Multiproduct Firms, Refunds and Product Returns*

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Abstract

Many e-commerce retailers adopt strategies that induce consumers to order multiple products at once, inspect their fit at home, and then decide which products to return. These policies introduce a trade-off as they result in consumers acquiring products that better fit their taste, at the expense of the private and social costs associated with products being returned. We determine the conditions under which retailers find it optimal to induce consumers to inspect products simultaneously or sequentially. We also analyze the efficiency properties of market outcomes and state conditions under which inducing simultaneous inspection (surprisingly) leads to fewer returns. An important part of the analysis characterizes the optimal alternative pricing policy that induces consumers to sequentially inspect products after ordering and finds that partial refunds facilitate the extraction of surplus from consumers.

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1 Introduction

Product returns play an increasingly important role in retail markets. A recent report of the National Retail Federation estimates that total returns in the retail industry of the USA reached $890 billion in 2024, which is around 16,9% of total annual sales. In the online segment of the retail market return rates were even 21% higher than their overall return rates.[1] Given the importance of product returns, retailers have started to treat returns strategically by developing optimal return policies. One of these developments is that e-commerce retailers like Amazon and Zalando offer(ed) consumers the possibility to order multiple items at the same time, inspect them at home to see whether they like them, and to return all items that are considered not to be a good fit.[2]

In this paper we show how *multi-product* firms, like many online retailers, should design their strategy towards pricing and refund policies to maximize profits. Refund policies can be an important source of a firm's profit. We also evaluate the welfare consequences of firms' optimal refund policies, with a special focus on how frequently products are returned. As returned products often cannot be easily resold in the market, there are potentially large environmental costs associated with product returns.[3] The increasing concern related to product returns is voiced by websites estimating that only 54 percent of all packaging gets recycled and that 5 billion pounds of returned goods end up in landfills each year.[4] One particular question we address in this paper is whether policies like 'Try Before You Buy' would inherently lead to more products being returned and if so, under which conditions would firms find it optimal to install such policies.

To study product returns, it is crucial to take a consumer search perspective. To learn their value for a product, consumers have to inspect it at a (time) cost. Roughly speaking, there are two ways consumers can perform inspections. First, they inspect before ordering

[1] See the NRF report available at `https://nrf.com/research/2024-consumer-returns-retail-industry`.

[2] Before discontinuing the service on 31 January 2025 Amazon offered it as 'Try Before You Buy' to 'Prime' customers. Before that it was called 'Prime Wardrobe'. Amazon motivates the discontinuation of the service by saying that customers increasingly use new AI-powered features like virtual try-on and personalised size recommendations; see, e.g., `https://nationaltechnology.co.uk/Amazon_Ends_Try_Before_You_Buy_Clothing_Service.php` and `https://www.livenowfox.com/news/amazon-try-before-you-buy-prime-program-ending` for more details. Below, we comment on how to account for these changes in the firm's policy and on their effects in terms of our framework.

[3] These environmental costs include greenhouse gas emissions, non-recycled packaging and products filling up landfills (see, e.g. Tian and Sarkis (2022)).

[4] See, e.g., https://www.akeneo.com/blog/the-environmental-impact-of-returns/, https://www.returnbear.com/resources/reducing-the-environmental-impact-of-returns or https://www.retailcustomerexperience.com/blogs/who-owns-the-retail-returns-process-the-impact-and-challenges-of-disjointed-strategies/.

and buy the product if they are satisfied with its features. This is the traditional way of shopping and the classic approach to consumer search in the literature (see, e.g., Wolinsky (1986) and Anderson and Renault (1999)). Alternatively, and second, consumers can order products straightaway and inspect them only after they have been delivered. As inspecting after ordering can usually be done in a more comfortable environment at a time that suits the consumer best, it is less costly for the consumer than inspecting before ordering. An important welfare consideration related to product returns is whether consumers eventually buy products that better fit their needs.

Whether consumers inspect before or after ordering depends not only on the difference in inspection costs, but also on the refunds firms offer (in case consumers learn after ordering that they do not sufficiently like the product). Offering generous refunds comes at a cost to the firm (as the salvage value, i.e., the value of the returned product, is smaller than the production cost), and whether they are willing to induce consumers to inspect after ordering depends on how much of additional consumer surplus they can extract by doing so.

In determining its optimal selling policy in terms of prices and refunds a multi-product online retailer may induce (or provide incentives to) consumers (i) to inspect products *before or after* ordering, and (ii) to inspect and buy (and return) multiple products *simultaneously or sequentially.* To consider the optimal strategy, it is important to note that online retailers can offer different prices and refunds for different products, but also condition these on whether or not a consumer orders multiple products simultaneously. The firm cannot, however, condition prices or refunds on whether or not a consumer inspected a product before ordering, as (certainly in online markets) firms do not observe this.[5] Thus, when designing a policy that induces consumers to inspect sequentially, it should take into account that they are free to inspect products before or after ordering.

We consider two important cases: (i) the difference in inspection cost before and after ordering is large and (ii) both inspection costs are small so that their difference is also small. In the first case, the option to search before ordering is not a credible threat as consumers never find it optimal to do so (simply because it is too difficult to learn a product's fit before having it at home). In contrast, in the second case, the option of inspecting before buying severely constrains the selling strategy of the firm. In both cases, we consider that consumers have a demand for one unit only. Before explaining our results in detail, it is useful to redefine the strategy of the firm as follows. The difference between a product's retail price and the

[5] An online retailer may observe that a consumer clicked on a product, and possibly observes that a consumer did not engage in any other activity online, but cannot distinguish between a consumer inspecting the product or being distracted.

refund is a “price” consumers always pay if they inspect the product after ordering, no matter whether they eventually buy/keep the product or not. We call this difference the *inspection fee* the firm chooses: it is the price the consumer pays for the right to inspect the product after ordering. The cost for the firm related to a consumer ordering, but not buying the product, is the *product degradation* cost. Once consumers have inspected the product, the relevant decision is whether or not they return it. The price the firm charges for *not* returning the product is the refund, while the cost for the firm of *not* returning the product is the salvage value of the product when it is returned. Thus, we consider that the firm chooses the refund and the inspection fee for its products.[6]

Large difference in inspection cost. With this redefinition in mind, we first consider that the difference in inspection cost is large. In this case, the *optimal selling policy that induces sequential search* is to set very different prices for the products and set them such that the consumer finds it optimal to inspect one particular product first (which we term the first product). For ease of exposition, we focus on the firm producing two products. The pricing policy is such that for the second product the refund is set equal to the salvage value and the inspection fee is chosen such that all consumer surplus from inspecting the second product is extracted. Thus, the refund for the second product is set efficiently as a consumer will only return the product if its value is smaller than the salvage value. For the first product, the refund is set equal to the opportunity cost of selling the second product, while the inspection fee is chosen to extract all surplus from the whole search process. The policy is such that (i) the first product has a higher refund than the second product and (ii) once consumers decide to inspect the second product, they never come back to buy the first product.

Under *sequential search*, the optimal selling policy has the flavour of a two-part tariff in the sense that the inspection fee is used to extract surplus, while the refund is chosen as a price reflecting the cost to the firm at different stages of the search process. There is one important difference with a traditional two-part tariff, however, in that the inspection fee for the second product also affects the consumers’ decision to inspect the second product. From a social efficiency perspective, the optimal selling policy sets this inspection fee for the second product (or equivalently, the refund for the first product) too high and this product is not inspected often enough.

Under *simultaneous inspection*, the optimal selling policy is such that the firm sets the refund equal to the salvage value and an inspection fee that extracts the expected maximum

[6] Thus, the retail price is implicitly defined as the inspection fee plus the refund, while the firm’s product cost equals the product degradation cost plus the salvage value.

consumer value (given that it is larger than the salvage value). From an efficiency perspective, the consumers' return decision for both products is optimal, but there is too much search, especially when the product degradation or inspection cost after ordering is relatively large.

Comparing the two candidate optimal policies in case the difference in inspection cost is large, we find that it is profitable for the firm to induce simultaneous inspection if the sum of product degradation and inspection cost after ordering is sufficiently small. This selling policy of inducing simultaneous inspection leads to more returns, however, and a regulation forbidding such policies would reduce the environmental costs related to returns (while in terms of their private well-being consumers are equally well off as they obtain zero surplus in both solutions).

Small inspection costs. In case both inspection costs (and their difference) are small, results are strikingly different. The main difference with the previous case is that to induce the consumer to inspect after ordering the firm cannot choose large inspection fees as otherwise the consumer inspects before ordering. This has two important implications.

First, in terms of the pricing strategy under sequential inspection, the refunds set for returning products should be close to each other and close to the joint monopoly price, i.e., the profit maximizing retail price of a two-product monopolist. The reason is that setting the joint monopoly price is optimal for the firm if consumers inspect before ordering and the inspection costs are small. Inspection fees should be small to induce consumers to inspect after ordering, in which case the firm does not want to deviate much from the joint monopoly price as revenues from product returns are small.

Second, even though consumers return at least one item for sure if they engage in simultaneous inspection, surprisingly the expected number of returns under simultaneous inspection may be lower than if consumers inspect sequentially. The reason is as follows. If consumers engage in sequential inspection, firms induce them to search the first product after ordering if the product degradation cost is small, while the second product will be inspected before or after ordering, depending on the value of the first product. Consumers continue to search the second product before inspection if the value of the first product is smaller than the refund. Thus, as the refund under sequential search is much higher than under simultaneous search they are more likely to return both products. This more than compensates for the possibility that under sequential inspection no product is returned.

This has important consequences for regulatory policies aiming to reduce the environmental impact of product returns. In particular, a regulation forbidding firms to induce consumers to simultaneously inspect multiple products and return what they do not like, may backfire and create more (socially wasteful) product returns. This is because it remains true that in the absence of such regulations the firm makes more profit when inducing consumers to inspect products simultaneously if the product degradation cost is sufficiently small. When these policies are in place, consumers will inspect products sequentially after ordering leading to more returns.

Finally, a comparison between the two cases (of large and small differences in inspection costs) reveals why some online retailers may abandon inducing consumers to search simultaneously many products at home. New technologies, such as "virtual try-on" and "personalized size recommendations" make it much easier for consumers in online markets to inspect products before purchasing them. The arrival of these technologies imply that there is a transition between the large and the small difference in inspection costs. When the difference in inspection costs is relatively small, online retailers may find it profitable to induce consumers to search sequentially before purchasing if the product degradation costs are relatively large. An important side effect of inspection before ordering is that it gives rise to (much) less product returns. *In our theoretical model where consumers learn their full value upon inspection, it leads to no products being returned.*

Related literature. The paper combines two strands of literature. The papers most closely related to ours are Janssen and Williams (2024), Jerath and Ren (2025) and Matthews and Persico (2007) in that they also study product returns in a consumer search setting. However, all these papers study a single product firm and consumers searching sequentially (where the first paper studies a competitive setting, while the last two analyze monopoly behavior). They find that the number of refunds is inefficiently high or low. None of these papers considers a firm that incentivizes consumers to search simultaneously among its multiple products.[7] Petrikaitė (2018a) studies search with returns in a duopoly setting, but also does not consider multiple products per seller or simultaneous search.

The second strand of literature is on multi-product search (Rhodes (2015), Shelegia (2012) and Zhou (2014)). The focus of these papers is on consumers searching for multiple products, creating a joint search effect: once consumers are at a store, they have a lower search cost for products at that store. These papers do not study product returns or simultaneous search.

The optimal behaviour of the firm if it wants to induce sequential search after ordering

[7] Another difference with Janssen and Williams (2024) is that we study a setting in which consumers learn the firm's prices and refunds at no cost. This feature our paper has in common with the recent literature on price directed search; see, e.g., Armstrong (2017), Choi, Dai, and Kim (2018). Bird, Garrod, and Wilson (2024) study a setting of experience goods where consumers can only learn the value of the good after purchase.

has features that also arise in Petrikaitė (2018b) and Gamp (2022) in that a multi-product firm has an incentive to obfuscate search among its products. These papers study a setting where consumers have to inspect products before purchasing one of them and where (together with prices) the firm chooses consumers' search cost directly. They show that the firm has an incentive to set a positive search cost and asymmetric prices so as to induce consumers to search the products in a particular order. In contrast, we allow consumers to order (or buy) products before inspecting them and have a setting where the firm cannot affect the inspection cost of consumers directly. However, by choosing a refund that is smaller than the price, the firm effectively sets an inspection fee that the consumer pays upfront when deciding to inspect. This inspection fee is part of the firm's profits, which makes for another important difference to the above mentioned papers.

More broadly, our paper contributes to the consumer search literature by extending the options consumers have, where in the seminal contributions by, for example, Wolinsky (1986), Anderson and Renault (1999) and Armstrong (2017) consumers can only learn their match value before ordering/purchasing. Morgan and Manning (1985) show that if agents can choose to search sequentially or simultaneously at the same terms, it is optimal to search sequentially if they are patient enough or if sequential search comes without delay. This result also applies to our setting if prices and refunds are identical across inspection modes. However, by offering different prices and refunds when ordering multiple items at once, the firm may induce the consumer to search simultaneously.

The remainder of the paper is organized as follows. The next section introduces the model. Section 3 discusses the case where the difference in inspection costs is large, while Section 4 considers the case where both inspection costs (and their difference) is small. Section 5 concludes with a discussion.

