expected price is increasing in η. Thus, for small enough s, expected price is decreasing in s, but this is not necessarily so for larger values of s. Figure 7 illustrates, however, that the expected price may well be decreasing in s for a large range of s values.[17]

5. HETEROGENEITY IN CONSUMER LINKS

In many markets, consumers differ in the number of connections they have. In this section, we show that our analyses in the previous sections extend to such markets. Although Atayev and Janssen (2023) contains a general analysis of random social networks, we restrict ourselves in this section to a simple, yet sharp way to model heterogeneity in consumer links, namely, to consider a so-called *star network*. A star network consists of $k+1$ consumers with $k \geq 2$, where one consumer, called the *core*, is connected to every other consumer, and all consumers in the *periphery* are only linked to the core. Hence, the core has k links and each member of the periphery has a single link. Let q_c and q_p be the respective search probabilities of the core and a periphery consumer. Star networks are frequently studied (see, e.g., Galeotti and Goyal, 2010) and observed in laboratory experiments (see, e.g., Goyal et al., 2016, and van Leeuwen et al., 2020).

As before, we focus on symmetric equilibria where consumers with the same number of links search with the same probability. In the current setting, consumers with a different number of links may search with different probabilities, however. Still, some of the equilibrium properties of active markets that we have discussed before remain valid. First, and similar to Lemma 2, we cannot have that $q_c = 1$ as the core is the only consumer who can compare prices and so with $q_c = 1$, there will be no price-comparing consumers, firms will set price equal to v, and no consumer will have an incentive to search, an inconsistency. A similar inconsistency arises if periphery consumers do not search for sure. Thus, it must be that $q_c < 1$ and $q_p > 0$. Second, from $q_c < 1$ and $q_p > 0$, it follows that the core is informed about both prices with a positive probability, implying there cannot be mass points in the symmetric price distribution: price dispersion is part of any equilibrium. Third, and similar to Lemma 3, it must be that $F(r) = 1$ as firms that charge prices above r will not sell to any consumer. Thus, we have the following lemma.

LEMMA 4. *Any equilibrium with positive sales must be an RPE with $F(r) = 1, q_c < 1$, and $q_p > 0$ and exhibits price dispersion.*

[17] We used the following parameter values for the figure: $v = 1$, $\delta = 0.9$, and $k = 5$.

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An individual firm's expected profit of setting a price equal to $p \leq \min\{r, v\}$ can then be written as

$$\Pi(p) = (\mu_0 + \delta\mu_1 + \delta\mu_2(1 - F(p)))p,$$

where $\mu_0 = (q_c + kq_p)/(2(k+1))$ is the share of consumers who search and buy from the firm under consideration. This share consists of two parts. A random consumer is a core one out of $k+1$ times. She searches with probability q_c and visits the firm under consideration half of the time. In k out of $k+1$ times, a random consumer is a member of the periphery and visits the firm and buys immediately with probability $q_p/2$.

The second part of the expected demand consists of consumers who wait and observe only the price of a firm under consideration:

$$\mu_1 = \frac{(1-q_c)\Big(\big(1-\frac{q_p}{2}\big)^k - (1-q_p)^k\Big) + \frac{q_c}{2}k(1-q_p)}{(k+1)}.$$

The numerator of this expression consists of two terms: one representing the core and the other, consumers at the periphery. The core waits with probability $1 - q_c$. She gets informed about the price of the firm only if all searching peripheries visit this firm, which happens with probability $(1 - q_p/2)^k - (1 - q_p)^k$. A periphery waits with probability $1 - q_p$ and is informed about the firm's price if the core searches the firm under question, which happens with probability $q_c/2$.

The final part of the demand consists of price-comparing consumers:

$$\mu_2 = \frac{(1-q_c)\Big(1 + (1-q_p)^k - 2\big(1-\frac{q_p}{2}\big)^k\Big)}{k+1}.$$

Since only the core has multiple links, she is the only consumer who may compare prices. This happens if she waits and is informed by members of the periphery about both prices. The former event happens with probability $1 - q_c$ and the latter with probability $1 + (1 - q_p)^k - 2(1 - q_p/2)^k$. As price-comparing consumers make a purchase from the firm under consideration if its price is lower than the rival firm's price, they buy with probability $1 - F(p)$.

