# Unobserved Wholesale Contracts*

Maarten C.W. Janssen† Santanu Roy‡

**Abstract**

A manufacturer with private information about product quality may earn higher expected profit selling through a retailer than selling directly to end consumers. For this to happen, it is important that their wholesale pricing contract with the retailer is unobserved by consumers. Secret wholesale contracts may prevent two types of distortions. It can prevent distortions due to price signaling by the manufacturer to end buyers and it can prevent double marginalization by the retailer. Reasonable pricing strategies exist where the manufacturer chooses wholesale prices that are independent of quality and yield higher expected profit relative to both direct selling as well as selling through a retailer with observable vertical contract; further, though consumers do not learn true quality they may earn higher expected consumer surplus. Because the retailer acts as an intermediary, the strategic interaction is different from standard signaling games and we show that the pooling equilibria we focus on satisfy a new equilibrium refinement that we develop in the spirit of the Intuitive Criterion.

**JEL Classification**: L13, L15, D82, D43.

**Key-words**: Asymmetric Information; Product Quality; Vertical Contracts; Wholesale Pricing.

*We thank Daniel Garcia, Renato Gomes, Patrick Rey, Andrew Rhodes, Yaron Yehezkel, participants of EARIE 2022 and SAET 2023 and seminar audiences in Vienna and Moscow for usefulness suggestions. Janssen acknowledges financial support from the Austrian Science Foundation under project number FG 6. This paper supersedes an earlier paper titled "Delegated Selling and Intermediated Signaling".

†University of Vienna and CEPR. E-mail: maarten.janssen@univie.ac.at.

‡Southern Methodist University. E-mail: sroy@smu.edu

## 1 Introduction

Manufacturing firms are often much better informed about quality attributes of their products than end users. Confidential product testing and customer feedback yield private information to producers about product durability, reliability, likelihood of being defective and potential health hazards. Producers may also be more informed about ethical dimensions of the production process such as the environmental footprint, the working conditions of employees (for instance, the use of child labor), employment discrimination, the use of genetically modified organisms, and "fairness" of prices paid to suppliers. In addition to manufacturers, owners of durable goods (such as land, cars or houses) acquire private information about the quality of the units they own (such as hidden mechanical problems with cars, issues with foundation of the house or toxicity and pollution of land).

Producers or current owners in these markets have a choice between selling directly to end users or through intermediaries[1] (retailers) that set the terms of trade (retail prices) faced by end users. Upstream contracts between manufacturers/owners and intermediary retailers are rarely observed by end users. However, manufacturers/owners have the option of publicly disclosing these contractual terms. In this paper, we argue that by selling through intermediary retailers and using contracts that are unobserved by end users, ($i$) producers/owners may hide their private information and that ($ii$) this may yield higher industry profit *and* consumer surplus than when upstream contracts between producers/owners and intermediaries are observed by end users. To stress the importance of the vertical structure in this interaction, in the rest of the paper, we use the terminology of manufacturer/retailer, but the analysis holds true for any other setting with intermediated selling.

The key idea is simple. Whether a manufacturer with private information sells through a retailer while making the wholesale contract observable to consumers or sells directly to consumers at a price that is known to consumers, he is able to signal information about quality to end users. Moreover, a high quality manufacturer has an incentive to choose a high (wholesale) price such that demand is sufficiently low and a low quality manufacturer has no incentive to imitate such a high

[1] Car owners can sell their used car directly or to used vehicle retailers such as Carmax that can then sell to final buyers (at prices set by the retailer). Likewise, home owners now have the option of selling their houses for cash to companies such as Opendoor, iBuyers, Homevestors etc; the latter then sell these houses to final buyers for profit.

price/low demand outcome. The resulting outcome potentially involves large inefficiencies due to these signaling distortions. Instead, when a manufacturer sells through a retailer and the contract between the manufacturer and the retailer is not observed by consumers, the manufacturer can *hide* information about quality by selling to the retailer at a price that is independent of quality. As a result, the retail price does not convey any information about quality to the buyers. Signaling does not work as the final buyer (Receiver) does not observe the actions of the manufacturer (Sender). Hiding information by choosing wholesale prices independent of quality eliminates the signaling distortions that arise when the manufacturer sells directly to buyers.[2]

As the outcomes that can be sustained under unobservable wholesale contracts may have relatively low prices and high demand on average, the manufacturer makes more profit than when his actions reveal information to end users and consumers are better off being in the dark about product quality and *not being able to infer* quality from prices. These gains in expected profit and consumer surplus disappear when consumers can observe the upstream contract. When uninformed consumers are able to directly observe and infer from wholesale prices set by the manufacturer, signaling distortions cannot be avoided and may be magnified by double marginalization at the retail level. We show that despite using two-part tariffs, observability of upstream contracts lowers expected profit and consumer surplus relative to both direct selling and to what may be achieved by selling through a retailer under a pooling equilibrium with secret wholesale contracts. Under secret wholesale contracts, separating equilibria also exist, but we do not focus on them, as they lead to similar signaling distortions at the retail level as the well-known signaling distortions under direct selling.

We also show that regulatory policies that enable credible disclosure of quality by firms through direct communication (for instance, "truth in advertising" regulation) may reduce consumer welfare and manufacturer profits as in the resulting full information outcome the manufacturer maximally exploits its market power (which is restrained by beliefs under unobservable wholesale contracts).

To make these points, we modify the well-known model by Bagwell and Riordan (1991) of price signaling product quality where a manufacturer with private information about product quality sells directly to consumers. The resulting signaling equilibrium may be quite inefficient as in order to

[2] We do allow for two-part tariffs. However, the fixed fee does not play an important role in our setting with unobservable wholesale contracts.

deter imitation by a low quality type, the high quality price should be sufficiently distorted upward.[3] In our framework, the manufacturer sells through an intermediary (a retailer) instead of selling directly to end users. Our results show that signaling distortions continue to play an important role when consumers observe wholesale contracts, but they may disappear when consumers do not observe these contracts.

Equilibrium refinements are a crucial part of the signaling literature. In particular, the strategic interaction falls within the class of standard signaling games where a Sender with private information sends a message that is observed by a Receiver who then chooses an action. A significant element in their analysis involves showing that pooling equilibria do not satisfy the Intuitive Criterion (Cho and Kreps 1987). When the privately informed manufacturer sells to consumers via a retailer, the game is no longer a standard signaling game and equilibrium refinements such as the Intuitive Criterion developed for standard signaling games cannot be applied. If consumers observe the wholesale contract, then the pay-off to the manufacturer depends on the beliefs of the consumer and the action taken by the retailer, which in turn depends on their belief about consumers' beliefs. If consumers do not observe the wholesale contract and only observe actions by the retailer, then they must consider whether an out-of-equilibrium retail price has to be attributed to a unilateral deviation by the manufacturer or the retailer. A methodological contribution of this paper is to develop refinements similar to the Intuitive Criterion for these two particular interaction structures and show that our result satisfies these refinements. Although this paper is about quality uncertainty, our methodological contribution applies more broadly to vertical industry structures where the manufacturer has some private information.

In the game where the manufacturer delegates setting the retail price to a retailer and upstream vertical pricing is not observed by consumers, there is a continuum of pooling equilibria that satisfy the new refinement. A key part of the argument is that even though buyers may rule out that certain high retail prices stem from a unilateral deviation by the low quality manufacturer, they may not rule out the real possibility that the retailer has unilaterally deviated to these prices. If the retailer does not know quality, this allows them to have the same beliefs about quality as the retailer. If the retailer knows quality, he cannot signal quality as his cost is independent of product

[3] Ellingsen (1997) analyzes price signaling of product quality by a seller when demand for every quality is perfectly inelastic. Janssen and Roy (2010) show that price signaling may also work in an oligopoly context where sellers are uninformed about the quality sold by their competitor.

quality implying that his incentives to choose actions are independent of whether quality is low or high.

Supply chain issues have been extensively studied in the management and economics literature, but there is no literature focusing on the manufacturer having private information and how it can beneficially use unobservable wholesale contracting with a retailer. A relatively recent literature studies the advantages of unobservable wholesale contracting in a context where the competition between multiple retailers in the retail market depends on whether or not they know their competitors' wholesale contract (see, among others, Arya and Mittendorf (2011), Pagnozzi and Piccolo (2012), Liu and Wang (2014), Rey and Verge (2020), Do and Miklos-Thal (2023)).[4] In the management literature the importance of vertical contracting has been studied to address, for example, inventory issues (see, e.g., Dong, Guo and Turcic (2018) and Qu and Raff (2019)).

Our paper also contributes to the literature on the role of intermediaries in markets with asymmetric information about quality that has largely focused on information or certification intermediaries that use their own information, skill or reputation to provide information to buyers (Biglaiser 1993, Lizzeri 1999, Albano and Lizzeri 2001 and Glode and Opp 2016).[5] In our framework, the intermediary retailer has no skill or market reputation and in fact, may have no more information about product quality than the uninformed consumer. In contrast to this literature, our key result is based on the beneficiary role of using a retailer to *hide* information from final consumers, and the result does not depend on the information the retailer has.

A number of papers have analyzed the role of leasing of new durable goods in reducing the extent of the lemons problem in the used goods markets. The leasing firm's opportunity cost of selling the used good (at the end of the lease) is determined prior to the realization of actual quality or performance of the used good and therefore independent of it; see, among others, Hendel and Lizzeri (2002) and Johnson and Waldman (2003). One may view leasing as selling the used good via an intermediary (the leasing firm). Further, the timing of actions rules out the possibility of signaling. Unlike this literature, our paper focuses on private information about producer's quality.

[4] This literature builds on an older literature focussing on the importance of retailer beliefs in constraining the incentives for the manufacturer to behave opportunistically (see, e.g., Bonanno and Vickers (1988), Katz (1989) and Hart and Tirole (1990)). Beliefs in the context of these classes of games have also been studied by In and Wright (2018). Fershtman and Kalai (1997) and Kockesen and Ok (2004), among others, study the effect of strategic delegation with unobservable contracts in games of perfect information.

[5] The welfare effects of intermediaries has been measured by Donna et al. (2022).

The manufacturer is informed about quality before he sets the terms under which the retailer acquires the good and he chooses whether or not the retailer's cost of acquiring the good varies with quality. Signaling by the manufacturer (to the retailer) is potentially possible when wholesale contracts are unobserved, but the manufacturer abstains from doing so. Observability of the terms of the vertical contract by final consumers affects the market outcome significantly in our setting, whereas this does not play a role in the leasing literature.

Our result on regulatory policies penalizing false advertising not being in consumers' interest contributes to a recently revitalized literature analyzing the role of false or deceptive advertising (see, e.g., Rhodes and Wilson 2018, Piccolo et al. 2015, Janssen and Roy 2022 and an earlier paper by Daughety and Reinganum 1997). These papers do not analyze vertical industries, however, and the mechanism we uncover is therefore very different from theirs.