2 The Model

A monopoly firm sells two products. Each product has a production cost $c \geq 0$ and a salvage value $\eta \in [0, c]$ to the firm in case the product is bought and then returned. We will define $k = c - \eta$ as the value lost if the product is returned after it is inspected and we will refer to k as the *product degradation* cost. The firm can set different retail prices and refunds for the different products $i = 1, 2$ and we denote retail price by $p_i \geq 0$ and refund by $\tau_i \in [0, p_i]$, which is the money the firm commits to return to the consumer in case the latter returns the product.[8] Consumers can learn the value of a product by inspecting it before or after ordering, where we interpret "inspecting after ordering" as the act of ordering the product and committing to pay the difference between retail price and refund in case the product is returned after having inspected it. A positive difference between p_i and τ_i can be interpreted as the firm giving a partial refund.[9]

The firm cannot condition its prices on whether consumers inspect products before or after ordering (as the firm does not observe this). It can only set prices and refunds such that it incentivizes consumers to inspect products in one way or the other. The firm does observe whether consumers order multiple products at once, and therefore it can offer consumers prices and refunds that are only valid if multiple products are ordered simultaneously. We denote these prices by (p_{sim}, τ_{sim}) with $p_{sim} \geq \tau_{sim}$.[10]

Consumers have unit demand. The two products are ex-ante identical to consumers with each product having a valuation that is independently and identically distributed by $v_i \sim F[\underline{v}, \bar{v}]$, with a density $f(v)$ that is positive, continuously differentiable and where f is logconcave.[11] To have an interesting model, we require $\bar{v} > c \geq \eta \geq \underline{v}$. Consumers know the prices and refunds the firm offers, but have to pay[12] an inspection cost of $s_B > 0$ to learn a product's value before ordering and a cost of $s_A > 0$ if they learn the product's value after ordering, with $s_A \leq s_B$.[13] The outside option of the consumer is normalized to 0. For future reference, it will be useful to write $\hat{v}^b$ as the reservation value of inspecting a product before ordering and $\hat{v}_i^a$ as the reservation value of inspecting product i after ordering. They are implicitly defined through the following equations:

$$\int_{\hat{v}^b}^{\infty} (v - \hat{v}^b) f(v) dv = s_B \quad \text{and} \quad \int_{\hat{v}_i^a}^{\infty} (v - \hat{v}_i^a) f(v) dv = s_A + p_i - \tau_i. \tag{1}$$

[8] Note that to prevent arbitrage the firm would never set a refund larger than the retail price.

[9] Even if firms formally give a full refund, consumers often face return or restocking fees. Return costs may consist of the cost of shipping the product back to the firm and/or time or "hassle" costs related to the return process. Similar to Janssen and Williams (2024) one can show that a model with an explicit return cost h is equivalent to our model without such a return cost if we redefine $\eta' := \eta - h$ and $\tau_i' := \tau_i - h$. Restocking fees can be substantial and up to 20 percent of the retail price (see e.g. `https://www.zonguru.com/blog/amazon-restocking-fee`).

[10] As the firm will not benefit from setting different prices under simultaneous search, we do not use subscripts for the price and refund of the different products.

[11] It is well-known that this implies that the associated distribution function F and $1 - F$ are then also logconcave; see, e.g., Bagnoli and Bergstrom (2005).

[12] Thus, in the main model we do not allow consumers to "buy blindly", that is without inspecting the product at all, as in Doval (2018). Qualitatively, our main results are not affected if we would allow for blind buying and in footnotes we do comment on how the optimal contract would be affected if consumers could also "buy blindly".

[13] Thus, if consumers simultaneously order two products they will always inspect them after ordering as this comes at a lower inspection cost.

Note that $\hat{v}_i^a$ is not only a function of exogenous parameters but also of p_i and τ_i, the two strategic variables of the firm for product i. When we write $\hat{v}_i^a$ we implicitly mean the function $\hat{v}_i^a(p_i - \tau_i)$.

Given the firm's choices, the consumer can take one of the following actions: (i) Inspect products sequentially, (ii) Inspect products simultaneously or (iii) Leave and take the outside option with a pay-off of 0. Under (i), the consumer decides in which order to inspect the products and can inspect each product either before ordering or after ordering. Inspecting a product before ordering entails paying the inspection cost of s_B to learn that product's value and then deciding whether to buy it at price p_i or, in case of the first product, continuing to inspect the second product. Inspecting a product after ordering entails paying the inspection cost of s_A to learn that product's value, deciding whether to keep it and pay the price p_i, or, in case of the first product, continuing to inspect the second product, and finally returning and paying $p_i - \tau_i$ for all products inspected after ordering that are not kept.[14] If consumers search sequentially, they have perfect recall.

Consumers inspecting simultaneously, i.e., option (ii), can only arise as an equilibrium outcome if they inspect after ordering. As firms cannot observe whether consumers inspect before ordering, they cannot price discriminate between simultaneous inspection before ordering and sequential inspection before ordering. But if the prices across these two options are the same, we can appeal to Morgan and Manning (1985) who showed that without delay consumers would prefer to inspect sequentially. This is different for inspection after ordering as the firm observes then whether or not the consumer orders multiple products. Thus, if (ii) arises in equilibrium the consumer inspects both products simultaneously after ordering at an inspection cost of s_A each and decides whether to buy at most one of the products at the contract (p_{sim}, τ_{sim}) and returns at least one.

It is important to note that it is possible to redefine inspection after ordering as a structurally simpler problem, which will facilitate the analysis. From the consumer's view inspection after ordering can be re-written as inspection before ordering with certain inspection costs and prices. In particular, at the moment consumers order product i to inspect it at inspection cost s_A, they commit to paying at least $p_i - \tau_i$, which is the part of the price they do not get back if they return the product. If they instead want to keep the product they additionally "pay" τ_i, as they forgoe the refund they could have received. Thus, we can redefine inspection after ordering as inspection before ordering with a redefined inspection

[14] Note that it does not matter whether p_i is paid at the time of ordering or at the checkout when the final decision is made on which product to keep. By ordering product i, the consumer commits in both cases to pay at least $p_i - \tau_i$ to the firm. See also the paragraph after the next one.

cost of $s_A + p_i - \tau_i$ and a redefined price of τ_i. Note that while s_A is lost, $p_i - \tau_i$ is the part of the redefined inspection cost that is paid to the firm. It is thus as if the firm was offering product i for inspection before ordering at an *inspection fee* of $\sigma_i := p_i - \tau_i$ and a *price* for keeping the product $\rho_i := \tau_i$. In line with this redefinition, we can also split the production cost c into two parts that the firm incurs when the consumer respectively *inspects* or *keeps* the product. When the consumer inspects the product, the firm incurs the product degradation cost k. When the consumer decides to keep the product, then the firm incurs the cost η, as it forgoes the product's salvage value. Overall, it is as if the firm chooses for each product an inspection fee σ_i with the associated opportunity cost k and a price (refund) ρ_i with the associated opportunity cost η.[15,16] Consumers may find it optimal to commit to pay (a relatively small) σ_i to inspect after ordering even if they do not keep the product as they benefit from a lower inspection cost. Firms may want to induce consumers to inspect after ordering and risk incurring the product degradation cost k when consumers do not keep the product as they may make profit from the return if $\sigma_i > k$.

3 When the Difference in Inspection Costs is Large

When the difference in inspection costs is large, and thus s_B is large, the consumer will never choose to inspect products before ordering. When designing the optimal contract conditional on the consumer searching sequentially, the firm does not need to worry about the consumer searching before ordering. Thus, its strategy focuses on a consumer inspecting the products sequentially after ordering. In this section, we first construct the optimal contracts for both simultaneous search and sequential search. We then compare profits under both contracts to determine the optimal contract for the firm, before we compare the number of returns under sequential and simultaneous search.

Sequential search. The next proposition states the optimal contract under sequential search.

Proposition 1 *If s_B is large[17] and the firm induces consumers to inspect sequentially the optimal contract has:*

$$(\sigma_1^*, \rho_1^*) = (\mathbb{E}[\max(v_1 - ES_2 - \eta, 0)] - s_A, ES_2 + \eta) \text{ and } (\sigma_2^*, \rho_2^*) = (ES_2 + k, \eta)$$

[15] To avoid confusion, we will refer to ρ_i as the "price" and to p_i as the "retail price".

[16] As discussed above, to explicitly account for a return cost h, the required transformations in this redefined model are $\eta' := \eta - h, \rho_i' := \rho_i - h, k' := k + h$ and $\sigma_i' := \sigma_i + h$, as c and p_i remain unchanged.

[17] It is clear that how large s_B should be for it not to impose a constraint on the contract the firm offers depends on the other parameters, most notably s_A. If s_A is relatively large itself, then s_B should be even larger for this to be true.

with profits $\pi^* = \mathbb{E}[\max(v_1 - \eta, ES_2)] - s_A - k$ *and where:*[18]

$$ES_2 = \mathbb{E}[\max(v_2 - \eta, 0)] - s_A - k. \tag{2}$$

The intuition behind the optimality of the strategy is as follows. Weitzman (1979) implies that the consumer first inspects the product with the higher net reservation value $\hat{v}_1^a - \rho_1 \geq \hat{v}_2^a - \rho_2$ and only inspects a product if it has a non-negative net reservation value $\hat{v}_i^a - \rho_i \geq 0$ (as this is a necessary condition for non-negative utility). Without loss of generality consider that product $i = 1$ is inspected first. Then, as the inspection fee σ_1 for the first inspected product is committed to be paid before inspection starts, the firm can increase it (without distorting consumer decisions) as long as the above inequalities are not violated. This implies that in the optimal contract we should have that $\hat{v}_1^a - \rho_1 = \hat{v}_2^a - \rho_2$, i.e. the net reservation values of the two products will be equal.[19] *If* the firm will choose the contracts for both products such that the net reservation values will be equal to zero $\hat{v}_i^a - \rho_i = 0$, implying that the consumer will buy the first product that has a positive observed net value, $v_i - \rho_i > 0$, then it is clear what the optimal contract is. For the last product in this order, the firm sets the refund (or the price for keeping the product) equal to the opportunity cost, i.e., $\rho_2 = \eta$ and the inspection fee σ_2 such that it extracts ES_2, the efficient surplus from inspecting the second product. Turning to the first product that is inspected, the firm's strategy follows the same principle, but here ρ_1 is priced at the "opportunity cost of selling the first product", which is the sum of its salvage value and the profit that the firm foregoes if the consumer does not inspect the second product. Thus, the firm (realizing it can make a profit of ES_2 and is getting the salvage value of the first product if the consumer continues to inspect the second product) will set the refund price such that $\rho_1^* = ES_2 + \eta$ and an inspection fee σ_1^* that extracts all remaining surplus, with $\sigma_2^* \geq \sigma_1^* \geq k$.[20,21]

What is less clear is why it is optimal to set $\hat{v}_i^a - \rho_i = 0$. At one level, this seems obvious as the firm extracts all consumer surplus. However, this is not the efficient surplus as (i) the inspection fee for the second product causes an inefficiency as the first product may be kept, ending search, even though the second product has a higher (net) value, while (ii) the difference in refunds for the first and second product also creates an inefficiency as it may

[18] If consumers could buy blindly, then the firm could alternatively extract the surplus of $\mathbb{E}(v) - c$ from the second product and would find it optimal to set the retail price p_2 equal to that surplus if, and only if, that is larger than ES_2. Note that thereby it would extract the full efficient surplus from both products.

[19] From (1) it follows that $\partial \hat{v}_i^a / \partial \sigma_i = -1/[1 - F(\hat{v}_i^a)] \leq -1$.

[20] $\sigma_1^* = \mathbb{E}[\max(v - ES_2 - \eta, 0)] - s_A = \mathbb{E}[\max(v - \eta, ES_2)] - \mathbb{E}[\max(v - \eta, 0)] + k \geq k$.

[21] It is easy to generalize this optimal solution to selling one out of n products.

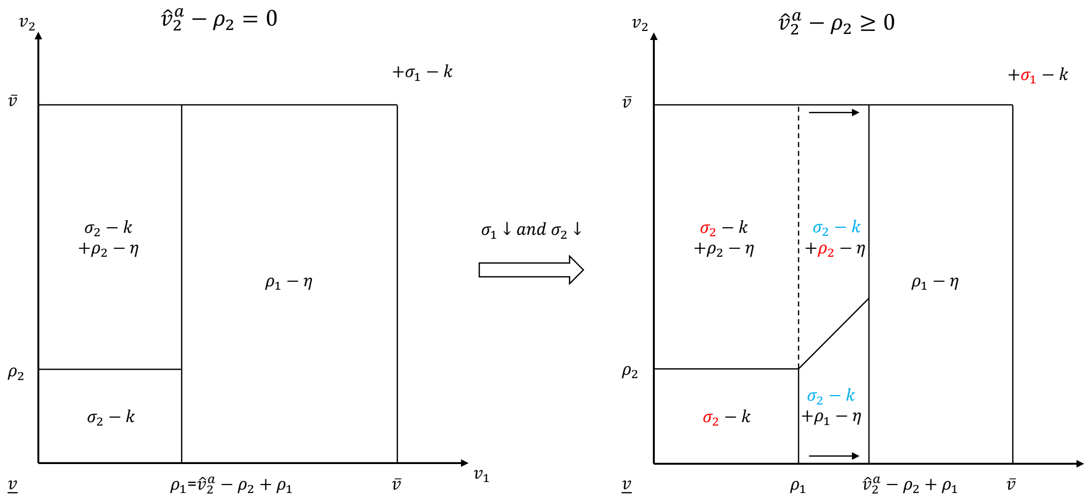


Figure 1: The left figure depicts the profits the firm makes in each of the three regions that the optimal contract induces in terms of consumer purchase behavior. The right figure depicts gains (in blue) and losses (in red) from a possible deviation.

well happen that the first product is returned, while the second product turns out to have a lower net value.[22]

The issue is illustrated by means of Figure 1. In the optimal solution, we have that the whole value area can be divided into three parts as in the left part of the figure: (i) if consumers have a value $v_1 > \rho_1$ they will buy product 1, (ii) if they have values $v_1 < \rho_1$ they will continue to search the second product and purchase that product if $v_2 > \rho_2$, and (iii) if they have a value $v_1 < \rho_1$ and $v_2 < \rho_2$, they will buy none of the products. In the right part of the figure, we indicate the different consumer behaviours for a possible deviation with $\hat{v}_i^a - \rho_i > 0$. Here, after inspecting the first product, consumers may decide not to buy the product immediately even if they discover that $v_1 > \rho_1$. Inspecting the second product delivers another inspection fee of σ_2 to the firm and consumers may still decide to buy product 1. The largest part of the proof in the appendix is dedicated to showing that deviations like this are not optimal and the firm indeed wants to set $\hat{v}_i^a - \rho_i = 0$ if $f(v)$ is logconcave.