As before, this profit must be equal to the profit that the highest price in the support of the price distribution yields. Thus, the equilibrium price distribution is given by (4), where now we have

$$\eta = \frac{\mu_0 + \delta\mu_1}{\delta\mu_2}.$$

We next argue that for s close enough to 0, the stable RPE has to be such that η is positive and finite. To see this note that, as the reservation price r continues to be defined by (1), it follows that $\lim_{s\to 0} r = E[p]$. Given that there are no mass points, it also follows that $\lim_{s\to 0} E[p] = E[\min\{p_1, p_2\}]$. Thus, in the limit when s goes to 0, price dispersion disappears, which happens if, and only if, $\eta \to \infty$ (which is equivalent to μ_2 approaching 0). Given the expression for μ_2 this implies that either $\lim_{s\to 0} q_c = 1$ or $\lim_{s\to 0} q_p = 0$. As, for the same reasons as in Section 3, the second possibility yields an unstable RPE, we focus on equilibria with $\lim_{s\to 0} q_c = 1$.

In the proof of the next proposition in the Appendix we show that in active markets the probabilities q_c and q_p cannot be the same. If they were the same, it would be the case that the payoff of waiting would be higher for the core than for the periphery (as the core has more links to benefit from), while the payoff of searching is identical across consumers. Following this reasoning that when facing identical situation the core may have more incentive to wait,

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we focus on the case where the core searches with a lower probability than the periphery, i.e., $q_c < q_p$. We prove that for small enough s, $q_p = 1$ if $q_c \in (0, 1)$, that this equilibrium exists and is determined by

$$\text{(7)} \qquad \begin{cases} v - E[p] - s = \delta\Big(v - E[p] + \Big(1 - \dfrac{1}{2^{k-1}}\Big)(E[p] - E[\min\{p_1, p_2\}])\Big), \\ v - E[p] - s > \delta q_c(v - E[p]). \end{cases}$$

We are now ready to state the main result of this section.

PROPOSITION 4. *Consider a star network with $k \geq 2$. For any $v > 0$, $\delta \in (0, 1)$, and sufficiently small search cost s, there exists an RPE that is given by $(q_c, q_p, r, F(p))$, which is determined by (1), (4), and (7). Furthermore, as $s \to 0$, price dispersion necessarily disappears with $\overline{p} = \underline{p} = v$.*

The proof is in the Appendix, but from the above exposition it is clear that the reasoning is very similar to that in Theorem 1. The equilibrium satisfying (7) exists as long as the core is indifferent between searching and waiting, while periphery consumers search with probability one.

In addition to this equilibrium, we show in the Appendix that there may be an equilibrium where the core searches with a higher probability than periphery consumers, that is, $q_c < q_p$. It is difficult to provide (sufficient) conditions on exogenous parameters, which would give rise to this equilibrium. Nonetheless we show that, if it exists, the equilibrium is determined by, along with (1) and (4),

$$\text{(8)} \qquad \begin{cases} v - E[p] - s = & \delta\big(1 - (1 - q_p)^k\big)(v - E[p]) \\ & +\delta\Big(1 + (1 - q_p)^k - 2\big(1 - \frac{q_p}{2}\big)^k\Big)(E[p] - E[\min\{p_1, p_2\}]), \\ v - E[p] - s = & \delta q_c(v - E[p]). \end{cases}$$

We also demonstrate that this possible equilibrium must be characterized by the price dispersion and as the search cost vanishes, the price level converges to the monopoly level.