There is also a large literature addressing the policy debate on whether financial kickbacks for financial intermediaries or advisors in other sectors should be made public so that consumers can understand the incentives involved. Our setting, the result and the mechanism we uncover are very different from that literature, however, as it is the manufacturer in our setting that has private information and they do not set the retail price the consumer pays.

The rest of the paper is organized as follows. Section 2 outlines the basic model of a privately informed manufacturer selling through a retailer. Section 3 considers the benchmark model where the wholesale contract is unobserved by consumers, whereas Section 4 covers our main result for markets where the wholesale contract is unobserved. Section 5 concludes with a discussion. Proofs and our methodological contribution related to equilibrium refinement for the model with unobservable contracts are contained in the Appendix.

## 2 The Model

Our basic framework is the one used by Bagwell and Riordan (1991) to analyze price signaling of product quality by a monopoly firm. The firm, who we shall henceforth refer to as the manufacturer, produces a good whose quality can be either high ($H$) or low ($L$). The unit cost of production is constant and depends only on the quality of the good. In particular, high quality has a unit cost $c > 0$, while the cost of low quality is normalized to zero. There is a unit mass of consumers. All consumers have unit demand. They have identical valuation $v_L > 0$ for the low quality product, while their valuation of high quality is uniformly distributed on $[v_L, 1 + v_L]$. Thus, if consumers face a price $p$ and assign common probability $\mu$ to high quality, then the quantity demanded $d(p, \mu)$ is given by:

$$\begin{aligned} d(p,\mu) &= 0, \text{ if } p \geq \mu + v_L \\ &= 1 - \frac{p - v_L}{\mu}, \text{ if } p \in [v_L, \mu + v_L] \\ &= 1, \text{ if } p \leq v_L. \end{aligned} \tag{1}$$

The prior probability that quality is high is common knowledge and denoted by $\alpha \in (0, 1)$. The realized quality of the good is observed only by the manufacturer.

The manufacturer sells his product exclusively through an intermediary retailer. The manufacturer offers a wholesale pricing contract to the retailer that takes the form of a two part tariff with a fixed fee $F \geq 0$ and and a per unit wholesale price $w \geq 0$. If the retailer accepts the contract, he determines the retail price $p$ at which he sells to the consumers. If the retailer does not accept the contract, the manufacturer does not sell. The retailer has no specific expertise, does not know the quality of the good provided by the manufacturer, and his outside option is zero. The only cost incurred by the retailer is what he pays the manufacturer for the good. His payoff is his expected profit net of this payment. The manufacturer's payoff is his expected profit and each consumer maximizes her expected net surplus.

We maintain the following assumption in the rest of this paper:

$$c + v_L < 1, \tag{2}$$

$$\alpha > \max\{c, v_L\}. \tag{3}$$

The first assumption guarantees that signaling involves a distortion from optimal pricing under full information, while the second assumption guarantees (as we will see) that interesting pooling equilibria exist when wholesale contracts are unobserved.

# 3 Benchmark: Observable Vertical Contracts

In this section, we consider the version of the basic model where in addition to the retail price, consumers are able to observe the vertical contract, i.e., the two-part tariff set by the manufacturer.

The extensive form is as follows. First, nature draws the type $\tau$ (product quality) of the manufacturer from a distribution that assigns probability $\alpha$ to $H$ (high quality) and $(1-\alpha)$ to $L$; only the manufacturer observes this move of nature. Next, the manufacturer chooses a two-part wholesale tariff $(w, F)$ at which he offers to sell to the retailer; here, $F$ is the fixed fee and $w$ is the marginal wholesale price; the two-part tariff is observed by the retailer and the consumers. Next, the retailer chooses his retail price $p$ which is also observed by consumers. Finally, consumers update their beliefs and make their purchase decisions. The expected payoff of the manufacturer of type $\tau$ is given by $(w - c_\tau)d(p, \mu) + F$ and that of the retailer is given by $(p - w)d(p, \mu) - F$.

We focus on symmetric pure strategy PBE with reasonable restrictions on the consumers' out-of-equilibrium beliefs. The manufacturer's strategy is given by $(w_\tau, F_\tau)_{\tau=H,L}$ where $w_\tau$ is the marginal wholesale price and $F_\tau$ is the fixed fee charged by manufacturer of type $\tau$. The retailer's strategy is described by a function $p(w, F)$ that indicates the retail price charged when $(w, F)$ is the two-part tariff set by the manufacturer.

This game is close to a standard signaling game where criteria like the Intuitive Criterion may apply. To apply such a criterion, the main question now is how to define the notion of equilibrium domination in view of the fact that whether a message is equilibrium dominated for the Sender (the manufacturer) may now depend on the action taken by the Intermediary (the retailer) and the effect this has on the Receiver's (buyers') response.

For our price signaling game, to see which type of manufacturer may have an incentive to deviate to some out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ what matters is consumer demand $d(p, \mu(\widehat{w}, \widehat{F}))$, which depends on consumer beliefs and on the price set by the retailer; this, in turn, depends on $(\widehat{w}, \widehat{F})$ and on the *second-order belief* of the retailer about what consumers would believe about product quality. Note that while in any PBE (both on and off-the-equilibrium path) the second-order beliefs of the retailer must necessarily coincide with consumers' first-order beliefs as specified in the equilibrium, a potential issue arises when we want to determine the reasonableness of *out-of-equilibrium* beliefs by looking at the relative incentives of different types of the manufacturer to choose an out-of-equilibrium wholesale price.

Whether or not a deviation is profitable depends on the relation between consumer beliefs and the retailer's *second-order belief* about consumer beliefs. Obviously, the more optimistic consumers are about product quality and the less optimistic the retailer believes the consumer is, the more incentive the manufacturer has to deviate. To give the Intuitive Criterion some bite, it is natural to impose that first- and second-order beliefs are symmetric, i.e., that the retailer holds correct beliefs about the beliefs of consumers: if after observing a deviation by the manufacturer, consumers believe with probability $\mu$ that the manufacturer sells high quality, then the retailer also believes that consumers have belief $\mu$. Symmetric beliefs, in this sense, are implied by, but weaker than, the retailer and the consumer having a "common prior" about the quality the manufacturer sells in the continuation game following the manufacturer's action $(w, F)$. Symmetric beliefs seem natural as the manufacturer cannot control these beliefs and there does not seem to be any reason why the manufacturer should entertain the possibility that consumers' beliefs about quality should be different from the retailer's second-order beliefs about consumers' beliefs.[6] Note that the retailer's own belief about quality does not play any role in determining his response to a deviation.

To define the Intuitive Criterion while requiring that first- and second-order beliefs are identical, suppose the manufacturer deviates from the equilibrium contract and chooses some out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ and that the retailer sets his retail price assuming that demand is $d(p, \mu)$ at any retail price $p$, where for notational simplicity we suppress that $\mu$ may depend on $(\widehat{w}, \widehat{F})$. The optimal response of the retailer, denoted by $p((\widehat{w}, \widehat{F}), \mu)$, is then given by:

$$p((\widehat{w}, \widehat{F}), \mu) = \arg\max_{p \geq \widehat{w}} \left[(p - \widehat{w}) d(p, \mu)\right].$$

Using this price reaction of the retailer to an out-of-equilibrium contract $(\widehat{w}, \widehat{F})$, and defining the equilibrium pay-off (at the equilibrium contract $(w_\tau^*, F_\tau^*)$) for type $\tau$ manufacturer as $\pi_\tau^* = (w_\tau^* - c_\tau) d(p^*(w_\tau^*)) + F_\tau^*, \tau = L, H$, we can directly apply the logic of the Intuitive Criterion and require that if for some $\tau \in \{H, L\}$ the out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ is equilibrium dominated,

[6] Note that symmetric beliefs (as defined here) would always arise in a perturbed game where strategies have full support. Requiring coordinated beliefs can therefore be viewed as a consequence of strategic stability where one studies our game as the limit of a sequence of such perturbed games.

i.e.,

$$\pi_\tau^* > (\widehat{w} - c_\tau)d(p(\widehat{w}, \mu), \mu) + \widehat{F} \text{ for all } \mu \in [0, 1],$$

while for $\tau' \in \{H, L\}, \tau' \neq \tau$ the out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ is *not* equilibrium dominated, i.e.,

$$\pi_{\tau'}^* < (\widehat{w} - c_{\tau'})d(p(\widehat{w}, \mu), \mu) + \widehat{F} \text{ for some } \mu \in [0, 1],$$

then the out-of-equilibrium belief $\mu(\widehat{w}, \widehat{F})$ should assign probability one to deviation by the manufacturer of type $\tau'$.

We now characterize all equilibria that satisfy the Intuitive Criterion with symmetric beliefs as defined above. We begin by showing that there is no such equilibrium where the low and high quality types of the manufacturer pool on the same two-part tariff contract with positive probability.

**Lemma 1** *When the manufacturer's upstream (two-part tariff) pricing is observable by consumers, there is no pooling or partially pooling equilibrium satisfying the Intuitive Criterion (with symmetric beliefs).*

The proof of this result argues that for any candidate pooling equilibrium contract $(w^*, F^*)$ when $w^* > \upsilon_L$, one can find deviations $(\widehat{w}, \widehat{F})$ such that, using the Intuitive Criterion with symmetric beliefs, consumers have to believe that they come from a high quality manufacturer, making these deviations profitable.[7] If $\widehat{w}$ is sufficiently large, low quality manufacturers would never have an incentive to deviate, while due to higher production cost one can still find wholesale prices in this range (and appropriately chosen fixed fees) that may be profitable for the high quality manufacturer. For $0 \leq w^* \leq \upsilon_L$, a low quality manufacturer gains by reducing the marginal wholesale price to zero and charging a fixed fee close to its full information monopoly profit regardless of the beliefs of buyers.

From Lemma 1, it follows that only fully separating outcomes are consistent with reasonable restrictions on out-of-equilibrium beliefs. Our next result outlines a separating equilibrium that satisfies our refinement criterion. Further, we show that this is the least distortionary separating equilibrium. The retail prices faced by buyers are never lower than in this equilibrium, while the

[7] This part of the argument is similar to the argument used by Bagwell and Riordan (1991) to eliminate pooling equilibria under direct selling, but in our setting the manufacturer also has to take double marginalization by the retailer into account.

manufacturer's profit and consumer surplus generated in the low and high quality states are never higher.

**Lemma 2** *Suppose the manufacturer's upstream (two-part tariff) pricing is observable by consumers.*

*(i) There exists a fully separating perfect Bayesian equilibrium satisfying the Intuitive Criterion (with symmetric beliefs) where the low quality manufacturer sets two-part tariff* $(w_L, F_L) = (0, \upsilon_L)$ *earning profit* $\upsilon_L$ *and the high quality manufacturer sets two-part tariff* $(w_H, F_H) = (1 - \upsilon_L, (\upsilon_L)^2)$ *earning profit* $\upsilon_L(1-c)$*. In this equilibrium, the high quality good is sold at a retail price* $p_H = 1$ *while the low quality good is sold at retail price* $\upsilon_L$*; the retailer earns zero profit, the ex ante expected manufacturer profit is*

$$\overline{\pi} = \upsilon_L(1 - \alpha c) \tag{4}$$

*and the ex ante expected consumer surplus is* $\overline{S} = \frac{\alpha}{2}(\upsilon_L)^2$.