We finalize the discussion of the optimal sequential contract after ordering with a numerical example and a few general remarks.

Example. The following example illustrates the nature of the optimal solution in Proposition 1 and shows why the optimal solution involves an asymmetric contract even if the products

[22] Note that even if the first product is returned only after the second is inspected, the consumer would still return the first product as it has a higher refund.

are ex ante symmetric. Suppose that $s_A = k = \eta = 0$ and that values are uniformly distributed over $[0, 1]$. If the firm would have one product to sell, it is clear that the optimal contract would have $\tau = \rho = 0$ and $p = \sigma = 1/2$. The firm sets the refund efficiently, namely equal to the salvage value, and then extracts all surplus by setting the retail price equal to the expected surplus of searching. This is also the optimal contract for the second product if the firm sells two products. Consider then the first product. The firm knows it can make a profit of $1/2$ and that the consumer gets an expected surplus of zero if the consumer continues to inspect the second product. It is then optimal to set the refund in the first period $\tau_1 = \rho_1 = 1/2$ as this is the opportunity cost of the refund: an even higher refund would make some consumers returning the product yielding a cost to the firm (the refund) that is larger than the expected profit the consume generates by inspecting the second product. Given the choice of the refund and a retail price p_1 in the first period consumers start searching if their expected surplus is nonnegative, which yields the following constraint: $-\sigma_1 + 1/2 * (3/4 - \rho_1) + 1/2 * 0 \geq 0$. It is optimal for the firm to set the largest retail price given this constraint, yielding $\sigma_1 = 1/8$. The total profit is thus equal to $5/8$ as the consumer pays the first inspection fee σ_1 of 1/8 and then pays the additional price τ_1 of 1/2 if the valuation is larger than 1/2 (which happens with probability 1/2) and if the valuation is smaller than 1/2 the consumer continues to search the second product, pays the inspection fee σ_2 of 1/2 and always keeps the product.[23]

Thus, the firm finds it optimal to make inspection costly by creating an inspection fee σ_i, or a partial refund. Consumers know that they commit to pay σ_i when they inspect a product. The example shows that even though the actual inspection cost equals 0, this optimal inspection fee can be quite large, especially for the second product.

It is also interesting to see that the resulting profit under sequential search equals $\mathbb{E}[\max(v - \eta, ES_2)] - s_A - k$, which is exactly identical to the efficient surplus if there was no recall. In addition, the firm makes this profit independent of whether the consumer eventually purchases product 1, 2 or no product at all, i.e., even if the consumer returns both products the firm makes the same profit as when it sells.

As in Petrikaitė (2018b), the profit maximizing strategy of the firm distorts the consumer's optimal search behavior in such a way as to remove their ability to recall any earlier inspected product. However, in our case it is further able to extract all resulting surplus by setting the inspection fees appropriately. The fact that the inspection fees are another source of revenue

[23] Note that the example continues to be valid if consumers would be allowed to buy blindly. Additionally, if we extend the example by allowing $k > 0$, then $ES_2 = 1/2 - k$ and the corresponding profit can be calculated to be equal to $5/8 - 3k/2 + k^2/2$. This expression is represented in Figure 2.

creates the technical complications alluded to above to show that indeed the firm wants to set $\hat{v}_i^a - \rho_i = 0$.

Simultaneous search. We now consider the optimal contract and profits when consumers search simultaneously after ordering so that the consumer pays the inspection fee σ_{sim} and the inspection cost s_A for both products upfront as long as their expected utility is non-negative.[24] Recall that consumers can buy at the terms of contract $(\sigma_{sim}, \rho_{sim})$ only if they choose to order both products simultaneously. The firm does not have to consider therefore a potential deviation of the consumer when choosing $(\sigma_{sim}, \rho_{sim})$ as it can in principle set very unattractive terms for the consumer to search sequentially to induce them to search simultaneously. When consumers search simultaneously, they will buy the product with the higher net value $v_i - \rho_{sim}$, as long as either of them is non-negative. So, the profit-maximizing contract is essentially a two-part tariff where the optimal price ρ^*_{sim} is set at marginal cost η and the optimal inspection fee σ^*_{sim} extracts all surplus. In particular, as the expected social surplus is given by

$$\mathbb{E}[\max(v_1 - \eta, v_2 - \eta, 0)] - 2(s_A + k) \tag{3}$$

the maximal profit $\pi^*_{sim} = 2(\sigma^*_{sim} - k)$ is equal to this expression.[25] From an efficiency standpoint, the number of inspections is too large, but products are returned at an efficient level: the product with the lowest valuation will always be returned and this is efficient as the consumer has no (additional) value for it, while the firm has a salvage value and the product with the highest valuation will be returned if its value is smaller than the firm's salvage value.

Example continued. Keeping the same parameter values, it is clear that under simultaneous search, the firm wants to set $\rho_{sim} = \eta = 0$. The firm then wants to set the retail price for the two products such that it extracts $\mathbb{E}[\max(v_1, v_2)] = 2/3$. Thus, it will set the inspection fees σ_i for each product equal to $1/3$.

[24] A different interpretation of simultaneous search is possible where consumers also pay σ_{sim} for both products upfront, but once they have both products "at home" they may inspect them sequentially. This is indeed optimal if some part of the inspection cost s_A comes from the effort of "testing the product at home" as sequential search is more efficient than simultaneous search (cf., Morgan and Manning (1985)). From an efficiency view, the maximum surplus is then realized if the firm sets $\rho_{sim} = \eta$, and the firm is able to extract all that surplus using σ_{sim}, which is larger than in (3). Qualitatively, Proposition 2 continues to hold, but the threshold below which $\pi^*_{sim} \geq \pi^*_{seq}$ would be somewhat "higher".

[25] Note that under blind buying the firm will be able to extract a maximum surplus of $\mathbb{E}(v) - c$ and therefore, the optimal contract we identified continues to be optimal if we allow for blind buying as long as the expression in (3) is larger than this. Note also that any contract with asymmetric inspection fees σ^i_{sim} satisfying $\sigma^1_{sim} + \sigma^2_{sim} = 2\sigma^*_{sim}$ qualitatively results in the same outcome.

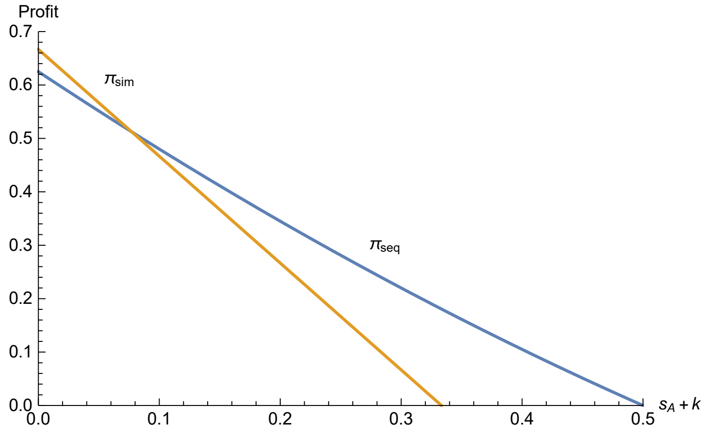


Figure 2: Profits π_{sim} and π_{seq} as functions of the sum of inspection and degradation costs $s_A + k$ for values distributed uniformly on $[0, 1]$ and $\eta = 0$.

Comparison. Finally, we are able to compare the profits and evaluate the impact of the candidate optimal contracts on the number of products returned. We find the following:

Proposition 2 *If s_B is large, then there exists a function $\underline{S}_A(\eta) > 0$ such that for all (s_A, k, η):*

$$s_A + k \leq \underline{S}_A(\eta) \Leftrightarrow \pi^*_{sim} \geq \pi^*_{seq}.$$

Moreover, the expected number of returns under simultaneous search is larger than the expected number of returns under sequential search.

The intuition behind the proposition is clear. Under both search protocols the firm extracts all surplus. However, the surplus is quite different. Under simultaneous search, the consumer inspects both products and chooses the one with the higher net value. The loss in surplus is due to inspection costs and product degradation related to the purchase and return of at least one product. Under sequential search, the consumer inspects the first product and keeps it if it has a higher net value than the expected value of the second inspection, including the inspection fee the firm imposes. Compared to simultaneous search, surplus is lower as it can happen that (i) the consumer decides not to inspect the second product even though it would have had a higher net value, or (ii) the consumer continues to inspect the second product, but does not keep the product with the highest value due to the difference in refunds. If the loss in surplus under simultaneous search due to unnecessary inspection costs and product degradation is relatively small, simultaneous search leads to higher profits. If, on the other hand, $s_A + k$ is relatively large, then inducing sequential search yields more profits as one can find a good fit already with the first product and save on inspection cost and product degradation. Figure 2 presents a numerical example.

What is interesting is that the inefficiencies that are outlined above are due to the multi-product nature of the firm and the associated search across different products. For a single product firm both solutions are identical and the outcome is efficient.

Thus, the firm induces consumers to search simultaneously if the sum of inspection and degradation costs $s_A + k$ is small and this leads to more product returns than when consumers search sequentially if $1 + F^2(\eta) > F(\rho_1^*)(1 + F(\eta))$. The proof of the proposition shows that this condition holds due to the logconcavity of $1 - F(v)$. Thus, a regulatory policy that forbids firms to induce consumers to order multiple products simultaneously would reduce the number of returns, and the associated environmental damage, if s_B is large enough.

Alternatively, a regulator could choose to impose that consumers get full refunds. In our framework this would imply that $\sigma_i = 0, i = 1, 2$. It is not difficult to see that in that case the firm's profits when setting a price p are equal to

$$(1 - F^2(p))(p - c - k) - 2F^2(p)k = (1 - F^2(p))(p - \eta) - 2k,$$

for the simultaneous search contract, and

$$(1 - F^2(p))(p - c - k) - 2F^2(p)k + F(\hat{v}_a)k = (1 - F^2(p))(p - \eta) - 2k + F(\hat{v}_a)k,$$

for the sequential contract, where $\hat{v}_a$ is defined as the usual reservation price relative to the search cost s_A (and $\sigma_i = 0$). Thus in both cases the firm optimally sets the price such that it maximizes joint monopoly profits given a cost η, and the profit in case of sequential search after ordering is higher as the firm may economize on the cost related to product degradation. Unless, the reservation value $\hat{v}_a < \rho_1^*$, it is clear that mandating full refunds leads to an increase in product returns as it implies higher refunds. Because of the absence of inspection fees, consumers enjoy higher surplus.

4 Small Inspection Costs[26]

When both inspection costs are relatively small, consumers could find it optimal to inspect a product before ordering while they search sequentially. For example, if the firm sets the same contract as in Proposition 1, consumers would deviate to inspecting before ordering. Therefore to induce consumers to search after ordering, the firm has to adjust its contract accordingly. Naturally, this implies reduced profits under sequential search compared to the

[26] Note that when inspection costs are small, blind buying is never optimal for the consumer. Thus, the results in this section continue to hold if we would allow the consumer to buy blindly.

previous section. However, that loss in profits is not the only implication of a relatively small s_B. Perhaps surprisingly, we will show that when s_B is relatively small, contracts inducing simultaneous inspection can be optimal for the firm and at the same time lead to a lower number of returns than those inducing sequential inspection. Thus, it may be that policies aiming at banning consumers from buying multiple items to try them at home before returning the ones they do not like are counterproductive. An important goal of this section is to identify conditions under which this is the case.

To characterize what type of search behavior the firm induces in the optimal contract, the following proposition provides a necessary and sufficient condition for consumers to inspect the first product before (or after) ordering. This result is important as the number of returns cannot be larger under sequential search than under simultaneous search if under sequential search the first product is inspected before ordering.

Proposition 3 *There exists an $\bar{s}_B > 0$ such that for all $s_A, \eta \geq 0$ with $s_A < s_B < \bar{s}_B$ there exists a threshold $\widetilde{k}$ such that under sequential inspection the optimal sequential contract $\{(\rho_i^*, \sigma_i^*)\}_{i=1,2}$ has consumers inspecting the first product after ordering if $k < \widetilde{k}$ and otherwise before ordering (if at all).*[27] *Moreover, $\widetilde{k} \geq s_B - s_A > 0$.*

Thus, for a given difference in inspection cost $s_B - s_A$ the product degradation cost k should not be too large for the firm to optimally induce consumers to inspect the first product after ordering. One way to understand this result is by noting that $s_A + k$ is the social cost of inspection after ordering, while s_B is the social cost of inspection before ordering. If $s_A + k < s_B$, social surplus is higher under inspection after ordering. Moreover, the firm is generically better able to extract surplus by setting inspection fees and refunds appropriately. If it induces consumers to inspect before ordering, it has fewer instruments (only the retail prices) to extract surplus.

Note that the condition $s_A + k < s_B$ is sufficient for inspection after ordering to be optimal, but not necessary. Even if $s_A + k > s_B$, the firm may benefit from offering a contract with a positive σ_1 and a lower ρ_1 than if it would induce inspection before ordering as a marginal change in ρ_1 would not affect profits significantly due to the envelope theorem (which effectively implies here that ρ_1 is chosen close to the joint monopoly price ρ^{JM}). Thus, the firm may induce consumers to inspect after ordering even if this is not socially optimal. In the rest of this section we first consider that the product degradation cost is small with $k < s_B - s_A$, so that consumers inspect the first product after ordering, and subsequently

[27] The phrase "if at all" refers to the possibility that k becomes so large that the firm sets prices such that consumers prefer not to search at all.

consider that $k > \widetilde{k}$ so that consumers inspect before ordering.