6. CONCLUSION

In this article, we have analyzed how WOM communication through social networks affects information acquisition and diffusion by consumers and how this impacts on the market power of firms. Without WOM communication, our model is prone to the Diamond paradox where the market breaks down due to the fact that no consumer makes price comparisons. WOM communication overcomes the Diamond paradox. Consumers that do not actively search themselves and free-ride on their friends in the social network may well be informed about different prices. The price comparisons they make provide positive externalities to the rest of the consumer population that actively searches as firms compete to be able to also sell to them.

As some consumers do compare prices, while others do not, the market is characterized by price dispersion. The level of prices and the nature of price dispersion depends on the network architecture, the search cost, and how quickly information is diffused in the social network. In the context of evaluating the impact of online markets and social networks, it is important to know how expected price, price dispersion and firms' profits react to (i) a decrease in search cost, (ii) an increase in the connectivity of the social network and (iii) an increase in the speed of information diffusion in the network. We find that there are opposing effects as the increased connectivity and speed of information diffusion lower expected market prices, whereas the decrease in search cost increases them. Importantly, price dispersion does not disappear even if all consumers are very well connected.

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We see our article as making a first step in analyzing how WOM communication and sequential search interact with each other. There are obvious ways our work can be extended in different directions. One direction that may be taken is to analyze markets with product differentiation à la Wolinsky (1986). In this case, consumers may not only communicate about prices, but also about the product match. In such markets, the degree of homophily (defined as the closeness of a consumer's preferences to those of his neighbors) will be important. Another direction for future research would be to model the incentives to share information directly. In some markets, consumers receive a financial benefit from firms for a successful referral and an important question is how consumers will react to such incentives and when such a financial incentive is optimal from a firm's perspective and to whom to give it.

APPENDIX: PROOFS

Proof of Lemma 1. Consider a candidate equilibrium where firms set price v and consumers do not search. We show that players do not have a profitable deviation. Consider first an individual firm. Its equilibrium payoff equals zero. If the firm deviates to another price, its payoff remains zero as consumers do not search so no consumer is informed about the deviation price. Now consider an individual consumer. Her equilibrium payoff is also zero. If she deviates to searching, she gets a negative payoff of $-s$ as the surplus from buying is zero. Thus, no one has an incentive to deviate.

Proof of Lemma 2. By contradiction, suppose there exists an equilibrium with $q = 1$. Then, the equilibrium price must be equal to v. First, we show that there cannot exist an equilibrium in mixed-strategy pricing. Assume it exists for contradiction. Two cases are possible: all prices are above r or some prices are lower than r. In the former case, all prices higher than r will be compared to another price, which means that only r must be charged in equilibrium, a contradiction. In the latter case, a contradiction arises because a firm has no incentive to charge a price equal to the lowest price in the support, as a firm setting this price can raise the price slightly without affecting its demand. Second, we show that there cannot be an equilibrium in pure strategies with $p_i \leq p_j \leq v$, $i \neq j$, where at least one of the inequalities is strict. If $p_i < p_j \leq v$, firm i has an incentive to raise its price as any consumer that observes this increased price will still buy from that firm. If $p_i = p_j < v$, consumers do not search beyond the first firm and consumers would continue to buy from firm i if it raises its prices to $\min\{p_i + s, v\}$, giving firm i again an incentive to raise its price. However, if firms set prices equal to v, then each consumer has an incentive to deviate and not search. Given that the firms' prices are equal to v, a consumer's payoff of searching equals $-s$, whereas if she deviates to not searching, her payoff is zero. Hence, $q = 1$ cannot be part of an equilibrium.

Proof of Lemma 3. We show that for $0 < q < 1$, it cannot be that $F(r) < 1$. Suppose, by contradiction, that it was, which implies that $\overline{p} > r$. Clearly, it has to be the case that in an equilibrium with sales, we have $\overline{p} \leq v$ as otherwise a firm charging $\overline{p}$ does not make any sales, while it would always get some consumers visiting the firm and buying if it sets a price equal to s. So, consider $v \geq \overline{p} > r$. First note that there cannot be an atom at $\overline{p}$. This is because consumers that observe this price would continue to search a rival firm (as $\overline{p} > r$) so that firms would increase their profit by undercutting this price as it leads to a discontinuous increase in a firm's demand (as a strictly positive share of consumers would compare prices). However, if there is no atom at $\overline{p}$, a firm charging this price does not make any sales as consumers observing this price would continue to search and with probability one observe a lower price. In addition, all waiting consumers will not be informed about $\overline{p}$ as all consumers who observed this price also observe another price. Thus, it cannot be that $\overline{p} > r$ and therefore it should be that $F(r) = 1$.