*(ii) In any fully separating perfect Bayesian equilibrium satisfying the Intuitive Criterion (with symmetric beliefs), the ex ante expected industry profit (and therefore, the ex ante expected manufacturer's profit) is bounded above by* $\overline{\pi}$ *and the ex ante expected consumer surplus is bounded above by* $\overline{S}$.

It is obvious that in any separating equilibrium, the low quality manufacturer must earn his full information profit $\upsilon_L$. In the equilibrium outlined in Lemma 1(i), the high quality manufacturer signals his type by setting a two-part tariff $(w_H, F_H)$ such that the incentive constraint of the low quality type is binding, i.e., the latter is indifferent between imitating the high quality type's action and sticking to his equilibrium action. As the two-part tariff $(w_H, F_H)$ reveals quality to be high for sure, the optimal retail price set by a retailer accepting this contract is $(1/2)(1 + \upsilon_L + w_H) = 1$ and the quantity sold by the high quality manufacturer is $\upsilon_L$. Note that under full information, the high quality manufacturer optimally sets the marginal wholesale price $w = c$ and the retail price is then $\frac{1+\upsilon_L+c}{2} < 1$ ($\upsilon_L + c < 1$ under assumption (2)). Thus, the signaling equilibrium involves an upward distortion of the marginal wholesale and retail prices with consequent loss of profit and consumer surplus in the high quality state (relative to the full information outcome). The extent of this distortion is higher when $c$ or $\upsilon_L$ is lower.

The high quality manufacturer would be eager to reduce the marginal wholesale price below $w_H$ (and set a higher fixed fee) if buyers would be optimistic about quality after the deviation. To deter such a deviation, for any observed deviation to a two-part tariff $(w, F)$ where $w \in (0, 1 - \upsilon_L)$, we choose out-of-equilibrium beliefs of buyers that assign probability one to the manufacturer being of low quality. These pessimistic beliefs are reasonable: we show that if the high quality manufacturer gains from such a deviation for some belief of buyers, then the low quality manufacturer can also gain from the same deviation. In particular, these beliefs meet the Intuitive Criterion with symmetric beliefs. The out-of-equilibrium beliefs assign probability one to high quality when the manufacturer deviates to a contract where the marginal wholesale price is higher than $w_H$. As the high quality manufacturer has higher marginal cost, this is a reasonable belief and is consistent with the Intuitive Criterion with symmetric beliefs.

It is important to note that the equilibrium outlined in Lemma 2(i) yields the same outcome in terms of prices paid by buyers and distortion in industry profits and consumer surplus as in the unique equilibrium satisfying the Intuitive Criterion when the manufacturer sells directly to buyers. The latter is characterized fully in Bagwell and Riordan (1991). Thus, direct selling and selling through a retailer with observed wholesale contracts share important features suggesting that what is important is that end users observe the actions chosen by the player with private information.

Part (ii) of Lemma 2 asserts that any separating equilibrium satisfying our refinement must generate at least as much distortion in profits and consumer surplus as the equilibrium outlined in part (i). In other words, the equilibrium described in part (i) is the least distortionary separating equilibrium which is not surprising as the low quality manufacturer's incentive constraint is binding in this equilibrium.

Combining Lemmas 1 and 2, we have the main result of this section:

**Proposition 1** *When the manufacturer's upstream (two-part tariff) pricing is observable by consumers, in any perfect Bayesian equilibrium satisfying the Intuitive Criterion with symmetric beliefs, the ex ante expected industry profit (sum of manufacturer and retailer's profit) is bounded above by* $\overline{\pi} = \upsilon_L(1 - \alpha c)$ *and the ex ante expected consumer's surplus is bounded above by* $\frac{\alpha}{2}(\upsilon_L)^2$.

## 4 Unobservable Wholesale Contracts

In this Section, we consider the situation where consumers do not observe the wholesale contracts set by the manufacturer. With unobservable wholesale contracts, pooling and separating equilibria exist. We focus on pooling equilibrium outcomes that yield higher profits for the manufacturer and may also lead to higher consumer surplus as when the manufacturer would sell directly to end consumers. Separating equilibria where the manufacturer offers different wholesale prices depending on product quality suffer from similar signaling distortions at the retail level as selling directly and are therefore generally not better for the manufacturer than selling directly.

The extensive form game is as follows. As in the previous section, nature draws the type $\tau$ (product quality) of the manufacturer from a distribution that assigns probability $\alpha$ to $H$ (high quality) and $(1-\alpha)$ to $L$. Only the manufacturer observes this move of nature. Following this, the manufacturer chooses a wholesale contract $(w, F)$ which he offers to the retailer. The key difference with the previous section is that this two-part wholesale tariff is not observed by consumers (although it continues to be observed by the retailer). Next, the retailer chooses his retail price $p$ which is observed by consumers. Consumers update their beliefs on the basis of the retail price and make their purchase decisions.

Note that this information structure is very different from a standard signaling game as the "Receivers" (consumers) do not observe the actions ($w$ and $F$) of the privately informed "Sender" (the manufacturer), but only the action $p$ of the retailer (an intermediary) who may himself not be informed. Deviations from the equilibrium path are observed by consumers ("Receivers") only if the retail price is different from the equilibrium retail price. In forming their beliefs in such situations, consumers consider whether the observed retail price is due to a deviation by the manufacturer or the retailer or both. It is easy to see that equilibrium refinements for standard signaling games cannot be readily applied here. In Appendix B, we outline a notion of equilibrium refinement for a class of games of this sort (which we call *Intermediated Signaling Games*), where a Sender with private information chooses an action that is only observed by an intermediary who himself chooses another action that is observed by the receiver and where the Sender's payoffs depend on the actions taken by the Intermediary and the Receiver. The refinement, the *Intuitive Criterion for Intermediated Signaling Games (IC-I)* is based on considerations similar to the Intuitive Criterion for standard signaling games.

Our goal is to show that when wholesale contracts are unobservable, we can construct reasonable pooling equilibria (with beliefs satisfying IC-I) that generate higher ex ante profit as well as consumer and social welfare than in any reasonable equilibrium (satisfying the intuitive criterion with symmetric beliefs) when wholesale contracts are observable.[8] We will do this in three steps. First, we construct a class of pooling PBE. Next, we show the out-of-equilibrium beliefs underlying these pooling equilibria satisfy our belief refinement IC-I (Proposition 2). To do this, we describe informally how IC-I works and why the beliefs satisfy this refinement. After this we show that some of these pooling equilibria satisfying IC-I generate higher ex ante expected profit, expected consumer surplus and expected social surplus than the upper bounds for these outcomes in the observable contracts case (Proposition 3 and 4). These steps require additional restrictions on the parameter space. Taken together, these restrictions show that our qualitative claim regarding comparison of observable and unobservable contracts can be rigorously demonstrated for a robust section of the parameter space.

## 4.1 Construction of Pooling PBE

Consider pooling equilibria where the manufacturer sets a wholesale price $w^*$ and a fixed fee $F^* = 0$ regardless of product quality. On the equilibrium path, the retailer follows up by selling at a retail price $p^* = w^*$.[9] After observing the retail price $p^*$, a buyer's updated belief is identical to her prior belief, i.e., $\mu(p^*) = \alpha$, while the manufacturer and the retailer sell quantity $d(p^*, \alpha)$. Recall that under assumption (3), $\alpha > \max\{v_L, c\}$. We focus on outcomes where

$$\alpha > p^* = w^* \geq \max\{v_L, c\}. \tag{5}$$

The second inequality above must be satisfied as otherwise the retailer wants to deviate (to $v_L$) if $p^*$ were smaller than $v_L$, or the high quality manufacturer would want to deviate (if $w^* < c$).

[8] Not all equilibria of the game studied in this section are pooling. There are separating equilibria where different types of the manufacturer offer different marginal wholesale prices and the retailer's pricing reflects (and reveals) this difference in wholesale pricing so that consumers infer quality from the retail price. Signaling distortions dampen the profits and surplus generated in these outcomes. As our objective is to show that unobservable contracts can generate *higher* profit and surplus than observable contracts, we naturally focus on the pooling equilibria that are free of signaling distortions.

[9] One can construct similar pooling equilibriawhere $p^* > w^*$ and $F^* > 0$ is such that the retailer makes zero profit on the equilibrium path.

The first inequality will be used later to ensure the low quality manufacturer has no incentive to reduce his wholesale price. Note that (5) implies $p^* = w^* < \alpha + \upsilon_L$ so that the manufacturer sells a strictly positive quantity.

We begin by specifying the symmetric out-of-equilibrium belief $\mu(p)$ of buyers that we use to sustain the above set of outcomes in an equilibrium of the game:

$$\begin{aligned} \mu(p) &= 0 \text{ if } p \in (\upsilon_L, p^*) \text{ or if } p \in (p^*, \alpha + \upsilon_L) \\ &= \alpha \text{ if } p \in [\alpha + \upsilon_L, 1 + \alpha). \end{aligned} \tag{6}$$

In the next subsection, we will discuss when these beliefs are reasonable (and satisfy our equilibrium refinement). We allow for any belief $\mu(p) \in [0, 1]$ for $p \geq 1 + \alpha$ as no buyer buys at such retail price regardless of belief. Similarly, we allow for any belief $\mu(p) \in [0, 1]$ for retail price $p \leq \upsilon_L$ as it is optimal for all buyers to buy regardless of their belief.

Given these beliefs, it is easy to see that the buyers' optimal strategy is summarized by the quantity $q(p)$ purchased where $q(p) = d(p, \mu(p)), p \neq p^*$, and $q(p^*) = d(p^*, \alpha)$. Thus,

$$\begin{aligned} q(p) &= 1 \text{ if } p \leq \upsilon_L \\ &= 0 \text{ if } p \in (\upsilon_L, p^*) \text{ or if } p > p^* \\ &= 1 - \frac{p - \upsilon_L}{\alpha} \text{ if } p = p^* \end{aligned} \tag{7}$$

Note that $q(p) = d(p, \alpha) = 0$ for $p \geq \alpha + \upsilon_L$ as given the belief $\mu(p) = \alpha$ if $p \in [\alpha + \upsilon_L, 1 + \alpha)$ buyers find the price too high. Also, buyers do not buy at retail price $p \in (p^*, \alpha + \upsilon_L)$ or $p \in (\upsilon_L, p^*)$ as they believe the product is of low quality for sure.