Small k: $k < s_B - s_A$. Our next result on consumer search behaviour shows that if consumers inspect the first product after ordering the inspection mode of the second product depends on the value of the first product the consumer learns upon inspection. In other words, the optimal contract is such that for some values of v_1 the consumer (weakly) prefers to inspect the second product before ordering and for others after ordering. If $v_1 \geq \rho_1$, searching the second product *after ordering* yields an additional benefit relative to v_1 of

$$\int_{v_1-\rho_1+\rho_2}^{\bar{v}} (1 - F(v_2))dv_2 - s_A - \sigma_2$$

while searching the second product *before ordering* yields an additional benefit relative to v_1 of

$$\int_{v_1-\rho_1+\rho_2+\sigma_2}^{\bar{v}} (1 - F(v_2))dv_2 - s_B.$$

When these expressions are equal, the consumer is indifferent between these two options and we will denote this value by $\widetilde{v}_1^a$, which is uniquely defined by:

$$\int_{\widetilde{v}_1^a-\rho_1+\rho_2}^{\widetilde{v}_1^a-\rho_1+\rho_2+\sigma_2} F(v_2)dv_2 = s_B - s_A. \tag{4}$$

Note that $\widetilde{v}_1^a$ is an implicit function of the firm's contract parameters and the inspection costs of the consumer. Thus, we state our next result as follows.

Lemma 1 *If $s_A + k < s_B$, then the optimal contract is such that the second product is inspected after ordering if $v_1 < \widetilde{v}_1^a$, before ordering if $\widetilde{v}_1^a < v_1 < \hat{v}^b - \sigma_2 - \rho_2 + \rho_1$ and not at all if $v_1 > \hat{v}^b - \sigma_2 - \rho_2 + \rho_1$, where $\rho_1^* \leq \widetilde{v}_1^a \leq \hat{v}^b - \sigma_2 - \rho_2 + \rho_1$.*

For any $v_1 \in (\rho_1^*, \widetilde{v}_1^a)$ the optimal contract induces consumers to inspect the second product after ordering it and, unlike the previous section, they may still buy the first product if the second product turns out to have a small value. The result can be intuitively understood as follows. Compared to inspecting before ordering, inspection after ordering comes at a lower inspection cost but implies paying part of the full price of the product upfront. The lemma shows that the firm will choose σ_2 such that $\sigma_2 + s_A > s_B$, i.e., the total upfront cost is larger when inspecting after ordering. This option is consequently better in situations where the outside option, i.e. the previously observed v_1, provides a low value, and it is therefore more likely that the second product will ultimately be bought. If instead the observed v_1 is already relatively large, then an improvement on it is unlikely, and the consumer will not be willing to pay upfront for inspecting the second product.

A consequence of this more complex search behavior and the fact that the firm's contract has four parameters it can choose is that it is difficult to explicitly solve for the optimal contract for general parameters. Instead, we identify the optimal contract for the limit case where $s_B = s_A = k = 0$ and utilize this contract to derive properties of the optimal contract that must hold in a neighborhood of these parameters.

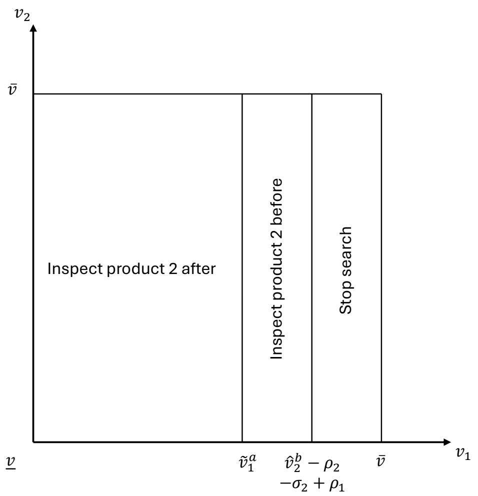
In the limit case $s_A = s_B = k = 0$ both inspection before and inspection after have the same inspection cost. Therefore the consumer is not willing to pay part of the price of the product upfront and thus it must be that in the optimal contract, where the first product is inspected after ordering, we have $\sigma_i = 0$ for both products. Log-concavity of the value distributions then implies that the optimal prices ρ_i^* will be symmetric (see Petrikaitė (2018b)). The firm thus maximizes $(1 - F^2(\rho))(\rho - \eta)$ and we denote the unique price that maximizes this expression as $\rho^{JM}(\eta)$, the joint monopoly price.

Example continued. If $s_A = s_B = k = 0$, setting a retail price $p = \rho$ for each of the products, the firm makes a profit of $(1 - p^2)p$. Maximizing this expression with respect to p yields the FOC $3p^2 = 1$, or $p = \sqrt{1/3} \approx 0.57$. Thus, the total profit of the firm is $\frac{2}{3}\sqrt{1/3} \approx 0.38$. Note that both the retail price and the profit are larger than for a single product monopolist (which are 0.5 and 0.25, respectively), but that the profit is considerably smaller than the profit of 0.625 under sequential search we derived in the previous section for large s_B.

Consider then a neighborhood of $s_A = s_B = k = 0$ where $s_A + k < s_B$. It is clear that for any $\eta < \bar{v}$ consumers are willing to start searching if s_B is small enough as the reservation value is larger than $\rho^{JM}(\eta)$. The following proposition then characterizes the optimal contract under sequential search.

Proposition 4 *In a neighborhood of $s_A = s_B = k = 0$ where $s_A + k < s_B$ the optimal sequential contract $\{(\rho_i^*, \sigma_i^*)\}_{i=1,2}$ is such that:*
(i) $\rho_i^* \approx \rho^{JM}(\eta)$ and $\sigma_i^* \geq k$ with $\lim_{k \to 0}\ \sigma_i^* = 0$
(ii) $F(\tilde{v}_1^a) \approx \frac{1+F^2(\rho^{JM})}{2} > \rho^{JM}$.

As $s_A + k < s_B$ it is socially optimal that consumers inspect the first product after ordering (see Proposition 3). As the firm is able to extract more of the surplus under this search protocol, the firm has an incentive to induce the consumer to do so. It can make a profit in case the product is returned by charging $\sigma_i^* \geq k$. However, as the inspection costs s_A, s_B are small, the firm can only charge an inspection fee that is small and therefore the optimal prices ρ_i will not be too different from $\rho^{JM}(\eta)$.



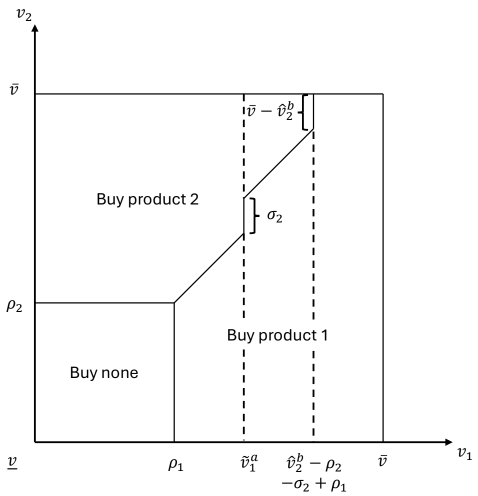


Figure 3: For $s_A + k < s_B \leq \bar{s}_B$ with $\bar{s}_B$ sufficiently small, the left figure shows consumers' search behavior for the second product given the observed value v_1, while the right figure shows the product the consumer eventually buys/keeps, if any.[28]

The expression in (ii) follows from (a) the fact that the firm wants to set the inspection fees so high that the consumer is just willing to inspect the first product after ordering, (b) the definition of $\widetilde{v}_1^a$ in (4) evaluated in a neighborhood of $\sigma_i \approx 0$, and (c) the fact that if consumers inspect the first product before ordering they pay (in addition to the inspection cost s_B and the price ρ in case of a purchase), the inspection fee $(1 + \frac{1}{2}\left(F^2(\widetilde{v}_1^a) - F^2(\rho^{JM})\right)$ times, while if they inspect the first product before ordering they pay (in addition to the inspection cost s_A and the price ρ in case of a purchase) the inspection fee $(1 + F(\widetilde{v}_1^a) + \frac{1}{2}\left(1 - F(\widetilde{v}_1^a)\right)^2)$ times. To understand these two expressions, Figure 3 visualizes the resulting search and purchase behavior given the optimal contract. The visible "jump" in the diagonal is due to consumers changing their inspection mode from inspecting after ordering to inspecting before, but note footnote 28. In case consumers inspect the first product before ordering, they always pay the inspection fee σ at least once, either because they inspect the second product after (if $v_1 < \widetilde{v}_1^a$) or because they buy at least one of the products (if $v_1 \geq \widetilde{v}_1^a$). In addition, they pay the inspection fee a second time if they inspect the second product after ordering, but nevertheless buy the first product. In case consumers inspect the first product after ordering, they also always pay the inspection fee σ at least once, but now they always pay the inspection

[28] Note that for clarity the proportions are exaggerated: For the considered parameters we have $\hat{v}_2^b \to \bar{v}$ and $\sigma_2 \to 0$ such that the consumer almost always inspects the second product and the jumps in the diagonal are small.

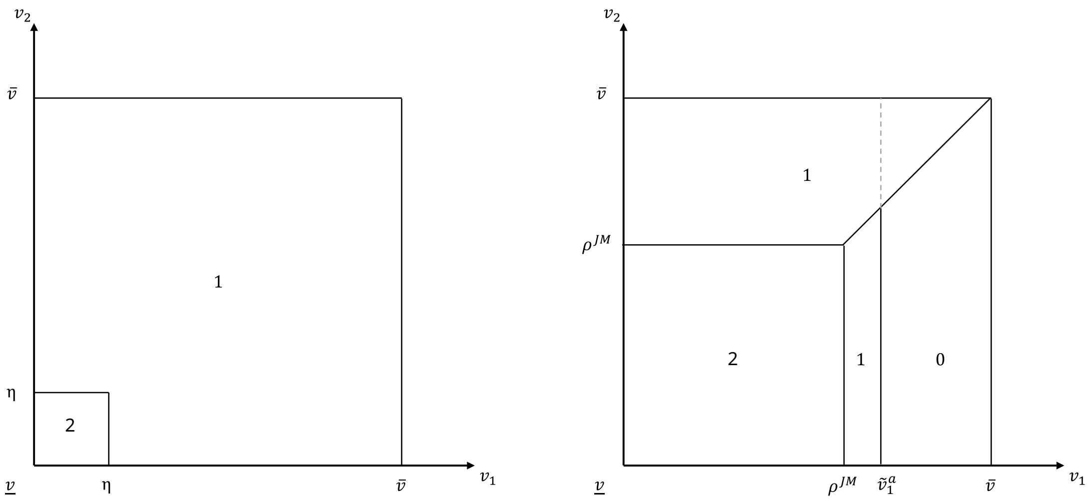


Figure 4: The number of returns under the simultaneous contract (left) and the sequential contract (right) for different realized values (v_1, v_2) for the uniform distribution and $\eta > \underline{v}$. The relatively bigger lower-left area where both products are returned under sequential search is responsible for the overall higher number of expected returns under the sequential contract.

fee a second time if they inspect the second product after ordering and also if they inspect the second product before ordering and buy the second product.

Comparing the number of products returned for small k. Given the optimal sequential contract, we now show that if the inspection costs are small, a pricing strategy that induces consumers to search products simultaneously, and return the products they do not want to keep can lead to fewer products being returned than pricing strategies that lead to consumers ordering and inspecting products sequentially. As we have shown in the previous section, in the optimal simultaneous contract, the firm sells one of the products at marginal cost and all its profits come from inspection fees. This is, in a sense, the exact opposite from the optimal sequential search contract as we have seen above. Thus, the expected number of returns under simultaneous search n_{sim} and sequential search n_{seq} are respectively given by

$$n_{sim} = 1 + F(\eta)^2 \quad \text{and} \quad n_{seq} \approx F(\tilde{v}_1^a) + F^2(\rho^{JM}) + \frac{1}{2}(1 - F(\tilde{v}_1^a))^2.$$

Figure 4 illustrates the number of returns under both pricing policies. The number of returns under simultaneous search (left Figure) is easily understood as both products are always inspected and one of them is returned with certainty. Both are returned only if both values are smaller than $\rho^*_{sim} = \eta$, the efficient return price the firm chooses. The number of returns under sequential search in the right figure is comprised of the following parts. The first two terms result from consumers inspecting the second product after ordering, which happens if $v_1 \leq \widetilde{v}_1^a$. In that case, they will certainly return one of the products, and they will return both products if both have a value $v_i < \rho_i \approx \rho^{JM}$. The third term results from consumers inspecting the second product before ordering, where they will return the first product in case the second has a higher net value, which happens in approximately half of the cases where both products have a value above $\widetilde{v}_1^a$.

Using these expressions for the expected number of returns and using point (iii) of Proposition 4, one can easily derive the condition under which the two contracts generate more returns.

Proposition 5 *If*[29]

$$F(\rho^{JM}(\eta)) > \sqrt{-5 + \sqrt{28 + 8F^2(\eta)}}, \tag{5}$$

then there exists an $\bar{s}_B > 0$ such that for all $s_A, k \geq 0$ with $s_A + k < s_B < \bar{s}_B$ the number of returns is larger under a contract inducing sequential inspection than under a contract inducing simultaneous inspection.

Condition (5) depends solely on the given distribution F and on the value of η. Inspecting the RHS, we see that it ranges from approximately 0.54 to 1 as η changes from $\underline{v}$ to $\bar{v}$. Thus, any distribution with a large enough joint monopoly value (at least larger than the 54th percentile of the value distribution) will fulfill the condition and results in contracts inducing sequential inspection leading to more returns than contracts inducing simultaneous inspection. It can be shown that for the uniform and the exponential distribution, the condition holds for any value of η, i.e., contracts inducing sequential inspection will always have more returns for small enough inspection and product degradation cost.

Figure 5 shows the difference in the expected number of returns for different values of η for these distributions. For the uniform distribution, we observe for values of η below the mean of the value distribution that sequential search leads to between around 5% and 13% more returns on average than simultaneous search. For the exponential distribution, that number lies between 18% and 32%. More generally, condition (5) is more likely to hold for distribution functions that are not particularly skewed to the left.