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Proof of Theorem 1. We start by noting the following facts. First, $E[p] = \overline{p} - \int_{\underline{p}}^{\overline{p}} F(p)dp = \eta\overline{p}\ln(1+1/\eta)$ and $E[\min\{p_1, p_2\}] = \overline{p} - 2\int_{\underline{p}}^{\overline{p}} F(p)dp + \int_{\underline{p}}^{\overline{p}} F^2(p)dp$. Second,

$$\begin{aligned} E[p] - E[\min\{p_1, p_2\}] &= \int_{\underline{p}}^{\overline{p}} F(p)dp - \int_{\underline{p}}^{\overline{p}} F^2(p)dp \\ &= \eta\overline{p}\left((1+2\eta)\ln\left(1+\frac{1}{\eta}\right) - 2\right). \end{aligned}$$

Finally, if $r \leq v$, we have that $\overline{p} = \min\{r, v\} = r = s/(1-\eta\ln(1+1/\eta))$.

As long as $r \leq v$, these facts allow us to rewrite (2) as

$$\text{(A.1)}\quad \frac{s}{v} = \frac{1-\delta\left(1-(1-q)^k\right)}{1+\frac{\eta}{1-\eta\ln\left(1+\frac{1}{\eta}\right)}\left[(1-\delta+\delta(1-q)^k)\ln\left(1+\frac{1}{\eta}\right)+\frac{\delta\mu_2}{1-q}\left((1+2\eta)\ln\left(1+\frac{1}{\eta}\right)-2\right)\right]}.$$

The RHS of (A.1) is clearly continuous in $0 < q < 1$. It is also positive for all $0 < q < 1$. To see this, note that as $\eta\ln(1+1/\eta) < 1$, the denominator is clearly positive if $\ln(1+1/\eta) > 2/(1+2\eta)$. As for $\eta \downarrow 0$, this latter inequality clearly holds, while the LHS and the RHS both approach 0 as $\eta \to \infty$, this inequality holds for all η if the derivative of the LHS is more negative than that of the RHS. The derivative of the LHS is $-1/(\eta(1+\eta))$, whereas the derivative of the RHS is $-2/((1+2\eta)^2)$. It is easy to see that the former derivative is smaller than the latter. Thus, as the numerator is also positive, the RHS is always positive.

We next show that there exist values of $0 < q < 1$ that solve (A.1) for small enough search costs. Given the above, it suffices to show that the RHS of (A.1) approaches 0 as $q \to 1$, which is associated with $\eta \to \infty$. In the limit, its numerator converges to $(1-\delta)$ and its the denominator goes to ∞. In order to establish the latter result, it is helpful to use the following three facts:

$$\lim_{\eta\to\infty} \eta\ln\left(1+\frac{1}{\eta}\right) = 1,$$

$$\lim_{q\uparrow 1}\frac{\delta\mu_2}{1-q} = \lim_{q\uparrow 1}\delta\left(1+(1-q)^k - 2\left(1-\frac{q}{2}\right)^k\right) = \delta\left(1-\frac{1}{2^{k-1}}\right),$$