We stipulate that the retailer's equilibrium pricing strategy if he accepts a wholesale contract with two part tariff $(w, F)$ is as follows:

$$\begin{aligned} p(w, F) &= w, \quad \text{if } w \geq p^* \\ &= p^*, \quad \text{if } w \leq p^* \end{aligned} \tag{8}$$

and further, the retailer accepts this contract if, and only if,

$$F \leq [p(w, F) - w]q(p(w, F)),$$

i.e., the retailer makes non-negative profit.

We have already noted that the buyers' strategy is optimal given the beliefs. Consider now the retailer's strategy (8). Note that the retailer's equilibrium profit is zero. If the per unit wholesale price $w \geq p^*$, the retailer can never sell a positive quantity by deviating to a retail price above $w$ (as $q(p) = 0$ for $p > p^*$). Hence, it is optimal for the retailer to set the retail price equal to $w$. In particular, the out-of-equilibrium beliefs eliminate double marginalization by the retailer. If the per unit wholesale price $w \in (\upsilon_L, p^*)$, the only retail price above $w$ at which the retailer can sell a positive quantity is $p^*$ and so it is optimal to charge $p^*$. If $w \in [0, \upsilon_L]$, the retailer can choose between charging $\upsilon_L$ (and selling to all buyers) and charging $p^*$ (selling $q(p^*) = d(p^*, \alpha)$). Using the fact that $p^* < \alpha$ one can check the latter is optimal for the retailer.[10]

We now argue that regardless of his type, the manufacturer has no incentive to deviate from the pooling two-part tariff ($w^* = p^*, F^* = 0$). The equilibrium payoffs of the low, respectively high, quality manufacturer are given by

$$\pi_L^* \equiv \left(1 - \frac{p^* - \upsilon_L}{\alpha}\right) p^* \text{ and } \pi_H^* \equiv \left(1 - \frac{p^* - \upsilon_L}{\alpha}\right) (p^* - c).$$

If the manufacturer deviates to a contract $(\widehat{w}, \widehat{F})$ where $\widehat{w} \in [0, p^*)$ and if such a contract is accepted, then (as per the retailer's equilibrium strategy) it leads to the same retail price as at $w = w^* = p^*$ and therefore cannot be more profitable. On the other hand, the manufacturer can never sell by deviating to a contract $(\widehat{w}, \widehat{F})$ where $\widehat{w} > w^* = p^*$ as the retailer, if he accepts such a contract, will set a retail price equal to $\widehat{w}$ and buyers are too pessimistic about product quality to buy at such a retail price. Thus, the strategies and beliefs outlined above constitute a perfect Bayesian equilibrium (PBE).

[10] For $w \in [0, \upsilon_L]$, $(p^* - w)[1 - \frac{p^* - \upsilon_L}{\alpha}] \geq \upsilon_L - w$ if $w \geq p^* - \alpha$ and the latter holds as $p^* \leq \alpha$.

### 4.2 Belief Refinement

The out-of-equilibrium beliefs (6) play an important role in sustaining the pooling PBE outcome outlined in Section 4.1. The IC-I refinement developed in Appendix B uses the notion of equilibrium domination (that is also at the heart of the Intuitive Criterion) to impose reasonable restrictions on beliefs. In particular, these restrictions apply to actions of the intermediary that can arise due to a *unilateral* deviation by the sender and/or the intermediary. We informally describe the main idea behind the refinement here and discuss how it applies to the beliefs (6) that sustain the pooling PBE described above.

In the pooling equilibrium described in Section 4.1, any observed off-equilibrium retail price $p > p^*$ can result from a unilateral deviation by the retailer (given the pooling equilibrium strategy $(w^*, F^*)$ of the manufacturer), but it can also be the consequence of a unilateral deviation by the manufacturer (given the equilibrium strategy (8) of the retailer). Our refinement IC-I gives any account that attributes a retail price $p > p^*$ to a unilateral deviation by one of the players priority over attributing the observed retail price to multiple deviations.

Given the equilibrium strategy (8) of the retailer, an off equilibrium retail price $p \in (p^*, \alpha + \upsilon_L)$ may be attributed by the buyers entirely to a unilateral deviation by the manufacturer. A low quality manufacturer can strictly gain by deviating to a two part tariff $(w = p, F = 0)$ which will lead to a retail price $p \in (p^*, \alpha + \upsilon_L)$ when buyers assign probability 1 to high quality if

$$\pi_L^* = \left(1 - \frac{p^* - \upsilon_L}{\alpha}\right) p^* < d(p, 1)p.$$

If the inequality holds, such a deviation is not equilibrium dominated for the low quality manufacturer and it is perfectly reasonable for the buyer to think the observed retail price is caused by a unilateral deviation of the low quality manufacturer. Thus, the buyer may assign $\mu(p) = 0$ for $p \in (p^*, \alpha + \upsilon_L)$ when it observes such a retail price. For this reason, the belief $\mu(p) = 0$ for $p \in (p^*, \alpha + \upsilon_L)$ meets our refinement (more precisely, satisfies part (iii) of condition IC-I in Appendix B) as long as $\pi_L^* < d(p, 1)p$ for all such $p$. It is easy to see the above inequality is satisfied for $p$ slightly above $p^*$. Using the concavity of the profit $d(p, 1)p$ one can see that the inequality holds for all $p \in (p^*, \alpha + \upsilon_L)$ as long as

$$\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right) p^* < d(\alpha + \upsilon_L, 1)(\alpha + \upsilon_L) \;= (1 - \alpha)(\alpha + \upsilon_L). \tag{9}$$

The next lemma states parameters for which this is the case.

**Lemma 3** *There exists $\underline{p} \in [\max\{\upsilon_L, c\}, \alpha)$ such that (9) holds for all $p^* \in [\underline{p}, \alpha]$ if, and only if, $\upsilon_L < 1 - \alpha$. Further, we can choose $\underline{p} = \max\{\upsilon_L, c\}$ if $\upsilon_L < \alpha(3 - 4\alpha)$.*

Let us now consider $p \in [\alpha + \upsilon_L, 1 + \upsilon_L)$. It is clear that not all these prices can be attributed to a unilateral deviation of the low quality manufacturer. In particular, prices close to $1 + \upsilon_L$ will yield a profit that is close to 0 and therefore lower than the equilibrium profit. Given the equilibrium strategy of the manufacturer, a unilateral deviation by the retailer to any $p \in [\alpha + \upsilon_L, 1 + \upsilon_L)$ can yield strictly positive profit to the retailer and is therefore strictly gainful if buyers assign sufficiently high probability to quality being high. In particular, no such retail price is equilibrium dominated for the retailer. Thus, it is reasonable for the buyers to attribute a retail price $p \in [\alpha + \upsilon_L, 1 + \upsilon_L)$ entirely to a unilateral opportunistic deviation by the retailer. As the retailer is uninformed and only observes a pooling contract from the manufacturer when he sets the retail price, his deviation does not provide any more information about product quality.[11] As buyers may attribute the deviation entirely to the retailer, belief $\mu(p)$ can be identical to the prior belief $\alpha$ and in particular, would satisfy part (ii) of condition IC-I in Appendix B.

Finally, consider off-equilibrium retail price $p \in (\upsilon_L, p^*)$. For any $p \in (\upsilon_L, p^*)$, it is clear that such a price cannot be attributed to a unilateral deviation by the retailer (as it is equilibrium dominated given the manufacturer's strategy), while given the strategy of the retailer it can also not be attributed to a unilateral deviation by the manufacturer. The refinement criterion IC-I we outline in Appendix B confines attention to incentives for unilateral deviation and does not impose any restriction on beliefs in the more complex case of joint deviations by multiple players. Thus, the belief $\mu(p) = 0$ for all $p \in (\upsilon_L, p^*)$ meets our refinement. However, for our specific game and equilibrium, one can informally argue that the low quality manufacturer has a stronger incentive

[11] If the retailer would be informed about quality, it is easy to see that in a candidate pooling equilibrium both the low and the high quality retailer would have the same incentives and therefore consumers would always be able to blame the low quality retailer for a deviation.

(than the high quality manufacturer) to deviate to a lower per unit wholesale price that would be needed for the downward deviation in the retail price so that the specified belief is intuitive.[12]

The next proposition summarizes the above discussion.

**Proposition 2** *Suppose the manufacturer chooses a wholesale contract that is not observed by consumers and that* $\upsilon_L < 1-\alpha$. *Then, there exists* $\underline{p} \in [\max\{\upsilon_L, c\}, \alpha)$ *such that for every* $p^* \in [\underline{p}, \alpha]$ *there is a pooling PBE satisfying IC-I where both types of the manufacturer offer identical two-part tariff* $(w^*, F^*)$*,where* $w^* = p^*, F^* = 0$ *and the retailer is fully squeezed.*

Proposition 2 provides a condition under which there is a continuum of pooling equilibria supported by reasonable beliefs. In the pooling PBE described above, the manufacturer is already able to fully extract the retailer's rent. It is easy to show that these pooling outcomes can also be sustained if the manufacturer uses a fixed fee $F > 0$ and set some $w^* < p^*$. The fixed $F$ can also be adjusted to give the retailer more rent if he would not otherwise participate.

Note that the belief refinement has bite and restricts the set of parameters for which a class of pooling PBE outcomes where the retailer is fully squeezed exists. Without refinement, one could simply stipulate $\mu(p) = 0$ for all $p > p^*$. This would immediately imply that neither the manufacturer nor the retailer would have an incentive to deviate upwards. Lemma 3 argues that there is non-empty interval of pooling prices where our belief refinement is satisfied (i.e., (9) holds) only if $\upsilon_L < 1 - \alpha$.

### 4.3 Profit and Welfare Comparison with Observable Wholesale Contract

Finally, we investigate whether some of the reasonable pooling equilibria under unobservable wholesale contracts characterized in Proposition 2 can yield higher ex ante expected profit, expected consumer surplus and, expected social surplus than when wholesale contracts are observable. Recall that Proposition 1 establishes that when the manufacturer's upstream (two-part tariff) pricing is observable by consumers, in any perfect Bayesian equilibrium satisfying the Intuitive Criterion with

[12] One may want to consider joint deviations of both the manufacturer and the retailer to investigate whether $\mu(p) = 0$ is a reasonable out-of-equilibrium belief. It is easy to check that if the high quality manufacturer gains by reducing his wholesale price from $w^*$ to $w \leq p < p^*$ for some belief of buyers $\mu'$ (after observing retail price $p$), *i.e.*, if $(w - c)d(p, \mu') \geq (p^* - c)d(p^*, \alpha)$, then the low quality manufacturer must strictly gain from this deviation, *i.e.*, $wd(p, \mu') > p^* d(p^*, \alpha)$ indicating that if some type of manufacturer would want to induce the retailer to set $p$ it is certainly the low quality manufacturer that has an incentive to do so. Thus, it seems that any restriction on beliefs based on joint deviations, should allow for buyers to hold the belief $\mu(p) = 0$ at $p \in (\upsilon_L, p^*)$.

symmetric beliefs, the ex ante expected industry profit (sum of manufacturer and retailer's profit) is bounded above by $\overline{\pi} = \upsilon_L(1-\alpha c)$ and the ex ante expected consumer's surplus is bounded above by $\frac{\alpha}{2}(\upsilon_L)^2$.