While Proposition 2 is stated for large values of s_B, the condition when inducing simultaneous search leads to higher profits than inducing sequential search is also sufficient for small s_B. The reason is that the profit from inducing sequential search will be strictly smaller for small s_B than what we derived in the previous section for large s_B, while the profit for

[29] Note that in the limit when $s_B, s_A \to 0$ this condition is both necessary and sufficient.

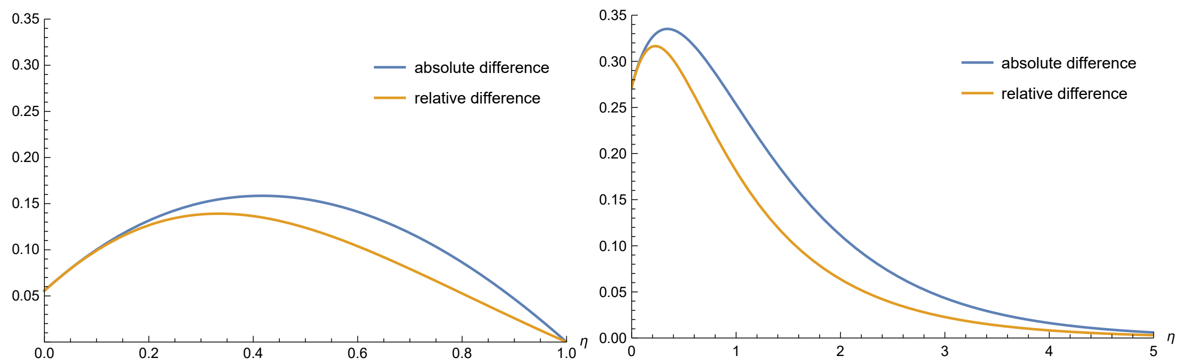


Figure 5: The difference in expected number of returns between sequential and simultaneous search for different values of the salvage value η for the uniform distribution $U[0, 1]$ (left figure) and the exponential distribution with $\lambda = 1$ (right figure).[30] The absolute difference is $n_{seq} - n_{sim}$ and the relative difference is $\frac{n_{seq} - n_{sim}}{n_{sim}}$.

inducing simultaneous search remains unchanged. Thus, Propositions 2 and 5 together imply the following:

Corollary 1 *There exists an* $\underline{S}_A(\eta) > 0$ *as defined in Proposition 2 and an* $\bar{s}_B > 0$*, such that for all* $s_A, s_B, k \geq 0$ *with* $s_B < \bar{s}_B$ *and* $s_A + k < \max[s_B, \underline{S}_A(\eta)]$ *and for all* (F, η) *that fulfill condition* (5)*, the profit maximizing strategy for the firm is to induce the consumer to inspect products simultaneously, leading to a fewer returns than when this policy would be banned.*

Thus, a regulation that bans firms from offering consumers to order many products simultaneously and returning those they do not want may lead to more rather than less returns. The intuition behind this result is the following. Under sequential search, the low inspection costs and fees make inspection of the second product attractive to the consumer, while the high refund price ρ_1 makes it unlikely that the consumer will consider the first product a good enough fit. Thus, there is a high chance that the second product will be inspected, in which case at least one product will be returned with certainty. Due to the similarly high refund ρ_2, it is however also likely that the consumer finds neither of the two products a good enough fit, implying that there is a much higher probability that both products would be returned than under simultaneous search as in that case the refund is set at the (much) lower η.

Large k: $k > \widetilde{k}$. When the product degradation costs are large, the firm does not want to induce consumers to search sequentially after ordering to avoid the product degradation cost.

[30] It can be shown that the figure for the uniform distribution is valid for arbitrary lower and upper bound values. The figure for the exponential distribution is valid for $\lambda = 1$, but it can be shown by means of a linear Taylor approximation of the profit function that it also approximates well the difference in returns for any other value of λ, if the x-axis is accordingly scaled by $\frac{1}{\lambda}$.

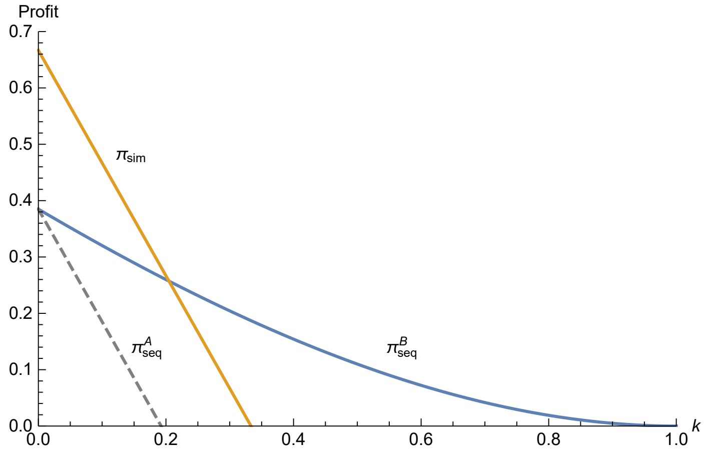


Figure 6: Profits π_{sim} and π^B_{seq} as functions of the product degradation cost k for values distributed uniformly on $[0, 1]$ and $s_A = s_B = \eta = 0$. For reference, the dashed line represents the firm's profit if the consumer would inspect the products sequentially after ordering.

In particular, if the inspection costs are the same, consumers are deterred from inspecting after ordering by any small inspection fee (or if there is some hassle cost of returning products).

Two observations are then immediate. First, as products are never returned when they are inspected before ordering, the number of product returns is strictly smaller under this traditional form of consumer search than under simultaneous inspection. Second, and related, if the production degradation costs are relatively large, the firm is better of inducing consumers to search sequentially than simultaneously. This is confirmed in Figure 6.

These observations may explain why now that new technologies emerge with which consumers can more easily inspect whether products fit their tastes, Amazon has canceled its 'Try Before You Buy' service for Prime members. Given that many products that are returned are not resold, k is relatively large and in such a world it is profitable for online retailers to invest in technologies that make consumers inspect products before they order them. This has the additional benefit of reducing environmental waste.

5 Discussion and Conclusion

The possibility retailers offer consumers of inspecting products after they have been ordered to see whether they are a good match introduces an interesting trade-off: due to the lower inspection costs at home, consumers will be able to find products that better match their preferences, but the associated product returns create an additional cost to firms. This paper adds an important element to the analysis of the desirability of refunds and product returns by considering how a multi-product retailer would optimally sell its products. Under what market conditions will retailers find it optimal to induce consumers to order many products simultaneously and return the products they do not want to keep? What prices and refunds will a retailer set? And how are welfare and the number of products returned affected by the optimal selling policy?

We reformulate the retailer's problem by interpreting the difference between the retail price and the refund offered for a returned item as an inspection fee. In case consumers decide to inspect products at home, they commit to pay this fee so as to be able to learn the product's match value at a lower inspection cost. Setting this inspection fee makes it clear that product returns can be a profit source, especially if it is larger than the cost of product degradation due to the return.

We have two sets of results, depending on the difference in inspection costs before and after ordering. If the inspection cost before ordering is large, then consumers will only consider inspecting products after ordering, even if the inspection fee the retailer charges is relatively large. In this case, the optimal selling policy inducing sequential inspection is to set asymmetric contracts and induce consumers to search the products in a certain order, with the first product to be inspected having a lower inspection fee and a higher refund. These contracts have the flavour of optimal obfuscation contracts as in Petrikaitė (2018b), but there are also important differences. In particular, the optimal contract in our setting has features of a two-part tariff with the inspection fee playing the role of a fixed fee that has to be paid independent of whether or not the product is purchased. This policy of inducing sequential inspection is optimal, if and only if, the social cost of inspecting after ordering is large, while the number of products returned is always higher under simultaneous search.

If instead the inspection cost before ordering is small, then consumers have a credible threat to search before ordering constraining the retailer in its choice of inspection fees if it wants to induce consumers to inspect sequentially. When the inspection costs are small, the retailer may still find it optimal to induce consumers to inspect after ordering if the product degradation cost is also small. For many distributions of consumers' match values, the number of products returned is, however, larger than under simultaneous inspection.

Accordingly, despite the appearance of creating unnecessary waste, inducing consumers to inspect many products simultaneously at home may actually lead to fewer (rather than more) products being returned. The implications for regulatory policies aiming to reduce the environmental impact of product returns and the externalities related to the number of returns are subtle. Our paper suggests that forcing retailers to abandon simultaneous inspection options will lead to more rather than less products being returned for products where the product degradation cost is relatively small and consumers can easily inspect the product match before ordering. In other markets where inspection before ordering is costly or the product degradation costs are large, the policy may have the desired effect.

References

Anderson, Simon P. and Régis Renault (1999). “Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model”. In: *The RAND Journal of Economics* 30.4, pp. 719–735.

Armstrong, Mark (2017). “Ordered Consumer Search”. In: *Journal of the European Economic Association* 15.5, pp. 989–1024.

Bagnoli, Mark and Ted Bergstrom (2005). “Log-Concave Probability and Its Applications”. In: *Economic Theory* 26.2, pp. 445–469.

Bird, Davina, Luke Garrod, and Chris M Wilson (2024). *Consumer protection versus competition: the case of mandatory refunds.*

Choi, Michael, Anovia Yifan Dai, and Kyungmin Kim (2018). “Consumer Search and Price Competition”. In: *Econometrica* 86.4, pp. 1257–1281.

Doval, Laura (May 2018). “Whether or Not to Open Pandora’s Box”. In: *Journal of Economic Theory* 175, pp. 127–158.

Gamp, Tobias (2022). *Guided Search.*

Janssen, Maarten and Cole Williams (2024). “Consumer Search and Product Returns in E-Commerce”. In: *American Economic Journal: Microeconomics* 16.2, pp. 387–419.

Jerath, Kinshuk and Qitian Ren (2025). “Consumer search and product returns”. In: *Marketing Science* 44.3, pp. 691–710.

Matthews, Steven A. and Nicola Persico (July 2007). *Information Acquisition and Refunds for Returns.* SSRN Scholarly Paper. Rochester, NY.

Morgan, Peter and Richard Manning (1985). “Optimal Search”. In: *Econometrica* 53.4, pp. 923–944.

Petrikaitė, Vaiva (May 2018a). “A Search Model of Costly Product Returns”. In: *International Journal of Industrial Organization* 58, pp. 236–251.

— (2018b). “Consumer Obfuscation by a Multiproduct Firm”. In: *The RAND Journal of Economics* 49.1, pp. 206–223.

Rhodes, Andrew (2015). “Multiproduct Retailing”. In: *The Review of Economic Studies* 82.1 (290), pp. 360–390.

Shelegia, Sandro (Mar. 2012). “Multiproduct Pricing in Oligopoly”. In: *International Journal of Industrial Organization* 30.2, pp. 231–242.

Tian, Xu and Joseph Sarkis (Jan. 2022). “Emission Burden Concerns for Online Shopping Returns”. In: *Nature Climate Change* 12.1, pp. 2–3.

Weitzman, Martin L. (1979). “Optimal Search for the Best Alternative”. In: *Econometrica* 47.3, pp. 641–654.

Wolinsky, Asher (1986). “True Monopolistic Competition as a Result of Imperfect Information”. In: *The Quarterly Journal of Economics* 101.3, pp. 493–511.

Zhou, Jidong (2014). “Multiproduct Search and the Joint Search Effect”. In: *The American Economic Review* 104.9, pp. 2918–2939.

A Appendix

A.1 Proof of Proposition 1

For clarity, we denote $\hat{v}_i^a$ as $\hat{v}_i$ in this proof. We further define $\overline{\rho} = \eta + \frac{1-F(\overline{\rho})}{f(\overline{\rho})}$, which due to the logconcavity of f is uniquely defined.

The proof is in several steps. First, note that as long as the consumer continues to inspect the first product first we can always increase σ_1 to increase profits. Thus, we should have $\hat{v}_1 - \rho_1 = \hat{v}_2 - \rho_2 \geq 0$. It is easy to show that *if* $\hat{v}_2 - \rho_2 = 0$, the optimal contract is as specified in the Proposition. If $\hat{v}_2 - \rho_2 = 0$, the firm's profit equals

$$\begin{aligned}
& \sigma_1 - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)(1 - F(\rho_2))(\sigma_2 + \rho_2 - c) + F(\rho_1)F(\rho_2)(\sigma_2 - k) \\
= \ & \sigma_1 - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)\left(\int_{\rho_2}(1 - F(v))dv - s_A + \rho_2 - c - F(\rho_2)(\rho_2 - \eta)\right).
\end{aligned}$$

where we used $\sigma_2 = \int_{\rho_2}(1-F(v))dv - s_A$, which is equivalent to $\hat{v}_2 - \rho_2 = 0$. The derivative wrt ρ_2 equals $-f(\rho_2)(\rho_2 - \eta)$. Thus, we should have $\rho_2 = \eta$ and it then follows from $\hat{v}_2 - \rho_2 = 0$ that $\sigma_2 = \int_\eta (1-F(v))dv - s_A$. Thus, the profit on the second product equals $\int_\eta(1-F(v))dv - s_A - k$ and overall profit is then equal to

$$\int_{\rho_1}(1-F(v))dv - s_A - k + (1-F(\rho_1))(\rho_1 - \eta) + F(\rho_1)\left(\int_\eta (1-F(v))dv - s_A - k\right).$$

The derivative wrt ρ_1 yields

$$-f(\rho_1)(\rho_1 - \eta) + f(\rho_1)\left(\int_\eta (1-F(v))dv - s_A - k\right),$$

which implies that the optimal ρ_1 is

$$\rho_1 = \eta + \int_\eta (1-F(v))dv - s_A - k.$$

Finally, $\hat{v}_1 - \rho_1 = 0$ implies that $\sigma_1 = \int_{\rho_1}(1-F(v))dv - s_A$. The rest of the proof shows that it cannot be the case that $\hat{v}_1 - \rho_1 = \hat{v}_2 - \rho_2 > 0$. This part of the proof is by contradiction. In particular, we show that if $\hat{v}_2 - \rho_2 > 0$ the firm can increase profits by increasing either (i) σ_2 and ρ_1 or (ii) ρ_2 and ρ_1 or (iii) σ_2 and σ_1 such that $\hat{v}_1 - \rho_1 = \hat{v}_2 - \rho_2 > 0$ continues to hold. By analyzing these joint increases in turn, we successively rule out different subcases that together imply that it cannot be that $\hat{v}_2 - \rho_2 > 0$.