$$\begin{aligned} \lim_{\eta\to\infty}\frac{\eta\left((1+2\eta)\ln\left(\frac{1+\eta}{\eta}\right)-2\right)}{1-\eta\ln\left(\frac{1+\eta}{\eta}\right)} &= \lim_{z\downarrow 0}\frac{\frac{1}{z}\left(\left(1+\frac{2}{z}\right)\ln(1+z)-2\right)}{1-\frac{\ln(1+z)}{z}} \\ &= \lim_{z\downarrow 0}\frac{(2+z)\ln(1+z)-2z}{z^2 - z\ln(1+z)} \\ &= \lim_{z\downarrow 0}\frac{\ln(1+z)+\frac{2+z}{1+z}-2}{2z-\ln(1+z)-\frac{z}{1+z}} \\ &= \lim_{z\downarrow 0}\frac{(1+z)\ln(1+z)-z}{z(1+2z)-(1+z)\ln(1+z)} \\ &= \lim_{z\downarrow 0}\frac{\frac{z}{(1+z)^2}}{\frac{z(3+2z)}{(1+z)^2}} = \lim_{z\downarrow 0}\frac{1}{3+2z} = \frac{1}{3}, \end{aligned}$$

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(where we used l'Hopital's rule to obtain third and final lines in the last limiting result), which implies that as $q \uparrow 1$, the second term in the denominator of the RHS of (A.1) goes to ∞, since η multiplied by the terms in the square brackets converges to a strictly positive finite value and $1 - \eta \ln(1 + 1/\eta)$ converges to zero.

The only thing left to show is that the above construction yields an RPE where indeed $r \leq v$. To see this, first note that if $s = 0$ there is an RPE where $r = 0$ and $q = 1$ and that r and q are continuous functions of s. Thus, $r < v$ for small enough values of s. Next, we argue that it cannot be the case that in an active market $r = v$. As long as $r \leq v$, we can rewrite (1) as $r = E[p] + s$ so that the expected benefit of actively searching is given by $v - E[p] - s = v - r$. Clearly, at $r = v$ this expected benefit of search equals 0. However, the expected benefit of waiting remains strictly positive, which implies that consumers have no incentive to actively search, or $q = 0$, a contradiction. In other words, (A.1) can only hold if $r < v$. Thus, as all endogeneously determined variables are continuous in s, there must exist a critical value of s, denoted by $\bar{s}$ such that an RPE exists for $s < \bar{s}$, whereas for all $s > \bar{s}$, there does not exist a $0 < q < 1$ such that (A.1) holds.

Now, we show the limiting price for $s \downarrow 0$. We know that s approaching zero is associated with $q \uparrow 1$, or $\eta \to \infty$ meaning that price dispersion vanishes. Then, it suffices to evaluate the limiting value of r. We note that

$$
\begin{aligned}
r = & \frac{s}{1 - \eta \ln\left(1 + \frac{1}{\eta}\right)} \\
= & \frac{\left(1 - \delta\left(1 - (1-q)^k\right)\right)v}{1 - \eta \ln\left(1 + \frac{1}{\eta}\right) + (1 - \delta(1 - (1-q)^k))\eta \ln\left(1 + \frac{1}{\eta}\right) + \left(\frac{\delta \mu_2}{1-q}\right)\eta\left((1 + 2\eta)\ln\left(1 + \frac{1}{\eta}\right) - 2\right)}.
\end{aligned}
$$

Notice that as $q \uparrow 1$, the numerator converges to $(1 - \delta)v$. The first two terms and the last term in the denominator converge to zero as $\eta \to \infty$, since

$$
\begin{aligned}
\lim_{\eta \to \infty} \eta\left((1 + 2\eta)\ln\left(\frac{1+\eta}{\eta}\right) - 2\right) &= \lim_{z \downarrow 0} \frac{\left((1 + \frac{2}{z})\ln(1+z) - 2\right)}{z} \\
&= \lim_{z \downarrow 0} \frac{((z+2)\ln(1+z) - 2z)}{z^2} \\
&= \lim_{z \downarrow 0} \frac{\ln(1+z) + \frac{2+z}{1+z} - 2}{2z} \\
&= \lim_{z \downarrow 0} \frac{(1+z)\ln(1+z) - z}{2z(1+z)} \\
&= \lim_{z \downarrow 0} \frac{\ln(1+z) + 1 - 1}{2 + 4z} = 0
\end{aligned}
$$

(where we used l'Hopital's rule to obtain the third and final lines) whereas the remaining term in the denominator converges to $1 - \delta$. Therefore, we have that $\lim_{s \downarrow 0} r = v(1 - \delta)/(1 - \delta) = v$.