**Proposition 3** *Suppose the manufacturer chooses a wholesale contract that is not observed by consumers and $\upsilon_L < 1-\alpha$. There exists a continuum of pooling PBE satisfying IC-I as described in Proposition* 2 *such that the ex ante expected consumer surplus in every such equilibrium is strictly higher than $\frac{\alpha}{2}(\upsilon_L)^2$, the upper bound of expected consumer surplus under observable wholesale contract. Further, the following holds:*

*(i) there exists $c_0 > 0$ such that if $c \in (0, c_0)$, there are pooling PBE satisfying IC-I as described in Proposition* 2 *where the ex ante expected profit for the manufacturer (and the industry) is strictly higher than $\overline{\pi}$;*

*(ii) there exists $\upsilon_0 > 0$ such that the conclusion in (i) holds if $\upsilon_L \in (0, \upsilon_0)$.*

For the pooling equilibria under unobservable contracts to yield higher profits than any separating equilibrium under observable contracts, we essentially need that the signaling distortions in the latter equilibria are large enough. In the previous section we have argued that this is the case if $\upsilon_L$ and/or $c$ are small enough. This is the key message of Proposition 3. The fact that consumers are also better off can be understood as follows. Under observable contracts, consumers only get a surplus from buying when the quality is high, but this surplus tends to be low as signaling distortions inflate the retail price. In fact, the least distortionary outcome characterized in Lemma 2 leads to a retail price equal to 1. In contrast, under unobservable wholesale contract, the retail price $p^*$ that can be sustained in a pooling equilibrium discussed above is smaller than $\alpha$, which in turn is restricted to be smaller than $1-\upsilon_L$. Thus, consumers gain substantially under unobservable contracts when quality is high. However, in case quality is low, they make a negative surplus as $\upsilon_L < p^*$. It turns out that this loss is dominated by the gain when quality is high. One way to see this is to note that the lowest *expected retail* price in the separating equilibria under observable wholesale contracts is equal to $\alpha \cdot 1 + (1-\alpha)\upsilon_L$. This is clearly larger than $\alpha$, the highest pooling price that can be sustained under unobservable contracts.

### 4.4 Comparison with the Full Information Outcome

At the end of this section we compare the pooling equilibrium outcomes described above with the outcome under full information. One can view the full information outcome as the market outcome if the manufacturer can credibly disclose product quality at sufficiently low cost. In particular, one can see it as the outcome of a regulatory policy that imposes severe fines for false advertising when advertising is fairly inexpensive. One may conjecture that the full information outcome is always better, but we show that this is not the case and that the inefficiencies created by monopoly power are such that welfare under the pooling equilibrium is higher.

To begin, we compare the consumer surplus under full information to that in the pooling equilibrium outcomes in Proposition 2. The retail prices in the low and high quality states under full information are $\upsilon_L$ and $\frac{1+c+\upsilon_L}{2}$, respectively. The expected consumer surplus under full information is therefore given by $\frac{\alpha}{8}(1+\upsilon_L-c)^2$. On the other hand, the expected consumer surplus in a pooling equilibrium outcome with retail price $p^*$ is $\frac{1}{2\alpha}(\alpha+\upsilon_L-p^*)^2$ and this exceeds the full information consumer surplus if

$$p^* < \frac{\alpha}{2}(1+c) + \left(1-\frac{\alpha}{2}\right)\upsilon_L. \tag{10}$$

The next proposition considers the situation where every $p^* \in [\max\{\upsilon_L, c\}, \alpha]$ can be sustained as the pooling price in an equilibrium described in Proposition 2 and shows that the inequality (10) is satisfied for all $p^*$ that lie below the pooling price that maximizes expected industry profits on this interval. For all such pooling prices, expected consumer surplus is higher than that under full information. Further, the expected total social surplus generated in some of these pooling equilibria can be higher than under full information.

**Proposition 4** *Suppose that* $\upsilon_L < \min\{1-\alpha, \alpha(3-4\alpha)\}$ *so that every* $p^* \in [\max\{\upsilon_L, c\}, \alpha]$ *can be supported as the retail price in a pooling equilibrium described in Proposition 2. Let* $\widetilde{p} \in [\max\{\upsilon_L, c\}, \alpha]$ *be the pooling price that maximizes expected industry profit across these equilibria.*

*(a) The expected consumer surplus generated in a pooling outcome with* $p^* \leq \widetilde{p}$ *is strictly higher than the expected consumer surplus under full information*

*(b) If $c$ is sufficiently small, pooling equilibrium outcomes with $p^*$ close to $\upsilon_L$ generate higher expected total social surplus than in the full information outcome. If, in addition, $\upsilon_L > \frac{2\alpha}{3(1+\alpha)}$, this holds for all $p^* \leq \widetilde{p}$.*

Thus, in a strong sense, consumers may benefit from not knowing quality. A regulatory policy that aims to enable credible disclosure of product quality can benefit the firms but lower consumer surplus as firms on average increase their prices and are able to achieve maximum profits under full information. The effect on consumer surplus can be so large that it more than offsets the decrease in industry profits. Thus, total surplus can also be higher in a pooling equilibrium; in the presence of market power, a vertical market with asymmetric information about product quality and unobserved vertical contracts may be more efficient than the full information market.

## 5 Discussion and Conclusion

In this paper we show that having an upstream contract with an intermediary that is unobserved by end users allows a manufacturer to hide his private information about product quality, creating the possibility of higher profits. If, on the contrary, such contracts are observed, the manufacturer always signals his private information resulting in potentially large distortions. Interestingly, consumers are typically also better off when product quality remains hidden as they buy (on average) at lower prices. Hiding information is only possible if the manufacturer sells through a third party (a retailer) who is free to determine the parameters (retail prices) that are of interest to consumers.

Observe that our results do not depend on whether or not we allow the manufacturer to set nonlinear wholesale prices. In particular, the pooling equilibrium we focus on in the case of unobservable wholesale contract is one where the manufacturer sets a per unit price (independent of quality) but no fixed fee. Beliefs allow the manufacturer to fully squeeze the retailer without using a fixed fee. Restricting the manufacturer to linear pricing would leave this outcome unchanged.[13]

Our key result also extends to a setting where the retailer is able to engage in first-degree price discrimination among consumers. Under first-degree price discrimination, the retailer would charge each consumer a price that is equal to their valuation. If the wholesale contract is separating i.e., wholesale prices depend on quality, then deterring imitation of the retail price charged to a consumer

[13] See, Rey and Vergé (2008) for an excellent exposition of nonlinear pricing and vertical restraints.

in the high quality state (by the retailer in the low quality state) would require the consumer to buy with sufficiently low probability at the higher price. This inefficiency and loss of profit can be prevented when the manufacturer offers a pooling contract in which case the retailer is only of one type and can extract the full surplus from each consumer. The manufacturer can then extract this profit from the retailer by setting an appropriate fixed fee. Thus, when the retailer can engage in first degree price discrimination, unobservable wholesale pricing can allow the manufacturer to extract maximal industry surplus and secure an efficient social outcome.

Our analysis indicates policies that incentivize firms to directly disclose product information credibly (for example by introducing severe penalties on false advertising) may backfire. With such policies in place, a high quality manufacturer will always directly communicate quality. This essentially results in a full information outcome that allows the manufacturers to exploit their market power more intensely and may adversely affect consumer welfare. Even the pooling equilibrium that maximizes expected profit in our setting with private information may yield higher consumer surplus than the full information outcome.

The methodology we develop on how to think of equilibrium refinement in "intermediate signaling games" where a Sender with private information sends an action to an Intermediary that is not observed by the Receiver should be of interest in a wide set of other strategic situations where the sender may potentially want to hide information by not interacting directly with the Receiver.

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## Appendix

### Appendix A (Proofs)

**Proof of Lemma 1.** We begin with some useful facts. Suppose that the (symmetric) belief is that quality is high with probability $\mu$. Then for any unit wholesale price $w \leq \mu + \upsilon_L$ the optimal price set by the retailer (if he accepts the contract) is

$$\begin{aligned} p(w,\mu) &= \frac{\mu + \upsilon_L + w}{2}, \text{ if } w \in [\upsilon_L - \mu, \upsilon_L + \mu] \\ &= \upsilon_L, \text{ if } w < \upsilon_L - \mu. \end{aligned} \tag{11}$$

For $w > \mu + \upsilon_L$, the retailer sells zero at any $p \geq w$ and so $p(w,\mu)$ is any price at least as large as $w$. The quantity sold by the retailer is then

$$\begin{aligned} d(p(w,\mu),\mu) &= \frac{\mu + \upsilon_L - w}{2\mu}, \text{ if } w \in [\upsilon_L - \mu, \upsilon_L + \mu], \\ &= 1, \text{if } w \leq \upsilon_L - \mu, \\ &= 0, \text{ if } w \geq \upsilon_L + \mu. \end{aligned}$$

Note that for $w \geq \upsilon_L$, $d(p(w,\mu),\mu)$ is non-decreasing in $\mu$ and for $w < \upsilon_L$, $d(p(w,\mu),\mu)$ is non-increasing in $\mu$.

Consider a perfect Bayesian equilibrium satisfying the Intuitive Criterion with symmetric beliefs where the $H$ and $L$ of the manufacturer pool with strictly positive probability and let $(w^*, F^*)$ be a pooling contract that can be offered in equilibrium by the manufacturer. Observe that

$$c \leq w^* \leq \alpha + \upsilon_L, \; F^* \leq (p^* - w^*)d(p^*, \alpha).$$

The retailer's equilibrium strategy $p_R(w, F)$ in such an outcome must be such that $p^* = p_R(w^*, F^*)$ is given by

$$\begin{aligned} p^* &= \frac{\alpha + \upsilon_L + w^*}{2}, \text{ if } w^* \in [\upsilon_L - \alpha, \upsilon_L + \alpha] \\ &= \upsilon_L, \text{ if } w^* < \upsilon_L - \alpha. \end{aligned}$$

Note that the equilibrium profit of the high type manufacturer is then

$$\begin{aligned}\pi^H &= \frac{1}{2\alpha}(w^* - c)(\alpha + \upsilon_L - w^*) + F^*, \text{ if } w^* \geq \upsilon_L - \alpha, \\ &= (w^* - c) + F^*, \text{if } w^* < \upsilon_L - \alpha;\end{aligned}$$

and the equilibrium profit of the low type manufacturer is

$$\begin{aligned}\pi^L &= \frac{1}{2\alpha}w^*(\alpha + \upsilon_L - w^*) + F^*, \text{ if } w^* \geq \upsilon_L - \alpha, \\ &= w^* + F^*, \text{ if } w^* < \upsilon_L - \alpha.\end{aligned}$$

Consider any unit wholesale price $w \in (w^*, 1 + \upsilon_L)$ and an associated fixed fee $F(w)$,where

$$F(w) = [p(w, 1) - w]d(p(w, 1), 1).$$

The profit earned by the low type manufacturer by deviating to such $(w, F(w))$ when buyers' belief is $\mu = 1$ equals

$$g(w) = \frac{1}{4}(1 + \upsilon_L + w)(1 + \upsilon_L - w).$$

Note that $g(w)$ is continuous (and strictly decreasing) in $w$ on $[\upsilon_L, 1 + \upsilon_L]$. As $w \downarrow w^*$,

$$\begin{aligned}g(w) &\rightarrow \frac{1}{4}(1 + \upsilon_L + w^*)\left(1 - (w^* - \upsilon_L)\right) > \frac{1}{4}(\alpha + \upsilon_L + w^*)\left(1 - \frac{w^* - \upsilon_L}{\alpha}\right) \\ &= p^* d(p^*, \alpha) = w^* d(p^*, \alpha) + (p^* - w^*)d(p^*, \alpha) \\ &\geq \frac{1}{2\alpha}w^*(\alpha + \upsilon_L - w^*) + F^* = \pi^L.\end{aligned}$$ [14]

On the other hand, as $w \uparrow (1 + \upsilon_L), g(w) \rightarrow 0$. Thus, there exists a unique $w_0 \in (w^*, 1 + \upsilon_L)$ such that $g(w_0) = \pi^L$.