First consider that we jointly increase σ_2 and ρ_1 such that $\hat{v}_1 - \rho_1 = \hat{v}_2 - \rho_2$. We can do that by changing them such that $(1 - F(\hat{v}_2))d\rho_1 = d\sigma_2$. The profit function is equal to

$$\begin{aligned}
&\sigma_1 - k + F(\hat{v}_2 + \rho_1 - \rho_2)(\sigma_2 - k) + \\
&\left[\int_{\rho_1}^{\hat{v}_2+\rho_1-\rho_2} F(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + 1 - F(\hat{v}_2 + \rho_1 - \rho_2)\right](\rho_1 - \eta) + \\
&\left[\int_{\rho_2}^{\hat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\hat{v}_2 + \rho_1 - \rho_2)\left[1 - F(\hat{v}_2)\right]\right](\rho_2 - \eta).
\end{aligned}$$

The increase in profits equals

$$\begin{aligned}
F(\hat{v}_1)\left(1 - F(\hat{v}_2)\right) + \quad &\int_{\rho_1}^{\hat{v}_1} F(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + 1 - F(\hat{v}_1) - \\
&\left[\int_{\rho_1}^{\hat{v}_1} f(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + F(\rho_2)f(\rho_1)\right](\rho_1 - \eta) + \\
&\left[\int_{\rho_2}^{\hat{v}_2} f(v_2 + \rho_1 - \rho_2)f(v_2)dv_2\right](\rho_2 - \eta),
\end{aligned}$$

which can be rewritten as

$$\begin{aligned} & \int_{\rho_1}^{\hat{v}_1} F(v_1-\rho_1+\rho_2)f(v_1)dv_1 + 1 - F(\hat{v}_1)F(\hat{v}_2) - F(\rho_2)f(\rho_1)(\rho_1-\eta) + \\ - & \left[\int_{\rho_1}^{\hat{v}_1} f(v_1-\rho_1+\rho_2)f(v_1)dv_1\right](\rho_1-\rho_2). \end{aligned}$$

This is equal to

$$\begin{aligned} & \int_{\rho_1}^{\hat{v}_1} \left[F(v_1-\rho_1+\rho_2)-F(\rho_2)\right] f(v_1)dv_1 + 1 - F(\hat{v}_2)F(\hat{v}_1) - F(\rho_2)(1-F(\hat{v}_1)) \\ & +F(\rho_2)\left[1-F(\rho_1)-f(\rho_1)(\rho_1-\eta)\right] - \int_{\rho_1}^{\hat{v}_1} f(v_2+\rho_1-\rho_2)(\rho_1-\rho_2)f(v_2)dv_2, \end{aligned}$$

which, as $1-F$ is logconcave and $1-F(\hat{v}_2)F(\hat{v}_1)-F(\rho_2)(1-F(\hat{v}_1)) = (1-F(\rho_2))(1-F(\hat{v}_1))+F(\hat{v}_1)(1-F(\hat{v}_2)) > 0$, is strictly larger than 0 if $\rho_1 \leq \min\{\rho_2, \overline{\rho}\}$. Thus, if $\hat{v}_2-\rho_2 > 0$ we should have $\rho_1 > \min\{\rho_2, \overline{\rho}\}$.

We next argue that if $\hat{v}_2-\rho_2 > 0$ we must have $\rho_2 > \eta$. If not, then a decrease in σ_2 and an increase in ρ_2 such that $\hat{v}_2-\rho_2$ is constant (so that $d\sigma_2 = -(1-F(\hat{v}_2))d\rho_2$) increases profits. Profits can be written as

$$\begin{aligned} & \sigma_1 - k + F(\rho_1)\left[\sigma_2 - k + (1-F(\rho_2)(\rho_2-\eta)\right] + \left(F(\rho_1+\hat{v}_2-\rho_2)-F(\rho_1)\right)(\sigma_2-k) + \\ & (\rho_1-\eta)\int_{\rho_1}^{\rho_1+\hat{v}_2-\rho_2} f(v)F(v+\rho_2-\rho_1)dv + (\rho_2-\eta)\int_{\rho_1}^{\rho_1+\hat{v}_2-\rho_2} f(v)(1-F(v+\rho_2-\rho_1))dv \\ & +(1-F(\rho_1+\hat{v}_2-\rho_2))(\rho_1-\eta) \end{aligned}$$

so that the increase in profits is equal to

$$\begin{aligned} & F(\rho_1)\left[-(1-F(\hat{v}_2)) + (1-F(\rho_2)) - f(\rho_2)(\rho_2-\eta)\right] - (F(\rho_1+\hat{v}_2-\rho_2)-F(\rho_1))(1-F(\hat{v}_2)) \\ & + \int_{\rho_1}^{\rho_1+\hat{v}_2-\rho_2} f(v)(1-F(v+\rho_2-\rho_1))dv + (\rho_1-\rho_2)\int_{\rho_1}^{\rho_1+\hat{v}_2-\rho_2} f(v)f(v+\rho_2-\rho_1)dv \\ = \; & F(\rho_1)\left[F(\hat{v}_2)-F(\rho_2)-f(\rho_2)(\rho_2-\eta)\right] \\ & + \int_{\rho_1}^{\rho_1+\hat{v}_2-\rho_2} f(v)(F(\hat{v}_2)-F(v+\rho_2-\rho_1))dv + (\rho_1-\rho_2)\int_{\rho_1}^{\rho_1+\hat{v}_2-\rho_2} f(v)f(v+\rho_2-\rho_1)dv \end{aligned}$$

which if $\hat{v}_2-\rho_2 \geq 0$ is clearly positive if $\rho_2-\eta \leq 0$ and $\rho_1 \geq \rho_2$. Thus, if $\hat{v}_2-\rho_2 \geq 0$ the optimal solution can only involve $\rho_2 \leq \eta$ and $\hat{v}_2-\rho_2 > 0$ if $\rho_1 < \rho_2 \leq \eta$. As $\rho_1 > \min\{\rho_2, \overline{\rho}\}$ this cannot be the case, however. Thus, if $\hat{v}_2-\rho_2 > 0$ we must have $\rho_2 > \eta$.

We finally argue that it cannot be that $\rho_2 > \eta$ by showing that an increase in σ_1 and σ_2 by keeping $\hat{v}_1-\rho_1 = \hat{v}_2-\rho_2 > 0$ increases profits. As $-(1-F(\hat{v}_i))\frac{\partial \hat{v}_i}{\partial \sigma_i} = 1$, this implies that

$\frac{(1-F(\hat{v}_1))}{(1-F(\hat{v}_2))} = \frac{d\sigma_1}{d\sigma_2}$. We write the firm's profit as

$$\sigma_1 - k + (1-F(\rho_1+\hat{v}_2-\rho_2))(\rho_1-\eta) + F(\rho_1+\hat{v}_2-\rho_2)(1-F(\hat{v}_2))(\sigma_2+\rho_2-c) +$$
$$F(\rho_2)F(\rho_1)(\sigma_2-k) + (\sigma_2+\rho_1-c)\int_{\rho_1}^{\rho_1+\hat{v}_2-\rho_2} f(v)F(v+\rho_2-\rho_1)dv +$$
$$(\sigma_2+\rho_2-c)\int_{\rho_2}^{\hat{v}_2} f(v)F(v-\rho_2+\rho_1)dv.$$

So that the increase in profit equals

$$\begin{aligned}
&\frac{(1-F(\hat{v}_1))}{(1-F(\hat{v}_2))} + \frac{f(\rho_1+\hat{v}_2-\rho_2)}{(1-F(\hat{v}_2))}(\rho_1-\eta) + F(\rho_1+\hat{v}_2-\rho_2)(1-F(\hat{v}_2)) + \\
&\frac{F(\rho_1+\hat{v}_2-\rho_2)f(\hat{v}_2) - f(\rho_1+\hat{v}_2-\rho_2)(1-F(\hat{v}_2))}{(1-F(\hat{v}_2))}(\sigma_2+\rho_2-c) + \\
&F(\rho_2)F(\rho_1) + \int_{\rho_1}^{\rho_1+\hat{v}_2-\rho_2} f(v)F(v+\rho_2-\rho_1)dv + \int_{\rho_2}^{\hat{v}_2} f(v)F(v-\rho_2+\rho_1)dv \\
&-(\sigma_2+\rho_1-c)\frac{f(\rho_1+\hat{v}_2-\rho_2)F(\hat{v}_2)}{(1-F(\hat{v}_2))} - (\sigma_2+\rho_2-c)\frac{F(\rho_1+\hat{v}_2-\rho_2)f(\hat{v}_2)}{(1-F(\hat{v}_2))}.
\end{aligned}$$

This can be rewritten as

$$\begin{aligned}
&\frac{(1-F(\hat{v}_1))}{(1-F(\hat{v}_2))} + \frac{f(\hat{v}_1)}{(1-F(\hat{v}_2))}(\rho_1-\eta) + F(\hat{v}_1) \\
&-(\rho_1-\rho_2)\frac{f(\hat{v}_1)F(\hat{v}_2)}{(1-F(\hat{v}_2))} - \frac{f(\hat{v}_1)}{(1-F(\hat{v}_2))}(\sigma_2+\rho_2-c) \\
= \; &\frac{(1-F(\hat{v}_1))}{(1-F(\hat{v}_2))} + \frac{f(\hat{v}_1)}{(1-F(\hat{v}_2))}(\rho_1-\eta) + F(\hat{v}_1) \\
&-(\sigma_2+\rho_1-c)\frac{f(\hat{v}_1)F(\hat{v}_2)}{(1-F(\hat{v}_2))} - \frac{f(\hat{v}_1)(1-F(\hat{v}_2)}{(1-F(\hat{v}_2))}(\sigma_2+\rho_2-c) \\
= \; &\frac{1-F(\hat{v}_1)}{(1-F(\hat{v}_2))} - (\sigma_2-k)\frac{f(\hat{v}_1)F(\hat{v}_2)}{(1-F(\hat{v}_2))} + F(\hat{v}_1) - f(\hat{v}_1)(\sigma_2-k-\rho_1+\rho_2) \\
= \; &\frac{1-F(\hat{v}_1)}{(1-F(\hat{v}_2))} - \frac{(\sigma_2-k)f(\hat{v}_1)}{(1-F(\hat{v}_2))} + F(\hat{v}_1) + f(\hat{v}_1)(\rho_1-\rho_2),
\end{aligned}$$

which because $\sigma_2 = \int_{\hat{v}_2}(1-F(v))dv - s_A$ is equal to

$$\frac{1-F(\hat{v}_1)}{(1-F(\hat{v}_2))} - \frac{(\int_{\hat{v}_2}(1-F(v))dv - s_A - k)f(\hat{v}_1)}{(1-F(\hat{v}_2))} + F(\hat{v}_1) + f(\hat{v}_1)(\hat{v}_1-\hat{v}_2).$$

This is positive if

$$\frac{1-F(\hat{v}_1)F(\hat{v}_2)}{f(\hat{v}_1)} - \left(\int_{\hat{v}_2}(1-F(v))dv - s_A - k\right) + (1-F(\hat{v}_2))(\hat{v}_1-\hat{v}_2) > 0. \tag{6}$$

That is certainly the case if $\hat{v}_2 = \overline{v}$. The derivative of this expression wrt $\hat{v}_2$ equals

$$-\frac{F(\hat{v}_1)f(\hat{v}_2)}{f(\hat{v}_1)} + (1 - F(\hat{v}_2)) - (1 - F(\hat{v}_2)) - f(\hat{v}_2))(\hat{v}_1 - \hat{v}_2).$$

This is clearly nonpositive if $-\frac{F(\hat{v}_1)f(\hat{v}_2)}{f(\hat{v}_1)} - f(\hat{v}_2))(\hat{v}_1 - \hat{v}_2) \leq 0$, which is the case if $\hat{v}_1 \geq \hat{v}_2 - \frac{F(\hat{v}_1)}{f(\hat{v}_1)}$. So, if we decrease $\hat{v}_2$ starting from $\hat{v}_2 = \overline{v}$, then (6) remains positive if $\hat{v}_1 \geq \hat{v}_2 - \frac{F(\hat{v}_1)}{f(\hat{v}_1)}$. So, the only possibility for an equilibrium with $\hat{v}_2 > \rho_2$ is that $\hat{v}_1 < \hat{v}_2 - \frac{F(\hat{v}_1)}{f(\hat{v}_1)}$.