The proof of the theorem is now complete.

Proof of Proposition 1. Note that the additional social benefit of a consumer searching is strictly larger than

$$
\left(1 - \delta + \delta(1 - q)^k\right)v - s
$$

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as firms also benefit from a sale without delay. Using (2), it follows that

$$
\begin{aligned}
&\big(1-\delta+\delta(1-q)^k\big)v-s \\
&= \big(1-\delta+\delta(1-q)^k\big)v-\big(1-\delta+\delta(1-q)^k\big)(v-E[p]) \\
&\quad +\delta\bigg(1+(1-q)^k-2\Big(1-\frac{q}{2}\Big)^k\bigg)(E[p]-E[\min\{p_1,p_2\}]) \\
&> \big(1-\delta+\delta(1-q)^k\big)E[p]+\delta\bigg(1+(1-q)^k-2\Big(1-\frac{q}{2}\Big)^k\bigg)(E[p]-E[\min\{p_1,p_2\}]) \\
&> 0.
\end{aligned}
$$

This means that from the perspective of social welfare, consumers search too little. This completes the proof of the proposition.

Proof of Proposition 2. Active markets with $0<q<1$ in the limit as $k\to\infty$ can exist only if (A.1) holds in the limit. For any $0<q<1$, as $k\to\infty$ μ_1 converges to 0, μ_2 to $(1-q)$ and so η converges to $q/(2\delta(1-q))$. As for $0<q<1$, the terms $(1-q)^k$ and $(1-q/2)^k$ on the RHS of (A.1) converge to zero when $k\to\infty$, the limiting indifference condition is approximately

$$
\frac{s}{v}=\frac{1-\delta}{1+\lim_{k\to\infty}\frac{\eta}{1-\eta\ln\left(1+\frac{1}{\eta}\right)}\Big[(1-\delta)\ln\Big(1+\frac{1}{\eta}\Big)+\delta\Big((1+2\eta)\ln\Big(1+\frac{1}{\eta}\Big)-2\Big)\Big]}.
$$

Since the RHS approaches 0 if $\eta\to\infty$ and $1-\delta$ if $\eta\downarrow 0$, whereas it is positive for any finite $\eta>0$, it is clear that this equation can always be satisfied for some strictly positive and finite η if $s<(1-\delta)v$. It follows that for any $s<(1-\delta)v$, one can find a critical value $\widehat{k}(s)$ such that for larger values of k, an RPE exists.

Price dispersion remains as $k\to\infty$ if η remains strictly positive and finite. However, we know that η remains finite and, moreover, it is strictly positive for $0<s<(1-\delta)v$.

Proof of Proposition 3. Observe that changes in s affect the LHS of (A.1) only. In particular, the LHS is increasing in s. As the RHS of the indifference equation (A.1) must be decreasing in q in a stable RPE, the optimal search probability of indifferent buyers q must be decreasing in s.

Proof of Proposition 4. Some parts of the proof are similar to the proof of Theorem 1. In order to avoid repetition, we omit some details here. We first argue that in an equilibrium with active sales it cannot be that $q_c=q_p$. Clearly, there are no active sales if $q_c=q_p=0$. Also, as argued in Lemma 2, it cannot be the case in equilibrium with active sales that $q_c=q_p=1$. To rule out that $0<q_c=q_p<1$, suppose to the contrary that $0<q_c=q_p<1$. This would imply that the core and a consumer in the periphery are both indifferent between searching and waiting in equilibrium. Searching yields a payoff equal to $v-E[p]-s$ to both. Since $q_c\le q_p$ implies that $1-q_c>(1-q_p)^k$ and $(1-(1-q_p)^k)(v-E[p])>q_c(v-E[p])$, waiting yields the core a payoff larger than $\delta(1-(1-q_p)^k)(v-E[p])$, which is larger than the payoff a consumer in the periphery obtains from waiting, which is $\delta q_c(v-E[p])$. This, however, implies that for $0<q_c=q_p<1$, it cannot be that both the core and a periphery are simultaneously indifferent between waiting and searching.