We now claim that the low type manufacturer loses by deviating to the contract $(w_0, F(w_0))$ for any belief $\mu \in [0, 1]$. As noted above, $w_0 > \upsilon_L$ implies $d(p(w_0, \mu), \mu)$ is non-decreasing in $\mu$. So, if the contract $(w_0, F(w_0))$ is feasible for belief $\mu$ (i.e., the retailer makes non-negative profit), the low type manufacturer's deviation profit

$$w_0 d(p(w_0,\mu),\mu) + F(w_0) = g(w_0) - w_0[d(p(w_0,1),1) - d(p(w_0,\mu),\mu)] \leq g(w_0) = \pi^L.$$

If the contract $(w^*, F^*)$ is not feasible for belief $\mu$, the low type manufacturer makes zero profit. Thus, regardless of the beliefs of buyers, the low type manufacturer can never gain by deviating to a contract $(w_0, F(w_0))$. Note that $g(w_0) = \pi^L$ implies

$$p(w_0,1)d(p(w_0,1),1) = w^* d(p^*,\alpha) + F^* \leq p^* d(p^*,\alpha) = \frac{\alpha + \upsilon_L + w^*}{2} d(p^*,\alpha).$$

As $\alpha < 1, w_0 > w^*$, $p(w_0,1) = \frac{1+\upsilon_L+w_0}{2} > \frac{\alpha+\upsilon_L+w^*}{2}$, it follows that

$$d(p(w_0,1),1) < d(p^*,\alpha).$$

If a high type manufacturer deviates to a contract $(w_0, F(w_0))$ and belief is $\mu = 1$ his deviation profit is:

$$\begin{aligned}(p(w_0,1) - c)d(p(w_0,1),1) &= g(w_0) - cd(p(w_0,1),1) = \pi^L - cd(p(w_0,1),1) \\ &= w^* d(p^*,\alpha) + F^* - cd(p(w_0,1),1) = \pi^H + c[d(p^*,\alpha) - d(p(w_0,1),1)] \\ &> \pi^H\end{aligned}$$

Note that $(w_0, F(w_0))$ must be an out-of-equilibrium contract.[15] Based on the above arguments, the Intuitive Criterion with symmetric beliefs requires that the out-of-equilibrium belief satisfy $\mu(w_0, F(w_0)) = 1$, which immediately implies that the high quality manufacturer has an incentive to deviate to $(w_0, F(w_0))$, a contradiction. This completes the proof.

**Proof of Lemma 2.**$(i)$ Observe that if $L$ type deviates to $(w_H, F_H) = (1 - \upsilon_L, (\upsilon_L)^2)$, the retail price would be 1, the quantity sold would be $d(1,1) = \upsilon_L$ and his deviation profit would be $\upsilon_L(1-\upsilon_L) + (\upsilon_L)^2 = \upsilon_L$ so that his incentive constraint holds with equality. If $H$ type deviates to

[15] $(w_0, F(w_0))$ cannot be a separating contract offered with positive probability by the $H$ $(L)$ type manufacturer in this equilibrium as it yields strictly higher (strictly lower) payoff to the $H$ $(L)$ type mamufacturer than the pooling contract $(w^*, F^*)$ when $\mu = 1$ $(\mu = 0)$. Further, it cannot be a pooling contract offered with positive probability both types of the manufacturer as it yields strictly lower payoff to the $L$ type manufacturer than the pooling contract $(w^*, F^*)$ when $\mu = \alpha$.

$(w_L, F_L) = (0, v_L)$, the quantity sold would be 1 and his deviation profit would be $v_L - c < (1-c)v_L$, his equilibrium profit. It is obvious that given the restriction on out-of-equilibrium beliefs outlined in the lemma (belief assigns probability one to low quality), neither type of the manufacturer can gain strictly by deviating to a two-part tariff $(w, F)$ where $w \in (w_L, w_H)$. So, consider deviation to $(w, F)$ where $w > w_H = 1 - v_L$; here, the out-of-equilibrium beliefs assign probability one to high quality; the retail price following such deviation is $\frac{1+v_L+w}{2}$. If the manufacturer is of high quality, his deviation profit is bounded above by the industry profit following this deviation which is $\frac{1}{4}(1+v_L-w)(1+v_L+w-2c)$; the latter is strictly increasing in $w$ for $w > c$; as $c < 1-v_L < w$, this industry profit is less than the equilibrium profit of $H$ type. Same holds for $L$ type. So, we only need to show that the out-of-equilibrium beliefs meet our refinement.

First, consider deviation two-part tariff $(w, F)$ where $w \in (w_L, w_H)$. It is sufficient to show that if such a deviation is gainful for the $H$ type manufacturer for some symmetric belief $\mu \in [0, 1]$, then it is not equilibrium dominated for the $L$ type manufacturer so that assigning probability one to $L$ type does not violate the Intuitive Criterion. For the deviation to be gainful for the $H$ type manufacturer for belief $\mu$,

$$(w-c)d(p(w,\mu),\mu) + F \geq (1-c)v_L, \tag{12}$$

where $p(w, \mu)$ and $d(p(w, \mu))$ are as indicated at the beginning of the proof of Lemma 1. Note that for $w \geq v_L$, $d(p(w, \mu), \mu)$ is non-decreasing in $\mu$ and for $w < v_L$, $d(p(w, \mu), \mu)$ is non-increasing in $\mu$. In particular, for $w \geq v_L$, (12) implies

$$(w-c)d(p(w,1),1) + F \geq (1-c)v_L,$$

so that

$$wd(p(w,1),1) + F - v_L \geq c[d(p(w,1),1) - v_L] = c[\frac{1+v_L-w}{2} - v_L] > 0,$$

as $w < w_H = 1 - v_L$. This implies that the $L$ type manufacturer strictly gains by deviating to the same contract if the symmetric belief puts probability one on high quality and thus the deviation is not equilibrium dominated for the $L$ type. Now, suppose $w < v_L$. Then, $d(p(w, \mu), \mu) \leq 1$ and

(12) implies $(w - c) + F \geq (1 - c)\upsilon_L$,i.e.,

$$w + F - \upsilon_L \geq c(1 - \upsilon_L) > 0,$$

so that the $L$ type manufacturer strictly gains by deviating to the same contract if the symmetric belief puts probability zero on high quality and thus the deviation is not equilibrium dominated for the $L$ type.

Next, consider out-of-equilibrium two-part tariff $(w, F)$ where $w > w_H = 1 - \upsilon_L$; here, our specified beliefs assign probability one to high quality; we need to show that this is consistent with our refinement. Suppose $w \geq \upsilon_L$. Then, the $L$ type's deviation profit at any symmetric belief $\mu$:

$$\begin{aligned} wd(p(w,\mu),\mu) + F &\leq p(w,\mu)d(p(w,\mu),\mu) \\ &= \frac{1}{4\mu}[(\mu + \upsilon_L)^2 - w^2] \leq \frac{1}{4}[(1 + \upsilon_L)^2 - w^2] \\ &< \frac{1}{4}[(1 + \upsilon_L)^2 - (1 - \upsilon_L)^2] = \upsilon_L \end{aligned}$$

(the last inequality follows from $w > w_H = 1 - \upsilon_L$). Thus, $(w, F)$ is equilibrium dominated for $L$ type manufacturer; assigning probability one to $H$ type for such an out-of-equilibrium contract is therefore consistent with our refinement. For $w \in (1 - \upsilon_L, \upsilon_L)$, $d(p(w,\mu),\mu)$ is decreasing in $\mu$ and bounded above by 1. If $(w, F)$ is not equilibrium dominated for the $L$ type manufacturer, there exists $\mu \in [0, 1]$ such that $\upsilon_L \leq wd(p(w,\mu),\mu) + F$ so that

$$(w - c)d(p(w,\mu),\mu) + F \geq \upsilon_L - cd(p(w,\mu),\mu) \geq \upsilon_L - c$$

and therefore, $(w, F)$ is not equilibrium dominated for the $L$ type manufacturer. So, assigning probability one to $H$ type for such an out-of-equilibrium contract is consistent with our refinement. This concludes the proof of part $(i)$ of the lemma.