To rule out that $\hat{v}_1 < \hat{v}_2 - \frac{F(\hat{v}_1)}{f(\hat{v}_1)}$ we finally consider that we increase σ_2 and decrease ρ_2 such that $\hat{v}_2 - \rho_2$ is constant. We can do that by changing them such that $-(1 - F(\hat{v}_2))d\rho_1 = d\sigma_2$. The profit function is equal to

$$\begin{aligned}
&\sigma_1 - k + F(\hat{v}_2 + \rho_1 - \rho_2)(\sigma_2 - k) + \\
&\left[\int_{\rho_1}^{\hat{v}_2+\rho_1-\rho_2} F(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + 1 - F(\hat{v}_2 + \rho_1 - \rho_2)\right](\rho_1 - \eta) + \\
&\left[\int_{\rho_2}^{\hat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\hat{v}_2 + \rho_1 - \rho_2)\left[1 - F(\hat{v}_2)\right]\right](\rho_2 - \eta).
\end{aligned}$$

The increase in profits equals

$$\begin{aligned}
&F(\hat{v}_1)\left(1 - F(\hat{v}_2)\right) - \left[\int_{\rho_2}^{\hat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\hat{v}_1)\left[1 - F(\hat{v}_2)\right]\right] - \\
&\left[\int_{\rho_1}^{\hat{v}_1} f(v_1 - \rho_1 + \rho_2)f(v_1)dv_1\right](\rho_1 - \eta) + \\
&\left[\int_{\rho_2}^{\hat{v}_2} f(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\rho_1)f(\rho_2) + F(\hat{v}_1)f(\hat{v}_2)\right](\rho_2 - \eta),
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
- &\int_{\rho_2}^{\hat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + \\
&\left[\int_{\rho_2}^{\hat{v}_2} f(v_2 + \rho_1 - \rho_2)f(v_2)dv_2\right](\rho_2 - \rho_1) + [F(\rho_1)f(\rho_2) + F(\hat{v}_1)f(\hat{v}_2)](\rho_2 - \eta) \\
\geq &\int_{\rho_2}^{\hat{v}_2} \left[\frac{f(v_2 + \rho_1 - \rho_2)}{F(v_2 + \rho_1 - \rho_2)}\frac{F(\hat{v}_1)}{f(\hat{v}_1)} - 1\right] F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + \\
&[F(\rho_1)f(\rho_2) + F(\hat{v}_1)f(\hat{v}_2)](\rho_2 - \eta).
\end{aligned}$$

As F is logconcave, f/F is decreasing and therefore as $\hat{v}_1 - \rho_1 = \hat{v}_2 - \rho_2$ and in the relevant area $v_2 < \hat{v}_2$ we have that $\frac{f(v_2+\rho_1-\rho_2)}{F(v_2+\rho_1-\rho_2)} > \frac{f(\hat{v}_1)}{F(\hat{v}_1)}$. Thus, the term in square brackets is strictly positive and the whole expression is strictly positive if $\rho_2 > \eta$. this would imply that the firm can increase profits if $\hat{v}_2 - \rho_2$, in contradiction to the fact that the firm chooses an optimal contract. $\square$

A.2 Proof of Proposition 2

The profits under the two search modes are

$$\pi^*_{sim} = \mathbb{E}[\max(v_1 - \eta, v_2 - \eta, 0)] - 2(s_A + k),$$

and

$$\pi^*_{seq} = \mathbb{E}[\max(v_1 - \eta, \mathbb{E}[\max(v_2 - \eta, 0)] - s_A - k)] - s_A - k$$

respectively, where in the second equation it is important to note that the second product is only inspected if inspection of the first product results in a low value. Thus, we have that $\pi^*_{sim} \geq \pi^*_{seq}$, if and only if,

$$\mathbb{E}[\max(v_1 - \eta, v_2 - \eta, 0)] \geq \mathbb{E}[\max(v_1 - \eta + s_A + k, \mathbb{E}[\max(v_2 - \eta, 0)])]$$

It is immediately evident that for $s_A + k = 0$ and for any value of η, simultaneous search leads to strictly higher profits. Thus, by continuity of the RHS in $s_A + k$, it follows that there exists a threshold $\underline{S}_A(\eta)$ such that simultaneous search yields larger profit if $s_A + k \leq \underline{S}_A(\eta)$. On the other hand, as the RHS of the above inequality is weakly increasing in $s_A + k$, and strictly increasing in $s_A + k$ if $s_A + k$ is large enough, it also follows that sequential search yields larger profit if $s_A + k > \underline{S}_A(\eta)$. This proves the first part of the proposition.

For the second part of the proposition we need to prove that for all $\eta < \overline{v}$ the expected number of returns under the simultaneous contract is larger than the expected number of returns under the sequential contract, which translates to

$$1 + F^2(\eta) > F(\rho_1^*)(1 + F(\eta)),$$

which is equivalent to showing that for all $\eta < \overline{v}$

$$\frac{1 - F(\rho_1^*(\eta))}{1 - F(\eta)} = \frac{S(\rho_1^*(\eta))}{S(\eta)} > \frac{F(\eta)}{1 + F(\eta)}.$$

Note that

$$\frac{\partial \ln \frac{S(\rho_1^*(\eta))}{S(\eta)}}{\partial \eta} = -\frac{f(\rho_1^*)}{S(\rho_1^*)} F(\eta) + \frac{f(\eta)}{S(\eta)} \leq \frac{f(\eta)}{S(\eta)}(1 - F(\eta)) = f(\eta),$$

where the inequality follows from the hazard rate $\frac{f(\rho_1^*)}{S(\rho_1^*)}$ being nondecreasing, due to logconcavity of $1-F$. It holds that $\ln \frac{S(\rho_1^*(\bar{v}))}{S(\bar{v})} = 0$, as with $\lim_{\eta\to\bar{v}} \rho_1^*(\eta) = \bar{v}$ and by applying l'Hôpital's rule

$$\lim_{\eta\to\bar{v}} \frac{S(\rho_1^*(\eta))}{S(\eta)} = \lim_{\eta\to\bar{v}} \frac{-f(\rho_1^*(\eta))F(\eta)}{-f(\eta)} = 1.$$

Then:

$$\begin{aligned} -\ln \frac{S(\rho_1^*(\eta))}{S(\eta)} &= \ln \frac{S(\rho_1^*(\bar{v}))}{S(\bar{v})} - \ln \frac{S(\rho_1^*(\eta))}{S(\eta)} \\ &= \int_\eta^{\bar{v}} \frac{\partial \ln \frac{S(\rho_1^*(v))}{S(v)}}{\partial v} dv \leq \int_\eta^{\bar{v}} f(v)dv = 1 - F(\eta), \end{aligned}$$

which is equivalent to

$$\frac{S(\rho_1^*(\eta))}{S(\eta)} \geq e^{-(1-F(\eta))}.$$

As $e^x \geq 1+x$, it follows that

$$\frac{S(\rho_1^*(\eta))}{S(\eta)} \geq F(\eta) > \frac{F(\eta)}{1+F(\eta)}$$

for all $\eta > \underline{v}$, while for $\eta \leq \underline{v}$, the original claim trivially holds as $F(\eta) = 0$. $\square$

A.3 Proof of Proposition 3

We first show that $\widetilde{k} \geq s_B - s_A$, or equivalently that the optimal sequential contract is such that the consumer inspects the first product after ordering for all k with $k < s_B - s_A$.

Assume that $k < s_B - s_A$ and suppose to the contrary that the consumer inspects the first product before ordering and the contract for the first product has (ρ_1, σ_1). If the firm would offer an alternative contract for the first product with

$$(\rho_1 + \sigma_1, s_B - s_A)$$

then the two contracts are identical from the consumers' perspective if they search the product afterwards using this alternative contract, i.e. searching before with the initial contract is identical to searching afterwards with the alternative contract. The consumer will never search before using the alternative contract as the conditions are worse. This also implies that the consumer does not want to switch the order of inspecting the two products, since for them the situation is effectively the same as with the initial contract. Now, importantly, the firm's expected profit from the second product is the same for both contracts, as neither its contract, nor the consumer's search behavior, nor the expected search result have changed. However, the firm's profits from the first product are strictly larger under the alternative contract, as it now receives additional profits from inspection of $s_B - s_A - k$, which is strictly positive due to our initial assumption. Because the firm strictly prefers the alternative contract and the consumer is indifferent, it must be the case that the first product is searched afterwards.

We now show that for k sufficiently large, the firm profits more from inducing the consumer to inspect the first product before ordering. In general, the firm can extract more of the efficient surplus from inspection after ordering than from before due to being able to raise an inspection fee in addition to the price. It is therefore possible that although inspecting the first product before ordering would lead to a larger efficient surplus, the firm is able to extract more profit by inducing the consumer to do so after ordering. However, a sufficient condition for the firm to make more profit from inspection before ordering is that the efficient surplus from inspecting one product after ordering, ES_A, is negative, while the efficient surplus from inspecting one product before ordering, ES_B, is positive.[31] The latter requires $\hat{v}^b > k + \eta$. For any $k > E[\max\{v - \eta, 0\}] - s_A$ it holds that $ES_A < 0$. We also have to confirm that at least at this threshold ES_B is indeed strictly positive. To see that, consider that $E[\max\{v-\eta, 0\}] - s_A = E[\max\{v, \eta\}] - \eta - s_A < \bar{v} - \eta$. As $s_B \to 0$ implies $\hat{v}^b \to \bar{v}$, there exists a sufficiently small s_B and a k for which it holds that $E[\max\{v, \eta\}] - \eta - s_A < k < \hat{v}^b - \eta$, where the first inequality guarantees $ES_A < 0$ and the second inequality guarantees that $ES_B > 0$ and thus profits are larger if the product is inspected before ordering compared to after.

We finally show that when inducing sequential search before ordering, an increase in k decreases profits more strongly than when inducing sequential search after ordering. The profit when inducing inspection after ordering is

$$\begin{aligned}
\pi_A^{seq} = {} & \sigma_1 - k + F(\widetilde{v}_1^a)(\sigma_2 - k) + F(\rho_1)(1 - F(\rho_2))(\rho_2 - \eta) + (1 - F(\rho_1))(\rho_1 - \eta) \\
& + (\rho_2 - \rho_1) \int_{\rho_1}^{\widetilde{v}_1^a} \int_{v_1 - \rho_1 + \rho_2} f(v_1) f(v_2) dv_2 dv_1 \\
& + (\sigma_2 + \rho_2 - \rho_1 - k) \int_{\widetilde{v}_1^a}^{\hat{v}^b + \rho_1 - \rho_2 - \sigma_2} \int_{v_1 - \rho_1 + \rho_2 + \sigma_2} f(v_1) f(v_2) dv_2 dv_1,
\end{aligned}$$

[31] ES_A is identical to ES_2 in section 3. $ES_A = E[\max\{v - \eta, 0\}] - s_A - k$ and $ES_B = E[\max\{v - \eta - k, 0\}] - s_B$.

while the profit when inducing inspection before ordering equals

$$\begin{aligned}\pi_B^{seq} &= F(\widetilde{v}_1^b)(\sigma_2 - k) + F(\rho_1+\sigma_1)(1-F(\rho_2))(\rho_2-\eta) + (1-F(\rho_1+\sigma_1))(\rho_1+\sigma_1-k-\eta)\\ &\quad + (\rho_2-\rho_1-\sigma_1+k)\int_{\rho_1+\sigma_1}^{\widetilde{v}_1^b}\int_{v_1-\rho_1-\sigma_1+\rho_2} f(v_1)f(v_2)dv_2dv_1\\ &\quad + (\sigma_2+\rho_2-\rho_1-\sigma_1)\int_{\widetilde{v}_1^b}^{\hat{v}^b+\rho_1+\sigma_1-\rho_2-\sigma_2}\int_{v_1-\rho_1-\sigma_1+\rho_2+\sigma_2} f(v_1)f(v_2)dv_2dv_1\end{aligned}$$

Thus, the respective partial derivatives with respect to k are:

$$\begin{aligned}\frac{\partial \pi_A^{seq}}{\partial k} &= -1 - F(\widetilde{v}_1^a) - \int_{\widetilde{v}_1^a}^{\hat{v}^b+\rho_1-\rho_2-\sigma_2}\int_{v_1-\rho_1+\rho_2+\sigma_2} f(v_1)f(v_2)dv_2dv_1\\ \frac{\partial \pi_B^{seq}}{\partial k} &= -1 - F(\widetilde{v}_1^b) + F(\rho_1+\sigma_1) + \int_{\rho_1+\sigma_1}^{\widetilde{v}_1^b}\int_{v_1-\rho_1-\sigma_1+\rho_2} f(v_1)f(v_2)dv_2dv_1\end{aligned}$$

As in a neighborhood of $s_B = 0$, σ_2 is very small and $\widetilde{v}_1^b = \widetilde{v}_1^a + \sigma_2 \approx \widetilde{v}_1^a$, it is clear that $\frac{\partial \pi_A^{seq}}{\partial k} < \frac{\partial \pi_B^{seq}}{\partial k} < 0$. Thus when s_B is smaller than some threshold, the profits under the optimal contract inducing inspection before ordering decrease less when k is increased than the optimal profits when inducing inspection after ordering.