Thus, in the sequel, we consider first the case where $q_c<q_p$ and then where $q_c>q_p$.

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Case: $q_c < q_p$. We show that for small search costs, the equilibrium has $q_p = 1$ and $q_c \in (0, 1)$. The conditions for this to be true are that

$$\begin{aligned} v - E[p] - s =& \delta\big(1 - (1 - q_p)^k\big)(v - E[p]) \\ &+ \delta\left(1 + (1 - q_p)^k - 2\left(1 - \frac{q_p}{2}\right)^k\right)(E[p] - E[\min\{p_1, p_2\}]), \\ v - E[p] - s >& \delta q_c(v - E[p]). \end{aligned}$$

First, from the above argument it follows that the equality and the inequality are consistent with each other for any $0 < q_c < q_p \leq 1$: searching yields the same payoff to both the core and a periphery, while we have argued above that $q_c < q_p$ implies that waiting yields the core a payoff, given by the RHS of the equation, that is larger than the payoff of waiting to a consumer in the periphery, given by the RHS of the inequality. Thus, in this case, if the core is indifferent between searching and waiting, the periphery consumers must prefer to search themselves. So, it must be that $q_p = 1$ for any $0 < q_c < q_p$.

As for s small enough $r \leq v$ and setting $q_p = 1$, we first rewrite the equation in (7) as

$$\frac{s}{v} = \frac{1 - \delta}{1 + \frac{\eta}{1 - \eta \ln\left(1 + \frac{1}{\eta}\right)}\left[(1 - \delta)\ln\left(1 + \frac{1}{\eta}\right) + \frac{\delta(k+1)}{1 - q_c}\mu_2\left((1 + 2\eta)\ln\left(1 + \frac{1}{\eta}\right) - 2\right)\right]},$$

where η only depends on q_c and hence the equation determines q_c. As in the proof of Theorem 1, one can establish that the RHS of the equation is positive for any $0 < q_c < 1$ and converges to zero as $q_c \uparrow 1$, which is associated with $\mu_2 \downarrow 0$ and $\eta \to \infty$. Thus, this equilibrium exists for s close to 0. Finally, applying reasoning similar to the proof of Theorem 1, one can evaluate the limiting result: $\lim_{s\downarrow 0} r = v$.

Case: $q_c > q_p$. Note that from the above argument, it follows that the equations in (8) can only hold simultaneously if $q_c > q_p$. Next, assuming that $r \leq v$, we first rewrite (8) as

$$\begin{aligned} \frac{s}{v} &= \frac{1 - \delta + \delta(1 - q_p)^k}{1 + \frac{\eta}{1 - \eta \ln\left(1 + \frac{1}{\eta}\right)}\left[(1 - \delta + \delta(1 - q_p)^k)\ln\left(1 + \frac{1}{\eta}\right) + \frac{\delta(k+1)}{1 - q_c}\mu_2\left((1 + 2\eta)\ln\left(1 + \frac{1}{\eta}\right) - 2\right)\right]}, \\ \frac{s}{v} &= \frac{1}{\frac{1}{1 - \delta q_c} + \frac{\eta \ln\left(1 + \frac{1}{\eta}\right)}{1 - \eta \ln\left(1 + \frac{1}{\eta}\right)}}. \end{aligned}$$

From these equations, we observe that the RHSs of both equations converge to zero as $\eta \to \infty$, which is the case if, and only if, $q_c \uparrow 1$. If such an equilibrium exists, employing a similar line of reasoning as in the proof of Theorem 1, one can show that in equilibrium with positive trade it must be that $r < v$ and $r \uparrow v$ as $s \downarrow 0$.

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Figure 1: Impact of δ on firm profit where firms do not discount future profits
Figure 2: Impact of δ on firm profit normalized by $1+\delta$

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