$(ii)$ Consider a fully separating perfect Bayesian equilibrium where the low and high type manufacturers set distinct two-part tariffs $(w_L, F_L)$ and $(w_H, F_H)$. In a separating equilibrium, consumers can infer quality from wholesale prices and the retailer can mark-up the retail price without affecting consumers' beliefs about quality. It is easy to see that in any such fully revealing equilibrium, the low type manufacturer will set $w_L \leq \upsilon_L$ and $F_L = (\upsilon_L - w_L)$; the retailer's optimal price must be $p(w_L, 0) = \upsilon_L$ and the low quality manufacturer's equilibrium profit is therefore $\upsilon_L$. Further, the retailer's optimal retail price after observing $(w_H, F_H)$ is $p(w_H, 1) = (1 + \upsilon_L + w_H)/2$. The condition that the low quality manufacturer should not have an incentive to imitate the high quality type is:

$$\frac{1}{2} w_H (1 - w_H + \upsilon_L) + F_H \leq \upsilon_L. \tag{13}$$

First, suppose that $w_H < 1 - \upsilon_L$.The ex ante expected profit of the high quality manufacturer in a separating equilibrium is

$$\begin{aligned} \pi_H &= \frac{1}{2}(w_H - c)(1 - w_H + \upsilon_L) + F_H \\ &\leq \upsilon_L - \frac{1}{2} c (1 - w_H + \upsilon_L) < \upsilon_L(1 - c) \end{aligned}$$

where the first inequality uses (13) and the second inequality follows from $w_H < 1 - \upsilon_L$ . Then, for $\epsilon > 0$ small enough

$$\pi_H < (\upsilon_L - \frac{\epsilon}{2})(1 - c + \frac{\epsilon}{2}). \tag{14}$$

Consider the following deviation contract $(\widehat{w}, \widehat{F})$ where $\widehat{w} = 1 - \upsilon_L + \epsilon, \widehat{F} = \left(\upsilon_L - \frac{\epsilon}{2}\right)^2$. Using an identical argument as in the last part of the proof of Lemma 1, one can show that if $\epsilon > 0$ is small enough and the manufacturer offers the deviation contract $(\widehat{w}, \widehat{F})$ the retailer rejects the contract if the symmetric belief $\mu < 1$ as he cannot break even (and so the deviating manufacturer gets zero profit); further, while the retailer can break even if $\mu = 1$, a low type manufacturer earns strictly lower than his equilibrium profit from this deviation at $\mu = 1$. Thus, this deviation is equilibrium dominated for the $L$ type manufacturer. However for belief $\mu = 1$ and when $\epsilon$ is sufficiently small, the $H$ type manufacturer can strictly gain from deviation to $(\widehat{w}, \widehat{F})$ which yields him profit $(\upsilon_L - \frac{\epsilon}{2})(1 - c + \frac{\epsilon}{2}) > \pi_H$ (using (14)). So using the Intuitive Criterion with symmetric beliefs, buyers should believe that the deviation comes from the manufacturer of type $H$ with probability one which then makes this deviation by the $H$ type manufacturer strictly gainful, a contradiction. Hence, $w_H \geq 1 - \upsilon_L$,which immediately implies that the retail price faced by buyers when the manufacturer is of $H$ type $p_H = p(w_H, 1) = \frac{1+\upsilon_L+w_H}{2} \geq 1$. Thus, the ex ante expected consumer surplus in the industry is bounded above by $\frac{\alpha}{2}(\upsilon_L)^2$.

Finally, as $(p-c)(1+v_L-p)$ is strictly decreasing in $p$ for $p \geq 1$ (using assumption (2)),

$$(p_H - c)(1 + v_L - p_H) \leq v_L(1-c)$$

where the left hand side is the sum of manufacturer and retailer's equilibrium profit i.e., the industry profit when manufacturer is of $H$ type. The ex ante expected industry profit is then bounded above by $\alpha v_L(1-c) + (1-\alpha)v_L = v_L(1-\alpha c) = \overline{\pi}$.

**Proof of Lemma 3.** At $p^* = \alpha$ and at $p^* = v_L$, the left hand side of (9) is equal to $v_L$ and the inequality in (9) is satisfied if, and only if, $v_L < 1-\alpha$. As $\left(1 - \frac{p^*-v_L}{\alpha}\right) p^*$ is concave and maximized at $p^* = \frac{\alpha+v_L}{2}$, (9) holds for all $p^* \in [v_L, \alpha]$ (and therefore, for all $p^* \in [\max\{v_L, c\}, \alpha]$) if, and only if, $\frac{1}{4\alpha}(\alpha+v_L)^2 < (1-\alpha)(\alpha+v_L)$, i.e., if, and only if, $v_L < \alpha(3-4\alpha)$. This establishes the second part of the lemma. If $v_L \geq \alpha(3-4\alpha)$, then there exist $z_1, z_2$,where $v_L < z_1 \leq z_2 < \alpha$ such that (9) is satisfied for $p^* \in [v_L, z_1] \cup [z_2, \alpha]$. Setting $\underline{p} = \max\{c, z_2\}$, we have the first part of the lemma. It follows that (9) is not satisfied for any $p^* \in [v_L, \alpha]$ if $v_L > 1-\alpha$.

**Proof of Proposition 3.** Note that in any pooling PBE satisfying IC-I described in Proposition 2 the expected consumer surplus is given by

$$\frac{1}{2\alpha}(\alpha + v_L - p^*)^2 \geq \frac{1}{2\alpha}(v_L)^2 > \frac{\alpha}{2}(v_L)^2$$

where the first inequality uses the fact that $p^* \leq \alpha$. Next, we investigate whether if there are $p^* \in [\underline{p}, \alpha]$ (where $\underline{p}$ is defined in Proposition 2) such that

$$\left(1 - \frac{p^* - v_L}{\alpha}\right)(p^* - \alpha c) > v_L(1-\alpha c) \tag{15}$$

First, consider $c$ small and in particular, $c \leq v_L$. Then, $\underline{p}$ as defined in Proposition 2 does not depend on $c$. Note that the function $f(p) = \left(\frac{\alpha+v_L-p}{\alpha}\right) p$ is strictly decreasing in $p$ for $p \in (\frac{\alpha+v_L}{2}, \alpha]$. Further, $f(\alpha) = v_L$. Fix $p^* \in (\max\{\underline{p}, \frac{\alpha+v_L}{2}\}, \alpha)$. Then, $f(p^*) > f(\alpha) = v_L$. As $c \to 0$, the right hand side of (15) converges to $v_L$ while the left hand side converges to $f(p^*) > v_L$. It follows that (15) holds for $c$ sufficiently small. This establishes (i).

Next, suppose that $v_L$ is small and in particular, $v_L < \alpha(3-4\alpha)$ in which case $\underline{p} = \max\{v_L, c\}$. It is easy to check that on the interval $[\max\{v_L, c\}, \alpha]$ the left hand side of (15) attains a maximum at $p^* = \max\left\{\frac{\alpha(1+c)+v_L}{2}, c\right\}$. Inequality (15) holds for some $p^* \in [\max\{v_L, c\}, \alpha]$ if and only if,

$$\begin{aligned}
\frac{1}{4\alpha}(\alpha(1-c)+v_L)^2 &> v_L(1-\alpha c), \text{ if } c \leq \frac{\alpha(1+c)+v_L}{2} \\
\frac{1}{\alpha}(\alpha+v_L-c)(1-\alpha)c &> v_L(1-\alpha c), \text{if } c > \frac{\alpha(1+c)+v_L}{2}
\end{aligned}$$

which is always satisfied for $v_L$ sufficiently small as $\frac{1}{4\alpha}(\alpha(1-c))^2 > 0$ and $\frac{1}{\alpha}(\alpha - c)(1-\alpha)c > 0$. This establishes (ii). This concludes the proof.

**Proof of Proposition 4.** From Lemma 3 and Proposition 2, $v_L < \min\{1-\alpha, \alpha(3-4\alpha)\}$ implies every $p^* \in [\max\{v_L, c\}, \alpha]$ can be supported as the retail price in a pooling equilibrium as described in Proposition 2. Let $\widetilde{p}$ be the pooling price that maximizes expected industry profit over $p^* \in [\max\{v_L, c\}, \alpha]$. If $\widetilde{p} \in (\max\{v_L, c\}, \alpha)$, then

$$\widetilde{p} = \frac{\alpha(1+c)+v_L}{2} < \frac{\alpha}{2}(1+c) + v_L\left(1 - \frac{\alpha}{2}\right)$$

so that (10) holds at $p^* = \widetilde{p}$ and therefore at $p^* \leq \widetilde{p}$. If $\widetilde{p} = \alpha$, then we must have $\frac{\alpha(1+c)+v_L}{2} \geq \alpha$ and using this one can check that $\alpha < \frac{\alpha}{2}(1+c) + v_L\left(1-\frac{\alpha}{2}\right)$ i.e., (10) holds at $p^* = \alpha$ and therefore at $p^* \leq \alpha$. Finally, consider the situation where $\widetilde{p} = \max\{v_L, c\}$ which requires

$$\frac{\alpha(1+c)+v_L}{2} \leq \max\{v_L, c\}.$$

As $\alpha > v_L$, this can only happen if $\max\{v_L, c\} = c$. It is easy to see that (10) holds at $p^* = c$ if $\frac{\alpha}{2} + \left(1 - \frac{\alpha}{2}\right)(v_L - c) > 0$ which is true. This establishes part (a). The expected total surplus under full information equals $(1-\alpha)v_L + \frac{3\alpha}{8}(1+v_L-c)^2$. The expected total surplus in a pooling equilibrium with $p^* \in [\max\{v_L, c\}, \alpha]$ is

$$\begin{aligned}
\frac{1}{2\alpha}(\alpha+v_L-p^*)^2 + \frac{p^*-\alpha c}{\alpha}(\alpha+v_L-p^*) &= \frac{1}{2\alpha}\left[(\alpha+v_L)^2 - p^{*2}\right] - c(\alpha+v_L-p^*) \\
&> (1-\alpha)v_L + \frac{3\alpha}{8}(1+v_L-c)^2
\end{aligned}$$

if, and only if,

$$\begin{aligned} p^{*2} - 2\alpha c p^* &< (\alpha + v_L)^2 - 2\alpha(1-\alpha)v_L - \frac{3\alpha^2}{4}(1 + v_L - c)^2 - 2\alpha c(\alpha + v_L) \\ &= \frac{\alpha^2}{4} + v_L^2 + \frac{\alpha^2}{2}v_L - \frac{3\alpha^2}{4}(v_L - c)^2 - \frac{\alpha^2}{2}c - 2\alpha c v_L. \end{aligned}$$

which holds at $p^* = v_L$ as $c \to 0$ and is therefore satisfied for $c$ small enough and $p^*$ close to $v_L$. Further, note that as $c \to 0$, $\widetilde{p} \to \frac{\alpha + v_L}{2}$. If $v_L > \frac{2\alpha}{3(1+\alpha)}$, the above inequality holds at $p^* < \frac{\alpha+v_L}{2}$ (and $c = 0$) and therefore, $p^* \leq \widetilde{p}$ when $c$ is small enough. This establishes (b).

**Appendix B: Belief Refinement for Intermediated Signaling Games (used in Section 4)**

In this appendix B, we develop a notion of equilibrium refinement that can be applied to our game of selling through a retailer with unobserved vertical pricing. It is based on considerations similar to the Intuitive Criterion in standard signaling games. The refinement we introduce is potentially useful in other contexts and can be applied to a general class of *intermediated signaling games*. These are dynamic games of incomplete information with three players: a Sender (S), an Intermediary (I) and a Receiver (R), with the following extensive form:

1. Nature draws a type $t$ for the sender from a finite set of types $T$ according to a probability distribution $\beta(t), t \in T$, where $\beta(t) > 0, \sum_{t \in T} \beta(t) = 1$.

2. The Sender observes $t$ and then chooses a message $m$ from a set $M$ of messages.

3. The Intermediary observes $m$ (but not $t$) and then chooses an action $a$ from a set of feasible actions $A$.

4. The Receiver observes $a$ (but not $m$ or $t$) and then chooses a response $r$ from a set of feasible responses $\rho$.

5. Payoffs for the Sender, the Intermediary and the Receiver are given by $U_S(t, m, r), U_I(m, a, r)$ and $U_R(t, a, r)$ respectively.