To conclude: *(i)* inducing inspection after ordering leads to higher profits for small k, *(ii)* to lower profits for large k and *(iii)* profits when inducing inspection after ordering decrease more strongly with k than when inducing inspection before ordering in a neighborhood of $s_B = 0$. Thus, there must exist a threshold for k, below which inducing inspection after ordering leads to higher profits and above which inducing inspection before ordering leads to higher profits. Note that our argument applies to the optimal contracts for both inspection modes. It is thus guaranteed that the consumer inspects the products in the intended order. □

A.4 Proof of Lemma 1

We first show that it cannot be that $\widetilde{v}_1^a > \hat{v}^b - \sigma_2 - \rho_2 + \rho_1$. If that were the case, then if the consumer would prefer to inspect the second product, he will strictly prefer to do so after ordering independent of the value v_1 observed by inspecting the first product. In that case the firm could increase σ_2, however, to increase profits. The profit function when both products are inspected after ordering is:

$$\sigma_1 - k + F(\hat{v}_2^a - \rho_2 + \rho_1)(\sigma_2 - k) + F(\rho_1)(1 - F(\rho_2))(\rho_2 - \eta) + (1 - F(\rho_1))(\rho_1 - \eta)$$
$$+(\rho_2 - \rho_1)\int_{\rho_1}^{\hat{v}_2^a - \rho_2 + \rho_1}\int_{v_1 - \rho_1 + \rho_2}^{\overline{v}} f(v_1)f(v_2)dv_2dv_1.$$

Using $\frac{\partial \widehat{v}_2^a}{\partial \sigma_2} = -\frac{1}{1-F(\widehat{v}_2^a)} < 0$, the derivative with regard to σ_2 yields:

$$F(\hat{v}_2^a - \rho_2 + \rho_1) - \frac{f(\hat{v}_2^a - \rho_2 + \rho_1)(\sigma_2 - k)}{1 - F(\hat{v}_2^a)} - (\rho_2 - \rho_1)f(\hat{v}_2^a - \rho_2 + \rho_1)$$

From (4) it follows that for $\tilde{v}_1^a > \hat{v}^b - \rho_2 - \sigma_2 + \rho_1$ to hold, σ_2 has to be small. In particular, in a neighborhood of $s_B = s_A = k = 0$ it has to be that $\sigma_2 \approx 0$. Similarly, it has to be that $\sigma_1 \approx 0$, as otherwise the first product is not inspected after ordering but before. When both $\sigma_i \approx 0$, the optimal values for both ρ_i are close to ρ^{JM}, as in the limit case, which implies $\rho_1 \approx \rho_2$. Finally, as both $s_A \approx 0$ and $\sigma_2 \approx 0$, we have that $\widehat{v}_2^a \approx \overline{v}$. Taken together, we can conclude that the second and third term in the above derivative are approximately equal to 0. This is immediately clear for the third term, while for the second term it follows from applying l'Hôpital's rule:

$$\lim_{\sigma_2 \to 0} -\frac{f(\hat{v}_2^a - \rho_2 + \rho_1)(\sigma_2 - k)}{1 - F(\hat{v}_2^a)} = \lim_{\sigma_2 \to 0} -\frac{\frac{-f'(\hat{v}_2^a - \rho_2 + \rho_1)(\sigma_2 - k)}{1 - F(\hat{v}_2^a)} + f(\hat{v}_2^a - \rho_2 + \rho_1)}{\frac{-f(\hat{v}_2^a)}{1 - F(\hat{v}_2^a)}} =$$
$$\lim_{\sigma_2 \to 0} -\frac{-f'(\hat{v}_2^a - \rho_2 + \rho_1)(\sigma_2 - k) + f(\hat{v}_2^a - \rho_2 + \rho_1)[1 - F(\hat{v}_2^a)]}{-f(\hat{v}_2^a)} = 0$$

Thus if $\tilde{v}_1^a > \hat{v}^b - \rho_2 - \sigma_2 + \rho_1$, it is profitable for the firm to raise σ_2 (which decreases $\tilde{v}_1^a$). Thus, in an optimal contract it cannot be that $\tilde{v}_1^a > \hat{v}^b - \rho_2 - \sigma_2 + \rho_1$.

We next show that it cannot be that $\widetilde{v}_1^a < \rho_1^*$. If this were the case then, conditional on inspecting it at all, consumers would effectively always inspect the second product before ordering, regardless of v_1. The reason is that for any $v_1 < \rho_1^*$ the consumer would never come back and purchase the first product. So, if for some $\widetilde{v}_1^a < v_1 < \rho_1^*$ consumers prefer to inspect the second product before ordering, they will do so for all $v_1 < \rho_1^*$. The profit of the firm would then be equal to

$$\sigma_1 - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)(1 - F(\rho_2 + \sigma_2))(\rho_2 + \sigma_2 - k - \eta) +$$
$$(\rho_2 + \sigma_2 - \rho_1 - k)\int_{\rho_1}^{\hat{v}^b - (\rho_2 + \sigma_2 - \rho_1)}\int_{\rho_2 + \sigma_2 + v_1 - \rho_1}^{\overline{v}} f(v_1)f(v_2)dv_2dv_1.$$

Note that this expression depends on the sum $\rho_2 + \sigma_2$, but not on its individual components

ρ_2 and σ_2.

If the firm would instead choose its contract such that $\widetilde{v}_1^a = \rho_1^*$, it follows from (4) that $\sigma_2 > s_B - s_A > k > 0$ and consumers would be indifferent between inspecting the second product before or after for all $v_1 \leq \rho_1^*$. The firm would, however, strictly prefer consumers to inspect the second product after ordering for all $v_1 \leq \rho_1^*$ as its profits would increase by

$$F(\rho_1)F(\rho_2)(\sigma_2 - k) + F(\rho_1)(F(\rho_2 + \sigma_2) - F(\rho_2))(\rho_2 + \sigma_2 - k - \eta) > 0.$$

In particular, the firm would get the same profits whenever consumers buy if they inspect the second product before, but now *(i)* it would also get a profit of $\sigma_2 - k$ when they do not, and *(ii)* the consumer would more frequently buy the second product. As this is a discontinuous increase in profits, the firm would find it profitable to set a contract such that $\widetilde{v}_1^a \geq \rho_1^*$. As it follows from (4) that $\widetilde{v}_1^a$ is monotonically decreasing in σ_2 if one holds $\sigma_2 + \rho_2$ constant, it can do so by lowering σ_2 and increasing ρ_2 such that $\sigma_2 + \rho_2$ is constant, provided that the search order of the consumer does not change.

We conclude the proof by showing that such a profitable deviation where the search order of the consumer does not change, is indeed possible. Consider the expected utility of the consumer when inspecting product i after ordering, followed by optimally inspecting product j:

$$U_{ij}^A = \Delta U_i^A(0) + F(\rho_i)\Delta U_j(0) + \int_{\rho_i}^{\bar{v}} \Delta U_j(v_i - \rho_i) f(v_i) dv_i.$$

Here the first term is the expected utility from inspecting product i on its own with $\Delta U_i^A(z) = \int_{z+\rho_i}^{\bar{v}} (1 - F(v_i))dv_i - s_A - \sigma_i$ where z is the net value of the so far best already inspected product (0 at the start of search). The two remaining expressions give the expected additional utility from inspecting product j after inspecting product i. Note that $\Delta U_j(z)$ may imply inspection before or after ordering, depending on whether z is below or above the threshold implied by $\tilde{v}_i^a$. However, for the following argument it is only relevant that $\Delta U_j'(z) < 0$. Similarly, when the first product is inspected before ordering:

$$U_{ij}^B = \Delta U_i^B(0) + F(\rho_i + \sigma_i)\Delta U_j(0) + \int_{\rho_i + \sigma_i}^{\bar{v}} \Delta U_j(v_i - \rho_i - \sigma_i) f(v_i) dv_i$$

with $\Delta U_i^B(z) = \int_{z+\rho_i+\sigma_i}^{\bar{v}} (1 - F(v_i))dv_i - s_B$.

We continue to denote product 1 as the product the firm intends to be inspected first by the consumer, followed by product 2. The utility of the initial contract with the firm's intended search order is then U_{12}^A. That contract must be such that the consumer is at least indifferent between inspecting the products in this intended order and the reverse order, i.e.,

$U_{12}^A \geq \max\{U_{21}^A, U_{21}^B\}$. Note that if the firm changes its contract for product 2 as described above, $\rho_2 + \sigma_2$ stays constant, implying that $U_{12}^A \geq U_{21}^B$ continues to hold. Thus, it suffices to show that $U_{21}^A < U_{21}^B$ by changing the contract as described above.

To this end, consider the difference:

$$\begin{aligned} U_{21}^A - U_{21}^B &= \Delta U_2^A(0) - \Delta U_2^B(0) \\ &\quad - \left[[F(\rho_2+\sigma_2) - F(\rho_2)]\Delta U_1(0) - \int_{\rho_2}^{\rho_2+\sigma_2} \Delta U_1(v_2 - \rho_2) f(v_2) dv_2\right] \\ &\quad - \int_{\rho_2+\sigma_2}^{\bar{v}} [\Delta U_1(v_2 - \rho_2 - \sigma_2) - \Delta U_1(v_2 - \rho_2)] f(v_2) dv_2 \end{aligned}$$

As $\Delta U_1'(z) < 0$, it is clear that the second and third term are always strictly negative for $\sigma_2 > 0$. Thus, as the initial contract has $\tilde{v}_1^a < \rho_1^*$ for which the consumer strictly prefers to inspect product 2 before ordering, (which is equivalent to $\Delta U_2^A(0) < \Delta U_2^B(0)$) we have $U_{21}^A < U_{21}^B$. By changing the contract in the way described above such that $\tilde{v}_1^a = \rho_1^*$, we have $\Delta U_2^A(0) = \Delta U_2^B(0)$ so that $U_{21}^A < U_{21}^B$ continues to hold. Thus, the firm can profitably deviate from the original contract to one where $\tilde{v}_1^a \geq \rho_1^*$. $\square$

A.5 Proof of Proposition 4

(i) We have already shown that in the limit of $s_B = s_A = k = 0$ the optimal contract is $\sigma_i^* = k = 0$ and $\rho_i^* = \rho^{JM}(\eta)$. We now show that for $s_A + k < s_B \leq \bar{s}_B$ the optimal contract will be approximately the same. As Lemma 1 shows that (4) must hold in the optimal sequential contract, it is straightforward to see that as the RHS goes towards zero, on the LHS σ_2 has to go towards zero as well. We have also argued in Lemma 1 that it is always possible for the firm to set $\sigma_2 > k$ for positive $s_B - s_A$. Therefore as $s_B \to 0$ the optimal σ_2^* will be $\sigma_2^* \approx k \approx 0$. The firm's profit is equal to:

$$\begin{aligned} &\sigma_1 - k + F(\tilde{v}_1^a)(\sigma_2 - k) + F(\rho_1)(1 - F(\rho_2))(\rho_2 - \eta) + (1 - F(\rho_1))(\rho_1 - \eta) \\ &+ (\rho_2 - \rho_1) \int_{\rho_1}^{\tilde{v}_1^a} \int_{v_1-\rho_1+\rho_2} f(v_1) f(v_2) dv_2 dv_1 \\ &+ (\sigma_2 + \rho_2 - \rho_1 - k) \int_{\tilde{v}_1^a}^{\hat{v}^b+\rho_1-\rho_2-\sigma_2} \int_{v_1-\rho_1+\rho_2+\sigma_2} f(v_1) f(v_2) dv_2 dv_1 \end{aligned}$$

It is immediately clear that the optimal σ_1^* will be as large as possible. Raising σ_1 too much relative to σ_2 would prompt the consumer to start inspection with the second product instead of the first. Therefore σ_1^* must similarly be approximately equal to k. Given that the profit from inspection is approximately zero, the optimal prices ρ_i^* are determined as in the limit, and take on the same value ρ^{JM}.

(ii) It is possible to derive the relation between $\widetilde{v}_1^a$ and $\rho^{JM}(\eta)$ near the limit. The relation follows from the consideration regarding when the consumer prefers to inspect the first product after ordering to inspecting it before ordering. We are again focusing on the case where the first product is always inspected after ordering, however, as we have argued before, in the limit of $s_B, s_A \to 0$ the difference between the two inspection modes vanishes as $\sigma_i^* - k \to 0$. Given that $\sigma_i - k$ is so small, the comparison between the cases where the first product is inspected after and before ordering is fully determined by considering when the consumer pays σ_i in both modes. The prices ρ_i are paid in nearly the same instances (whenever a product is paid or kept), and the expected gain through the product values is also nearly the same. The main difference is then that under inspection after ordering, the consumer pays σ_1 upfront in a significant number of cases. Therefore, to determine when the consumer weakly prefers to inspect the first product after ordering, we consider when the expected expenditure in inspection costs is smaller under inspection after ordering.

If consumers inspect the first product before, then if $v_1 > \widetilde{v}_1^a$ they inspect the second product also before, but they always buy one of the products so they pay either σ_1 or σ_2 as part of the price of the product.[32] If $v_1 < \widetilde{v}_1^a$ they inspect the second product after ordering, so they always pay σ_2, but also pay σ_1 (in addition) as part of the price of the first product if $v_1 > \max\{v_2, \rho_1\}$. In part (i) we have shown that in the limit the firm's optimal contract is approximately symmetric, i.e. $\sigma_1 = \sigma_2 = \sigma$ and $\rho_1 = \rho_2 = \rho^{JM}$. Then consumers implicitly or explicitly pay approximately $\sigma(1 + \frac{1}{2}[F^2(\widetilde{v}_1^a) - F^2(\rho^{JM})])$ when starting search by inspecting the first product before ordering.

If consumers inspect the first product afterwards, then if $v_1 > \widetilde{v}_1^a$ they inspect the second product before ordering and buy the second product if $v_2 > v_1$. If $v_1 < \widetilde{v}_1^a$ they also inspect the second product after ordering, so they always pay $\sigma_1 + \sigma_2$. Then, with again $\sigma_1 = \sigma_2 = \sigma$, consumers implicitly or explicitly pay approximately $\sigma(1+F(\widetilde{v}_1^a)+\frac{1}{2}[1-F^2(\widetilde{v}_1^a)])$ when starting search by inspecting the first product after ordering.

To weakly prefer inspecting the first product after ordering it must hold that

$$s_B + \sigma(1 + \frac{1}{2}\left(F^2(\widetilde{v}_1^a) - F^2(\rho^{JM})\right) \geq s_A + \sigma(1 + F(\widetilde{v}_1^a) + \frac{1}{2}\left(1 - F(\widetilde{v}_1^a)\right)^2),$$

[32] Note that if the first product is inspected before, then the relevant threshold value is $\widetilde{v}_1^b$ which solves $\int_{\widetilde{v}_1^a-\rho_1-\sigma_1+\rho_2}^{\widetilde{v}_1^a-\rho_1-\sigma_1+\rho_2+\sigma_2} F(v_2)dv_2 = s_B - s_A$ and thus $\widetilde{v}_1^b = \widetilde{v}_1^a + \sigma_1$. But as in the limit σ_1 is approximately zero, it holds that $\widetilde{v}_1^a \approx \widetilde{v}_1^b$.

or

$$\sigma \leq \frac{2(s_B - s_A)}{1 + F^2(\rho^{JM})}.$$

Consider that for $s_B \to 0$ the consumer receives positive surplus from inspecting the products. Then for the firm's profit maximization when inducing inspection after ordering the only constraint is the above inequality. As we have argued before, the firm optimally raises the σ_i as much as possible, which then implies that in the optimal contract, the above weak inequality will hold with equality.

Finally, with the mean value theorem for integrals (4) becomes for $\sigma \to 0$ approximately $\sigma F(\widetilde{v}_1^a) = s_B - s_A$, which together with the above yields

$$F(\widetilde{v}_1^a) = \frac{1 + F^2(\rho^{JM})}{2},$$

concluding the proof. $\square$