The standard signaling game analyzed widely in the existing literature has two players - a Sender and a Receiver. The Sender has private information about its type and chooses a message which is observed directly by the Receiver who then chooses a response. The payoffs of both players may depend on the type of the Sender, the message and the response of the Receiver.

The intermediated signaling game outlined above differs from this standard signaling game in several respects. First, there are three players including an Intermediary who moves after the Sender and before the Receiver. Like the Receiver, the Intermediary does not observe the type of the Sender. Second, the message sent by the Sender is observed only by the Intermediary, while the Receiver only observes the action chosen by the Intermediary. Third, the Receiver's payoff does not depend directly on the message sent by the Sender to the Intermediary, but only depends on the action chosen by the Intermediary. Finally, the Sender's payoff does not depend directly on the action chosen by the intermediary (though the latter may influence the Sender's payoff through the Receiver's response).

Focusing on pure strategy Perfect Bayesian Equilibrium (PBE), the equilibrium strategies are as follows. (a) The Sender's equilibrium strategy is a function $m^* : T \to M$ with a Sender of type $t$ choosing message $m^*(t), t \in T$, (b) The Intermediary's equilibrium strategy is a function $a^* : M \to A$, following any message $m$ from the Sender the Intermediary chooses action $a^*(m)$. (c) The Receiver's strategy is a function $r^*(a) : A \to R$, following any action $a$ by the Intermediary, the Receiver chooses response $r^*(a)$.

On the equilibrium path, the message sent by the Sender lies in the set $M^*$ where

$$M^* = \{m^*(t) : t \in T\}$$

and the action chosen by the intermediary lies in the set $A^*$

$$A^* = \{a^*(m^*(t)) : t \in T\}.$$

Let $U_s^*(t)$ denote the equilibrium payoff of type $t$ sender. For $m^* \in M^*$, let $U_I^*(m^*)$ denote the equilibrium payoff of the Intermediary in the continuation game after the manufacturer chooses $m^*$. For $a \in A/A^*$, let the probability distribution $\mu_a(t) \geq 0, t \in T$, $\sum_{t \in T} \mu_a(t) = 1$, be the out-of-equilibrium belief of the Receiver when it observes action $a$ of the intermediary.

Consider any out-of-equilibrium action $a \in A/A^*$of the Intermediary. Let $\Sigma_T$ be the set of all probability distributions on $T$. Further, for any $\widehat{\mu} \in \Sigma_T$ let

$$BR(\widehat{\mu}, a) = \arg\max_{r \in \rho} \sum_{t \in T} U_R(t, a, r)\widehat{\mu}(t)$$

be the set of best responses of the Receiver after observing $a$ when it has belief $\widehat{\mu}$; let $BR(T,a)$ be the set of all such best responses for all possible beliefs of the Receiver i.e.,

$$BR(T,a) = \cup_{\widehat{\mu}\in\Sigma_T} BR(\widehat{\mu},a).$$

In line with the Intuitive Criterion (Cho and Kreps, 1987) for the standard signaling game and in accordance with the principle of focusing on unilateral deviations when accounting for an observed out-of-equilibrium action, we introduce two definitions of equilibrium domination, one for the Sender and one for the Intermediary:

**Definition 1** *Given the equilibrium strategy* $m^* : T \to M$ *of the Sender, an out-of-equilibrium action* $a \in A/A^*$ *is said to be equilibrium dominated for the Intermediary who has observed message* $\widetilde{m} \in M^*$ *if*

$$\max_{r\in BR(T,a)} U_I(\widetilde{m},a,r) < U_I^*(\widetilde{m}).$$

**Definition 2** *Given the equilibrium strategy* $a^* : M \to A$ *of the Intermediary, for* $t \in T$*, a message* $\overline{m} \in M/M^*$ *is said to be equilibrium dominated for the Sender of type* $t$ *if*

$$\max_{r\in BR(T,a^*(\overline{m}))} U_S(t,\overline{m},r) < U_S^*(t).$$

The belief formation process by the Receiver after observing an out-of-equilibrium action $a \in A/A^*$ has two components. First, the Receiver assigns a probability $x_a^I \in [0,1]$ that the observed out-of-equilibrium action $a$ results from a unilateral deviation by the Intermediary (given the equilibrium strategy of the Sender) and a probability $x_a^S = 1 - x_a^I$ that the out-of-equilibrium action $a$ results from a unilateral deviation by the Sender (given the equilibrium strategy of the Intermediary). Second, conditional on a unilateral deviation by the Intermediary, the Receiver assigns a probability $y_a^I(t)$ to the Sender being of type $t, y_a^I(t) \geq 0, \sum_{t\in T} y_a^I(t) = 1$, while conditional on a unilateral deviation by the Sender, the Receiver assigns a probability $y_a^S(t)$ to the Sender being of type $t, y_a^S(t) \geq 0, \sum_{t\in T} y_a^S(t) = 1$. Note that even when considering a unilateral deviation by the Intermediary, one assigns probabilities $y_a^I(t)$ to the Sender being of type $t$ as only the Sender and not the Intermediary can be of different types. Nevertheless, as explained in more detail below, depending on the type of equilibrium (pooling, semi-separating or separating), a deviation by the

Intermediary can reveal some information to the Receiver.

Next, we use the above definitions of equilibrium domination to define when an out-of-equilibrium action can be attributed to a unilateral deviation by the Intermediary and/or the Sender.

**Definition 3** *An out-of-equilibrium action* $a \in A/A^*$ *of the Intermediary can be attributed to a* unilateral *deviation by the Intermediary if given the equilibrium strategy* $m^* : T \rightarrow M$ *of the Sender,* $a$ *is* not *equilibrium dominated for the Intermediary for some equilibrium message* $\widetilde{m} \in M^*$ *from the Sender.*

**Definition 4** *An out-of-equilibrium action* $a \in A/A^*$ *of the Intermediary can be attributed to a* unilateral *deviation by the Sender if* $a = a^*(\overline{m})$ *for some* $\overline{m} \in M$ *and further, given the equilibrium strategy of the Intermediary, there exists* $t \in T$ *such that* $\overline{m}$ *is* not *equilibrium dominated for the Sender of type* $t$.

Note that the above two definitions incorporate an asymmetry in that any out-of-equilibrium action $a$ can, in principle, be attributed to a unilateral deviation of the Intermediary, whereas this is not the case for the Sender. In particular, if $a$ is not in the reach of the equilibrium strategy of the Intermediary, then no unilateral deviation by the Sender can account for $a$.

We now outline a set of restrictions on the out-of-equilibrium belief $\mu_a(t), t \in T$, that adapt the Intuitive Criterion for standard signaling games to our game of intermediated signaling. The criterion follows the principle that if observations can be rationalized by unilateral deviations, they should get priority (see, e.g., Bagwell and Ramey 1991). This implies, among other things, that if $a$ is not in the reach of the equilibrium strategy of the Intermediary and $a$ is not equilibrium dominated for the Intermediary, then the Receiver should blame the Intermediary for the deviation from equilibrium play.

**Intuitive Criterion for Intermediated Signaling Games (IC-I)**: *Consider an out-of-equilibrium action* $a \in A/A^*$ *of the intermediary that can be attributed to a unilateral deviation by either the Sender or the Intermediary (or both). Then, the out-of-equilibrium belief* $\mu_a(t), t \in T$ *satisfies the Intuitive Criteria for Intermediated Signaling Games (IC-I) if the following restrictions are satisfied:*

*(i)*

$$\mu_a(t) = x_a^I y_a^I(t) + (1 - x_a^I) y_a^S(t), t \in T$$

*and further, if action a cannot be attributed to a unilateral deviation by the Sender (Intermediary), then* $x_a^I = 1$ ($x_a^I = 0$).

*(ii) Suppose that action a can be attributed to a unilateral deviation by the Intermediary. Let* $T_0(a) = \{t \in T : a$ *is equilibrium dominated for the Intermediary when the Sender is of type t and sends equilibrium message* $m^*(t)\}$*. Then,* $y_a^I(t) = 0$ *for all* $t \in T_0(a)$*. Further, for any* $\widehat{t} \in T/T_0(a)$, $\tau(\widehat{t}) = \{t \in T/T_0(a) : m^*(t) = m^*(\widehat{t})\}$*, the following holds:*

$$\frac{y_a^I(\widehat{t})}{\sum_{t\in\tau(\widehat{t})} y_a^I(t)} = \frac{\beta(\widehat{t})}{\sum_{t\in\tau(\widehat{t})} \beta(t)}.$$

*(iii) Suppose that the action a can be attributed to a unilateral deviation by the Sender. If* $a = a^*(\overline{m})$ *and further, given the equilibrium strategy of the Intermediary, the message* $\overline{m}$ *is equilibrium dominated for the Sender of type t, then* $y_a^S(t) = 0$.

The first part of the definition considers when to attribute an out-of-equilibrium action $a \in A/A^*$ exclusively to the retailer or the manufacturer or whether unilateral deviations by both players can account for the deviation (given the equilibrium strategy of the other). For any such attribution to a unilateral deviation by a player, the second and third requirements impose restrictions on assignment of beliefs to different types of the Sender. The third requirement simply adjusts the implications of the original Intuitive Criterion to the setting of intermediated signaling where the Receiver does not observe the Sender's message and can only infer which Sender could have deviated given the Intermediary's equilibrium strategy.

The second requirement is more involved and a few examples may clarify. If, for example, one considers a candidate pooling equilibrium and an out-of-equilibrium action $a \in A/A^*$ that is not equilibrium dominated for the retailer, then $y_a^I(\widehat{t}) = \beta(\widehat{t})$, i.e., conditional on a unilateral deviation by Intermediary, as the Receiver cannot infer any information from the incentive of the Intermediary to deviate (given the deviation is not based on any learning of the type of the Sender), the Receiver should assign the beliefs to be identical to the prior for all types. In other words, as the Intermediary does not acquire any additional information after observing the Sender's message, to the extent that the Receiver blames out-of-equilibrium action on a unilateral deviation by the Intermediary it should assign the same beliefs as the Intermediary has at that stage. A similar logic applies if a subset of Sender types pool on a message and one looks at the event of unilateral deviation by the intermediary conditional on this pooled message: the relative likelihood of each type that pools should be as in the prior belief. Finally, if an out-of-equilibrium action $a \in A/A^*$ is equilibrium dominated for the Intermediary given the Sender's equilibrium strategy $m^*(t)$ for only a strict subset of Sender types, then conditional on attributing attributing $a$ to a unilateral deviation by the Intermediary, the Receiver should assign probability zero to this subset and probability one to the complement of this subset. Overall, the second requirement is a conservative way to implement the considerations underlying the Intuitive Criterion: the Receiver attributes deviations in such a way that they are consistent with the information the Intermediary may have had when deviating.

The criterion outlined above confines attention to incentives for unilateral deviations. If an out-of-equilibrium action cannot be accounted for by unilateral deviations, then our criterion does not impose any restriction on the out-of-equilibrium belief.